POZNAN UNIVERSITY OF TECHNOLOGY ACADEMIC JOURNALS
No 69 Electrical Engineering 2012

Krzysztof BUDNIK*
Wojciech MACHCZYŃSKI*

MUTUAL INDUCTANCE OF FINITE LENGTH TWISTED-WIRE PAIR

Twisting of a bifilar lead is commonly used in various fields of electrical instruments and measurement systems in order to reduce the electromagnetic interference (EMI). Knowledge of inductances for helical conductors is needed for fundamental electromagnetic calculations, e.g. in electromagnetic compatibility studies. In the paper, a calculation method based on the Neumann's formula is applied for the mutual inductance calculation of a twisted-wire pair of finite length. The exemplary calculation is also presented.

1. INTRODUCTION

Twisting of conductors pairs is a well known method used in telephone communications to minimize crosstalk among lines in bundled cables. Generally, the effect of twisting can be utilized when the magnetic field is a problem. Already practiced is twisting of insulated high-voltage three phase power cables and single-phase distribution cables as well [1 – 7].

Fig. 1. Twisted-wire pair

Twisting of a bifilar lead as in Fig.1 is commonly used in various fields of electrical instruments and measurement systems in order to reduce the electromagnetic interference (EMI).

In the twisted-wire pair two individual insulated conductors are twisted together and each conductor can be represented by a helical line. A realistic model for the twisted-pair cable is a bifilar helix (double helix) that consists of two helices having the same radius and pitch; the helices are located 180 spatial degree from each other. Inductance calculations for helical conductors are needed for fundamental electromagnetic calculations.

* Poznan University of Technology.

In the paper, a calculation method based on the Neumann's formula is applied for the mutual inductance calculation of a twisted-wire pair of finite length. The exemplary calculation is also presented.

2. MUTUAL INDUCTANCE OF COAXIAL HELICAL THIN CONDUCTORS WITH FINITE LENGTH

Consider a general case when two different helical conductors are located coaxially in the Cartesian co-ordinate system (x,y,z), Fig.2.

The mutual inductance between the helical current lines, can be computed using the double integral Neumann's formula [7]:

$$M = \frac{\mu_0}{4\pi} \oint_{c_1} \oint_{c_2} \frac{\vec{dl_1} \cdot \vec{dl_2}}{r} \tag{1}$$

Here, as shown in Fig. 2, c_1 and c_2 are the contours of the filament structures, respectively, $\vec{dl_1}$ and $\vec{dl_2}$ are infinitesimally small integration elements, r is the distance between line elements dl_1 and dl_2; the symbol μ_0 denotes the magnetic permeability of the vacuum ($4\pi\times10^{-7}$ H/m).

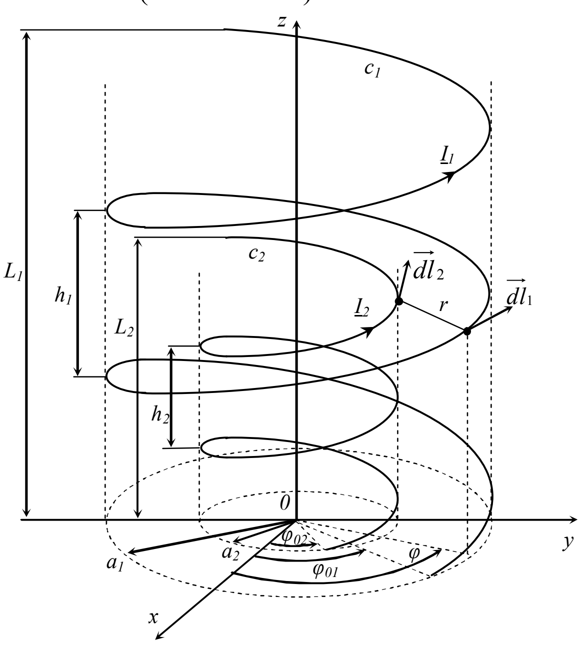

Fig. 2. Two coaxial helical current lines

Denoting the winding radius of the i-th (i=1, 2) helix by a_i, the pitch distance of the helix by h_i and the φ co-ordinate of the point where the helix intersects the plane $z=0$ by φ_{0i}, the parametric equations of the i-th helical line with respect to the parameter φ_i and with $\varphi_{0i}\neq 0$ are:

$$\begin{aligned} X_i(\varphi_i) &= a_i \cos\varphi_i \\ Y_i(\varphi_i) &= a_i \sin\varphi_i \\ Z_i(\varphi_i) &= \frac{h_i}{2\pi}(\varphi_i - \varphi_{0i}) \quad , \quad i=1,2 \end{aligned} \tag{2}$$

In order to apply the formula (1), we have to find suitable expressions $\overrightarrow{dl}_i(\varphi_i)$ and $r(\varphi_i)$. By looking at Fig.2, and taking into account the eqn.(2):

$$\overrightarrow{dl}_i(\varphi_i) = -a_i \sin\varphi_i d\varphi_i 1_x + a_i \cos\varphi_i d\varphi_i 1_y + \frac{h_i}{2\pi} d\varphi_i 1_z \ , \quad i=1,2 \tag{3}$$

where $1_x, 1_y, 1_z$ are rectangular unit vectors.

