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Optimization of an EPQ model in an imprecise environment with defuzzification by the centroid method under inflation

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Abstract

The awareness of making decisions in an imprecise environment has resulted in considering the inventory system under a fuzzy approach. The effects of uncertain demand have been finding increased application in many inventory systems. Uncertainty creates complicated situations for the manufacturer in making decisions. Markets have become more competitive as a result of technological advancements. The effect of inflation on the overall cost of the inventory system is useful in providing a tool for the analysis of inventory decisions. This study intended to estimate the effect of different fuzzy numbers on a manufacturer's annual joint expected total cost. The comparative study of this proposed model has been considered for two different fuzzy numbers with the defuzzification technique as the centroid method. The optimization technique has been used to minimize the producer's joint expected total cost under the condition mentioned earlier, and the model is validated numerically.

Keywords: *fuzzy production inventory model, selling price dependent demand, inflation; triangular fuzzy number, trapezoidal fuzzy number; centroid method*

1. Introduction

In production planning and management, uncertainty poses significant challenges. Normally, the demand rate is assumed to be constant by researchers when developing economic order quantity models. However, in most cases, these amounts will deviate slightly from the exact value because of order changes, suppliers' erratic ability, uncontrollable weather, system breakdowns, and human errors. As a result, these variables can be considered fuzzy variables in real-world scenarios. Shekarian et al. [43]

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studied the comprehensive review of the fuzzy inventory models. In fuzzy inference, Chen et al. [9] studied an inventory model taking trapezoidal fuzzy numbers for uncertain cost parameters. Kazemi et al. [21] develop a new fuzzy EOQ inventory model with backorders that consider human learning over the planning horizon. De and Rawat [12] explained their model with vagueness in data. They applied fuzziness to control inventory-related issues. Generally, when a product's price raises, the demand for the goods falls and when the price of a commodity falls, the demand for products rises. Thus, the demand for products is expected to fluctuate in response to the selling price. Shah and Vaghel [41] developed a cost-effective production quantity model for declining products that includes both upstream and downstream trade credits. Concerning the sale price, the corresponding benefit function is maximized. Thus, a manufacturing company must have the best possible inventory levels. Inventory holding costs, such as room rent, decision-making costs, old unused stock costs, damages, and theft, can rise if there is too much on hand. Inadequate inventory costs a lot of money because customers may avoid supporting you and go to your competitors instead.

Tripathi and Kumar [48] present an optimal inventory strategy for declining goods based on an exponential demand rate with an appropriate payment delay. In two scenarios, a mathematical model was developed. In case I, the cycle time is greater than or equal to the allowable delay period; in case II, the cycle time is less than or equal to the allowable delay period. Customer satisfaction is the primary goal of successful inventory management and provides critical customer support while keeping inventory costs down daily. Hemapriya and Uthayakumar [18] investigate the possibility of lowering ordering costs and revenue lost due to stock-outs because of the uncertainty, the lead time and the rate of missed sales are decision variables. They even take into account that the lead time and the cost of crashing are proportional to the negative exponential lead time. Thus, ordering the right amounts of stock at the right time results in good customer service. In the production inventory system aim of the manufacturer is to calculate the optimal production cost with the lowest price of goods. The relationship between inventory management and inventory control was studied by Nemtajela and Mbohwa [28].

2. Literature review

The uncertain demand for fast-moving consumer goods was investigated using a survey questionnaire to gather primary data from five fast-moving consumer goods organizations in the Johannesburg manufacturing industry. Inventory optimization software aids businesses in making clear product orders. Making the right choices improves productivity by allowing for more accurate forecasts of future demand. Lower inventory costs are a sure advantage for a company that effectively manages its inventory. There are costs to consider when making inventory management decisions of any kind. Effective inventory management reduced these costs by reducing the overall amount of inventory needed to run the company. Inventory management monitors and records inventory amounts, and orders in sufficient quantities to prevent obsolescence and degradation. Inflation is an important factor influencing production models. The effectiveness of a marketing campaign is determined by the rate of inflation caused by public demand and the availability of materials. Considering inflation on various costs is providing more reliable results due to real-life problems. The rate at which the overall price level of goods and services increases is referred to as inflation. Buzacott [6] stated that as a result of the high rate of inflation, it is crucial to look at how various inventory strategies are affected by the time value of money. Inflation is defined as the average change in the price of a basket of goods and services over a given period. When the cost of goods and services rises, a currency unit's buying power erodes. Inflation is commonly associated with rapidly increasing prices, which reduce the purchasing power, which varies greatly depending on the period. In other words, inflation will help companies raise their profit margins by giving them more purchasing power. If profit margins are growing, it means that the prices businesses charge for their goods are rising higher than production costs.