The scalar product when we are given the Cartesian components of the two vectors in (1) takes the form:

$$\overrightarrow{dl}_1 \cdot \overrightarrow{dl}_2 = dl_{1x}dl_{2x} + dl_{1y}dl_{2y} + dl_{1z}dl_{2z} \tag{4}$$

where

$$\begin{aligned} dl_{ix} &= -a_i \sin\varphi_i d\varphi_i \\ dl_{iy} &= a_i \cos\varphi_i d\varphi_i \\ dl_{iz} &= \frac{h_i}{2\pi} d\varphi_i \ , \qquad i=1,2 \end{aligned} \tag{5}$$

Remembering that

$$r = \left\{[X_1(\varphi_1) - X_2(\varphi_2)]^2 + [Y_1(\varphi_1) - Y_2(\varphi_2)]^2 + [Z_1(\varphi_1) - Z_2(\varphi_2)]^2 \right\}^{1/2} \tag{6}$$

it follows from (2) that:

$$r = \left\{ \begin{aligned} &(a_1\cos\varphi_1 - a_2\cos\varphi_2)^2 + (a_1\sin\varphi_1 - a_2\sin\varphi_2)^2 + \\ &+ [\frac{h_1(\varphi_1-\varphi_{01}) - h_2(\varphi_2-\varphi_{02})}{2\pi}]^2 \end{aligned} \right\}^{1/2} \tag{7}$$

Finally, the mutual inductance between two coaxial helical conductors with different finite lengths L_1 and L_2, different pitch distances h_1 and h_2 and different winding radii a_1 and a_2, takes the form:

$$M = \frac{\mu_0}{4\pi} \int_{\varphi_{01}}^{\frac{2\pi l_1}{h_1}+\varphi_{01}} \int_{\varphi_{02}}^{\frac{2\pi l_2}{h_2}+\varphi_{02}} \frac{[a_1 a_2 \cos(\varphi_1-\varphi_2) + \frac{h_1 h_2}{4\pi^2}]d\varphi_1 d\varphi_2}{\left\{ \begin{aligned} &a_1^2 + a_2^2 - 2a_1 a_2\cos(\varphi_1-\varphi_2) + \\ &+ [\frac{h_1(\varphi_1-\varphi_{01}) - h_2(\varphi_2-\varphi_{02})}{2\pi}]^2 \end{aligned} \right\}^{1/2}} \tag{8}$$

The integral formula (8) has to be solved numerically.

It should be noted that the formula derived allows calculations of the external inductance of single helical conductor with the finite radius as well.

3. EXAMPLE OF CALCULATIONS

Mutual inductance of twisted-pair cable

The calculations have been curried out for 3 m long twisted-pair cable with ¼ inch diameter and a 3-inch pitch length, formed by no.18 copper wires located 180 spatial degrees from each other as in [8]. The mutual inductance has been calculated numerically according to the formula (8) for following parameters: $a_1 = a_2 = 0.32$ cm, $h_1 = h_2 = 7.62$ cm, $L_1 = L_2 = 300$ cm, $\varphi_{01} = 0$, $\varphi_{02} = 180°$, giving: $M = 3.5$ μH.

The mutual inductance is numerically obtained basing on the adaptive Simpson quadrature provided by Matlab.

4. FINAL REMARKS

In the paper, a calculation method based on the Neumann's formula is applied for the mutual inductance calculation of a twisted-wire pair of finite length.

The formula derived allows calculations of the mutual inductance of a two-wire helix, as well as the external inductance of single helical conductor with finite radius, and can be used by a software tool when the magnetic field is a problem, e.g. in electromagnetic compatibility studies.

REFERENCES

[1] T. Tominaka, "Inductance calculation for helical conductors", *Superconductor Science and Technology*, 18 (2005), pp. 214 – 222.

[2] T. Tominaka, "Vector potential for a single helical current conductor", *Nuclear Instruments and Methods in Physics Research*, A 523 (2004), pp. 1 – 8.

[3] T. Tominaka, Y. Chiba, "Low frequency inductance for a twisted bifilar lead", *Journal of Physics*, D: *Appl. Phys.* 37 (2004), pp. 1592 – 1595.

[4] T. Tominaka, "Self- and mutual inductances of long coaxial helical conductors", *Superconductor Science and Technology*, 21 (2008), pp.1 –11.

[5] T. Tominaka, "Vector potential for a single helical current conductor", *Nuclear Instruments and Methods in Physics Research*, A 523 (2004), pp. 1 – 8.

[6] J. Franek, M. Kollar, "Determination of self and mutual inductances of double-helix coil", *Journal of Electrical Engineering,* vol. 60, no. 5, 2009, pp. 268 – 272.

[7] D.J. Griffiths: Introduction to Electrodynamics. Prentice Hall, Englewood Cliffs, N.J., 1989.

[8] J.R. Moser, R.F. Spencer, "Predicting the magnetic fields from a twisted-pair cable", *IEEE Trans. on Electromagnetic Compatibility*, vol. EMC-10, no. 3, September 1968, pp. 324 – 329.

IMPEDANCJA WZAJEMNA SKRĘTKI JEDNOPAROWEJ O SKOŃCZONEJ DŁUGOŚCI

Skrętka jednoparowa jest zwykle wykorzystywana w różnego rodzaju urządzeniach elektrycznych i elektronicznych systemach pomiarowych w celu redukcji niekorzystnych sprzężeń elektromagnetycznych. Impedancja wzajemna skrętki jednoparowej (układu dwóch wzajemnie skręconych przewodów o kształcie helisy) jest podstawową wielkością wykorzystywaną w obliczeniach np. z zakresu kompatybilności elektromagnetycznej. W pracy do wyznaczenia impedancji wzajemnej skrętki jednoparowej o skończonej długości wykorzystano wzór Neumanna. Zaprezentowano przykład obliczeniowy.