The common inventory model is the economic order quantity model (EOQ). The simple model of EOQ was developed by Harris [17]. Generally, in the classical EOQ models, most of the parameters are considered deterministic type. However, this is quite different due to ambiguity or vagueness. Therefore, researchers are used to fuzzify the parameters for the inconclusive condition in EOQ models. Primarily, the work on fuzzy parameters is introduced by Zadeh [52]. After that, fuzziness in inventory models has subsequently been considered by several researchers. Lee and Yao [25] look at a programming schema for the EPQ. After that, Chen and Hsieh [8] proposed a new method of ranking the generalized L R form of fuzzy numbers and prove some ranking properties. De Kumar et al. [11] derived a practical EPQ inventory model with a finite production rate, a fuzzy demand rate, and a fuzzy deterioration rate. Their research considers the impact of a loss in output quantity due to a faulty/old device. Chen et al. [10] represented the graded mean integration method for a trapezoidal and triangular fuzzy number of generalized forms. They explained the method for the multiplication of fuzzy numbers. The right centroid formulae for fuzzy numbers were presented by Wang et al. [49]. For trapezoidal and triangular fuzzy numbers, they developed simpler expressions. By the function principle, a model of imperfect production in the fuzzy environment is discussed in the research study of Chen and Chang [7]. They explained their model with the defuzzification of total production cost by the representation of graded mean integration. Then the technique of Kuhn Tucker is discussed to evaluate the optimal economic production quantity. Kazemi et al. [20] considered fuzzy parameters in their inventory model. Planned back ordering is also taken with decision variables. Their model is solved for trapezoidal and triangular fuzzy numbers with Kuhn–Tucker's conditions. In the field of inventory control, Appadoo et al. [2] derived an economic order quantity model for uncertain carrying costs and ordering costs. They used adaptive trapezoidal fuzzy numbers for the uncertain cost parameters. Then, this quantity is defuzzified with the middle of the maxima method. Borzecka [5] discussed the fuzzy multi-criteria decision-making problem based on a linguistic approach of P-optimality and P-dominance. Dutta and Kumar [14] proposed a fuzzy inventory system without backorder. In which the relevant cost is assumed as a trapezoidal fuzzy number. Signed-distance technique is used to convert fuzzifying total cost as a single-valued function. Kosinski et al. [22] determine an economic order quantity (EOQ) with a fuzzy approach for a problem originating from administrative accounting under a variable competitive environment with imprecise and vague data.

Roy [38] focuses on using fuzzy set theory to manage inventory. A triangular fuzzy number is used to represent and define an unknown cycle time. The demand rate as a component of the sale price is also considered. A signed distance approach is used to defuzzify the best results. In terms of quantity and time, Agrawal et al. [1] developed a model for the prediction of product returns to the company for recycling. A graphical assessment and analysis technique was used for the development of the product return forecasting model. The commodity returns are stochastic, unpredictable, and uncertain for its recycling. To tackle the randomness, volatility, and stochastic existence of commodity returns, the project considered all potential flows of sold goods from the consumer to their reuse, storage, recycling or landfill, and managers should use this project for better forecasting to help them plan the reverse supply chain effectively. Rao [36] considered a deterministic inventory model of demand that is influenced by the selling price and allows for shortages, which is partially backlogged. Sahoo et al. [39] discussed a model of inventory with shortages and backlogs. The market price of the product determines demand. i.e. if the selling price rises, consumer demand falls, and vice versa. Garai et al. [15] have considered an EOQ inventory model with a price-dependent demand. Trapezoidal fuzzy numbers are used to establish a time-varying keeping expense in fuzzy settings. To find the estimation of the benefit function in the fuzzy sense, the estimated value method of defuzzification is used by Maheswari et al. [26] to present a new defuzzification centroid formula for octagonal fuzzy numbers. Yazdi et al. [19] developed a strategy for a multi-item production model under grey environment and space constraints using uncertain demand in fuzzy environments. By analysis of their model using crisp and fuzzy data, the authors compared the results. After that, a two-level partially trade-credit inventory model with inflation is introduced by Pramanik and Maiti [32]. Under some circumstances and strategic decisions and obtain maximum profit. They describe the suitable strategy for the business through their proposed model using the artificial bee colony method, a hybrid algorithm, a genetic algorithm, artificial bee genetic algorithm. Then the benchmark test function is applied to test the efficiency of the algorithm. In the research paper of Rajput et al. [34] for defuzzification of total cost, graded mean and singed distance methods are used. They compare the result in both a fuzzy sense and a crisp sense. The paper's perplexing factors are the cost of shortages, the cost of buying, the cost of delivery deteriorating cost and holding cost. In inventory modelling, the fuzzy set theory is the closest possible approach to reality, because reality is not precise and can only be measured to some degree. In the same way, Fuzzy theory seeks to integrate unpredictability into the model's architecture, thereby closing it to reality. Rajput et al. [33] compared using fuzzy and crisp parameters in three cases of demand for pharmaceutical inventory. Rani et al. [35] worked on a green supply chain and tried to give a spotlight on manufacturing with low carbon discharge and reduced wastage by developing a model of inventory for deteriorating items by taking out some parameters as triangular fuzzy numbers. They take recycling, remanufacturing and reverse logistics with carbon-dependent and imprecise parameters. For defuzzification of the total cost function, the signed distance method is applied by them.

Table 1. Basic overview of the literature review

Authors	Inventory system	Demand pattern	Effect of inflation	Fuzzified parameter	Defuzzification	Solving method
Shekarian et al. [44]	EOQ	fuzzy	not	all parameters and decision variables	graded mean integration	Karush–Kuhn–Tucker
Shekarian et al. [46]	SC	fuzzy	not	demand rate and collection rate	signed distance and graded mean integration	Hessian matrix /calculus optimization
Kazemi [20]	EOQ	fuzzy	not	all parameters	graded mean integration	Karush–Kuhn–Tucker
Kazemi [21]	EOQ	fuzzy	not	demand lead time	possibility/necessity measure; degree of fuzziness	Iterative algorithm
Shekarian et al. [42]	EPQ	fuzzy	not	all parameters	graded mean integration	Lagrangian function
Bera et al. [4]	EPQ	fuzzy	not	demand decreasing rate marketing cost	signed distance and centroid method	Lagrangian function
Pal. et al. [29]	EPQ	price discounted promotional demand	not	demand	centroid method	genetic algorithm
Yadav et al. [51]	EPQ	demand under screening error	not	demand	centroid method	calculus optimization
Glock et al[16]	EPQ	fuzzy	not	demand	centroid method	calculus optimization
Yadav et al. [52]	EOQ	with learning effect	not	all parameters	centroid method	calculus optimization
Pathak et al. [30]	EOQ	fuzzy	not	all parameters	possibility necessity measure	Calculus method Lingo software
De Kumar [13]	EOQ	randomised l-fuzzy	not	all parameters	fuzzy duality optimization	Lingo software
Basu et al. [3]	EOQ	exponentially declining market	yes	not	not	calculus optimization
Tripathi, Kumar [48]	EOQ	exponential demand	yes	not	not	Mathematica software
Shah, Veghila [41]	EPQ	price-dependent demand	not	not	not	classical optimization
Salas-Navarro et al. [40]	EPQ	probabilistic demand	not	not	not	calculus approach
Rout et al. [37]	EPQ	constant	not	deterioration rate	linear sum weightage method	Pareto optimization type-2 fuzzy approach
This paper	EPQ	selling price dependent	yes	demand and production rate	centroid method	Mathematica software

Salas Navarro et al. [40] developed an EPQ model with consideration of faulty products and probabilistic demand for a supply chain of two echelons. The method of manufacturing is imperfect, and products of poor quality are omitted from the lot size. The inventory system's demand rate is random with a probability density function with an exponential distribution. The findings show that collective strategy enables the supply chain to reap greater benefits and the demand on the market reflects the variable behaviour and uncertainty created by supply chain replenishment. In the non-collaborative approach, each supply chain member may be the leader and take decisions to maximize income and all members make collective decisions in the collaborative method regarding demand, supply, distribution, and inventory to maximise supply chain profits. The calculus method is used to determine the overall benefit associated with supply chain participants.

Teixeira et al. [47] look at the impact of service providers and business-to-business customers interacting frequently based on customer satisfaction, taking into account technical services of various degrees of complexity. Service sophistication affects the frequency of interactions. According to their research, buyer satisfaction is linked to low and medium levels of service complexity. The study of Kumar [23] deals with an EOQ model for deteriorating items with time-dependent exponential demand rate, penalty cost and partial backlogging. The backlogging rate of unsatisfied demand is a function of waiting time. The purpose of their study is that the total cost of the product can be reduced by maximizing the demand rate and minimizing the penalty cost. Since 1975, several articles have been published that look at the impact of inflation on inventory management. Before the 1990s, earlier attempts were thought to be straightforward cases. Buzacott [6] stated that the inventory carrying cost that is used in the EOQ formula is determined by the pricing strategy of the company. If rates fluctuate regardless of replenishment order timing, the inventory charge should be low and affected by inflation. Practical experience reveals that inflation is non-deterministic and unpredictable in many real-life circumstances. Mirzazadeh et al. [27] discussed that the rate of demand is linked to the rate of inflation over the time horizon, with varying probability density functions under stochastic inflationary conditions. Basu et al. [3] consider the EOQ model of multi-item demand under inflation and the demand rate is an exponentially decreasing function.

3. Discussion of the problem

Uncertain demand is always a worrying issue for a manufacturer. If the selling price of a commodity affects such volatile demand, the manager of a production system faces crucial challenges as a result. Thus, choosing an appropriate selling price is a challenging task for such type of inconclusive situation. Furthermore, if the sale price is random, determining the selling price becomes a difficult task. In many aspects, our research varies from the previous studies. First, rather than examining the seller's best option based on all available data, we take a scenario in which due to volatility in demand the relevant parameter is inconclusive. Under such an inconclusive situation we consider the demand parameter as a triangular and trapezoidal fuzzy number. Second, rather than concentrating exclusively on the manufacturer's decision which disregards the effect of inflation that represents asymmetric information. We developed a fuzzy production model for decision-makers to take strategic decisions that should be interactive. Finally, in terms of the outcome, to concentrate on the best choices for each process, we also lay out the conditions under which the decision-maker selects the most profitable mechanism. This section discusses how the demand curve can be used to identify the optimal price. A production inventory model for deteriorating goods is established in this research having time and price-sensitive demand under the influence of inflation over a finite planning horizon. Due to tough competition in the market manufacturer has to provide good quality items to the customers at the lowest reasonable rate. Shortages are permitted and partially backlogged. To minimize the optimal cost of the proposed model a computational algorithm has been applied numerical.

4. Model formulation

4.1. Preliminaries

Definition 3.1. Fuzzy point [33]. An $\tilde{F}$ fuzzy set is referred to be a fuzzy point on $R = (\infty, \infty)$ if the membership function is as follows:

$$\mu_{\tilde{F}}(\lambda) = \begin{cases} 1 & \text{if } \upsilon \in \tilde{F} \\ 0 & \text{if } \upsilon \notin \tilde{F} \end{cases}$$

Definition 3.2. Fuzzy set [26]. If X is a set of objects denoted generically by x, then a fuzzy set $\tilde{F}$ of ordered pairs in X is defined as $\tilde{F} = \{x, \mu_{\tilde{F}}(\lambda): x \in X\}$ where $\mu_{\tilde{F}}(\lambda)$ is called the membership function of A.

Definition 3.3. Fuzzy number [26]. Consider the fuzzy set $\tilde{F}$ on R of a real number. The membership grade of these functions has the form $\mu_{\tilde{F}}: R \to [0, 1]$. Then $\tilde{F}$ act as a fuzzy number if:

(i) $\tilde{F}$ must be a normal fuzzy set, i.e., $\mu_{\tilde{F}}(\lambda)=1$ for at least one $\lambda \in \tilde{F}$,

(ii) $\alpha_{\tilde{F}}$ must be closed for every $\alpha \in [0, 1]$, i.e., $\alpha \neq 0$,

(iii) the support of $\tilde{F}$ must be bounded, i.e., a fuzzy number must be 1,

(iv) $\tilde{F}$ is convex fuzzy set.

Definition 3.4. **Triangular fuzzy number [26].** Let a fuzzy set $\tilde{F} = (l, m, n)$, $l < m < n$, on the set of real number R. It is a triangular fuzzy number if the membership functions of $\tilde{F}$ is defined as:

$$\mu_{\tilde{F}}(\lambda) = \begin{cases} \dfrac{\lambda - l}{m - l}, & l \leq \lambda \leq m \\ \dfrac{m - \lambda}{m - n}, & m \leq \lambda \leq n \\ 0 & \text{otherwise} \end{cases}$$

Definition 3.5. **Trapezoidal fuzzy number [26].** Let a fuzzy set $\tilde{F} = (l, m, n, o)$, $l < m < n < o$, on a set of real number R. It is a trapezoidal fuzzy number if the membership functions of $\tilde{F}$ is defined as:

$$\mu_{\tilde{F}}(\lambda) = \begin{cases} \dfrac{\lambda - l}{m - l}, & l \leq \lambda \leq m \\ 1, & m \leq \lambda \leq n \\ \dfrac{o - \lambda}{o - n}, & n \leq \lambda \leq o \\ 0 & \text{otherwise} \end{cases}$$

Definition 3.6. Centroid method for defuzzification [26]. Let a trapezoidal fuzzy number $\tilde{F} = (l, m, n, o)$, $l < m < n < o$, on set of real number R. Then the formula for centroid is

$$\text{Centroid } \tilde{F} = \frac{\int_l^m \frac{\lambda - l}{m - l} \lambda d\lambda + \int_m^n \lambda d\lambda + \int_n^o \frac{o - \lambda}{o - n} \lambda d\lambda}{\int_l^m \frac{\lambda - l}{m - l} d\lambda + \int_m^n d\lambda + \int_n^o \frac{o - \lambda}{o - n} d\lambda}$$

The centroid formula is obtained by solving the above centroid equation for trapezoidal fuzzy number as

$$\text{Centroid } \tilde{F} = \frac{(n^2 + o^2 + no) - (l^2 + m^2 + lm)}{3\big((n + o) - (l + m)\big)}$$

Since the fuzzy numbers of triangle are special cases of fuzzy numbers of trapezoidal with $m = n$. The centroid of any piecewise linear membership function on a triangular fuzzy number can be calculated by using the formula

$$\text{Centroid } \tilde{F} = \frac{(l + m + n)}{3}$$

Definition 3.7. α-Cuts of fuzzy numbers [33]. For a triangular fuzzy number, the α cut of $\tilde{F} = (l, m, n)$ is

$$F(\alpha) = [F_L(\alpha), F_R(\alpha)] \text{ where } F_L(\alpha) = l + (m - l)\alpha, \; F_R(\alpha) = n - (n - m)\alpha \text{ and } \alpha \in [0, 1].$$

For a trapezoidal fuzzy number, the α cut of $\tilde{F} = (l, m, n, o)$ is

$$F(\alpha) = [F_L(\alpha), F_R(\alpha)], \text{ where } F_L(\alpha) = l + (m - l)\alpha, \; F_R(\alpha) = o - (o - n)\alpha \text{ and } \alpha \in [0, 1]$$

Definition 3.8. Fuzzy arithmetical operations [26]. Let $\tilde{\varphi} = (\varphi_1, \varphi_2, \varphi_3, \varphi_4)$ and $\tilde{\sigma} = (\sigma_1, \sigma_2, \sigma_3, \sigma_4)$ are trapezoidal fuzzy numbers, where $\varphi_1, \varphi_2, \varphi_3, \varphi_4, \sigma_1, \sigma_2, \sigma_3, \sigma_4$ are positive real numbers, then fuzzy arithmetical operations are defined as:

(i) $\tilde{\varphi} \oplus \tilde{\sigma} = (\varphi_1 + \sigma_1, \varphi_2 + \sigma_2, \varphi_3 + \sigma_3, \varphi_4 + \sigma_4)$

(*ii*) $\tilde{\varphi} \otimes \tilde{\sigma} = (\varphi_1\sigma_1, \varphi_2\sigma_2, \varphi_3\sigma_3, \varphi_4\sigma_4)$

(iii) $\tilde{\varphi} \varnothing \tilde{\sigma} = (\varphi_1 - \sigma_4, \varphi_2 - \sigma_3, \varphi_3 - \sigma_2, \varphi_4 - \sigma_1)$

(iv) $\tilde{\varphi} \varnothing \tilde{\sigma} = (\frac{\varphi_1}{\sigma_4}, \frac{\varphi_2}{\sigma_3}, \frac{\varphi_3}{\sigma_2}, \frac{\varphi_4}{\sigma_1})$

(v) $\alpha\tilde{\varphi} = (\alpha\varphi_1, \alpha\varphi_2, \alpha\varphi_3, \alpha\varphi_4)$, where α is any real number.

4.2. Notations and assumptions

Assumptions

The rate of deterioration is a time-dependent linear function, i.e., ζt.

The deteriorated items are not replaced.

Demand is selling price dependent, i.e., $\widetilde{D}(k) = \tilde{m}/k^n$.

The production rate is directly relative to the function of demand rate, i.e., $\tilde{P} = a\tilde{D}$.

The model is designed for a time span of finite length.

It is acceptable to have shortages, which are partially backlogged.

Inflation is taken into consideration of this model.

Notations

$S(t)$ –inventory at time t in interval $[0, T]$
P –maximum inventory at time t_1
ζ –deterioration parameter of the item $(0 < \zeta < 1)$
$\tilde{m}$ –fuzzy parameter of demand
n –constant parameter of demand
k –selling price per unit
a –production coefficient
ρ –set-up cost factor/production cycle
β –holding cost factor/unit/cycle
γ –deterioration cost factor/unit/cycle
δ –production cost factor/unit
Ψ –rate of backlogging
η –shortage cost/unit/cycle
λ –Lost sale cost/unit/cycle
T –cycle time (in months)
u –time when inventory is zero
t_1 –time for the production period
$\widetilde{TC}$ –fuzzy total cost

4.3. Model formulation

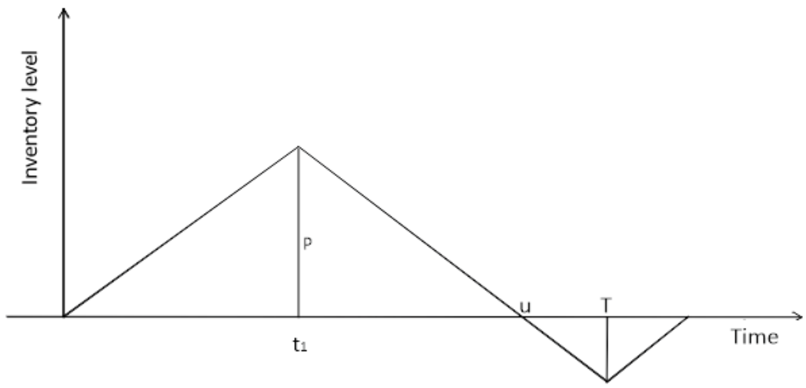

Figure 1. Representation of inventory of deteriorated objects with time T

Figure 1 illustrates the inventory model for production with perishable objects having allowable shortages and partially backlogged. Production started at $t = 0$ and continued until t_1. Throughout the time interval [t_1, u] the inventory rates are declining due to the degradation and demand of the object in the market. Then at u, the inventory becomes zero and shortages occur. Firstly, formulate the inventory at time t.

The produced items in [0, t_1] increase the inventory at a rate of $a\tilde{m}/k^n$ and decrease as a result of customer demand and deteriorating items. As a result, the inventory high point during this time period can be calculated by the differential equation

$$\frac{d\tilde{S}(t)}{dt} + \zeta t\tilde{S}(t) = (a-1)\frac{\tilde{m}}{k^n}, 0 \le t \le t_1 \tag{1}$$

The inventory reduces in the period [t_1, u] due to the demand function and deterioration. Therefore, the inventory is

$$\frac{d\tilde{S}(t)}{dt} + \zeta t\tilde{S}(t) = -\frac{\tilde{m}}{k^n},\ t_1 \le t \le u \tag{2}$$

In period [u, T], there will be no stock on hand for the manufacturer as a result, no deterioration has occurred. Thus, the shortage arises in this period. Then the differential equation is

$$\frac{d\tilde{S}(t)}{dt} = -\frac{\tilde{m}}{k^n},\ u \le t \le T \tag{3}$$

The initial conditions are

$$\tilde{S}(0) = 0; \tilde{S}(t_1) = P; \tilde{S}(u) = 0; \tilde{S}(T) = 0 \tag{4}$$

The solution of equation (1) with condition $S(0) = 0$ is

$$\tilde{S}(t) = (a-1)\frac{\tilde{m}}{k^n}\left(t + \frac{\zeta t^3}{6}\right)e^{-\frac{\zeta t^2}{2}}$$

By expanding $e^{-\frac{\zeta t^2}{2}}$ up to two terms for a small value of ζ, we get

$$\tilde{S}(t) = (a-1)\frac{\tilde{m}}{k^n}\left(t - \frac{\zeta \mathrm{t}^3}{3} - \frac{\zeta^2 \mathrm{t}^5}{12}\right) \tag{5}$$

Similarly, the solution of the equations (2) with the boundary condition $S(u) = 0$

$$\tilde{S}(t) = \frac{\tilde{m}}{k^n}\left((u-t) + \frac{\zeta}{6}(u^3 - t^3) - \frac{\zeta t^2}{2}(u-t) - \frac{\zeta^2 t^2}{12}(u^3 - t^3)\right) \tag{6}$$

In interval [t_1, u] the maximum production quantity can be calculated with boundary condition is $\tilde{S}(t_1) = P$ as

$$P = \frac{\tilde{m}}{k^n}\left[(u - t_1) + \frac{\zeta}{6}(u^3 - t_1^3) - \frac{\zeta t_1^2}{2}(u - t_1) - \frac{\zeta^2 t_1^2}{12}(u^3 - t_1^3)\right] \tag{7}$$

Equation (3) solution with condition $S(T) = 0$

$$S(t)=\frac{\tilde{m}}{k^n}(T-t) \tag{8}$$

The producer's inventory model includes the set-up cost, production cost, deteriorating cost and holding cost. These elements are formulated below:

$$\text{Production cost} = \text{production cost factor} \times \text{total produced units}$$

$$=\alpha\int_0^{t_1}\frac{a\tilde{m}}{k^n}\,\mathrm{e}^{-Rt}dt=\frac{a\alpha\tilde{m}}{k^n}\left(t_1-\frac{Rt_1^{\,2}}{2}\right) \tag{9}$$

$$\text{Set-up cost} = \beta \tag{10}$$

$$\begin{aligned}\text{Holding cost} &= \gamma\left(\int_0^{t_1}S(t)\mathrm{e}^{-Rt}dt+\int_{t_1}^{u}S(t)\mathrm{e}^{-R(t_1+t)}dt\right)\\
&=\gamma(a-1)\frac{\tilde{m}}{k^n}\left(\frac{t_1^{\,2}}{2}-\frac{Rt_1^{\,3}}{3}-\frac{\zeta t_1^{\,4}}{12}-\frac{\zeta Rt_1^{\,5}}{30}-\frac{\zeta^2Rt_1^{\,6}}{72}\right)\\
&\quad+\frac{\gamma\tilde{m}}{k^n}\left(\frac{u^2}{2}+\frac{\zeta u^4}{12}-R\left(\frac{t_1u^2}{2}+\frac{u^3}{6}\right)-\frac{\zeta R}{4}\left(\frac{u^4}{2}t_1+\frac{u^5}{5}\right)-\frac{\zeta^2u^6}{72}\right.\\
&\quad+\left(ut_1-\frac{t_1^2}{2}\right)-\frac{\zeta}{6}\left(\frac{u^3t_1}{3}+\frac{t_1^4}{2}\right)+\frac{3uRt_1^2}{2}-\frac{5Rt_1^3}{6}\\
&\quad\left.+\frac{\zeta Ru^3t_1^5}{4}-\frac{3\zeta Rt_1^5}{40}+\frac{\zeta ut_1^3}{6}+\frac{\zeta u^3t_1^3}{36}-\frac{\zeta^2t_1^6}{72}\right)\end{aligned} \tag{11}$$

Deterioration cost = total deteriorated units

$$=\text{total production} - \text{total demand} = \int_0^{t_1}\frac{a\tilde{m}}{k^n}\,dt-\int_0^{u}\frac{\tilde{m}}{k^n}\,dt=\frac{\tilde{m}}{k^n}(at_1-u)$$

Thus, total deterioration cost with inflation effect can be calculated as

$$\frac{\delta\tilde{m}}{k^n}\int_0^{u}\left(at_1-u\right)\mathrm{e}^{-Rt}dt=\frac{\delta\tilde{m}}{k^n}\left(at_1-u\right)\left(u-\frac{Ru^2}{2}\right) \tag{12}$$

$$\text{Shortage cost} = \eta\psi\int_u^{T}\frac{\tilde{m}}{k^n}\mathrm{e}^{-Rt}dt=\frac{\eta\psi\tilde{m}}{k^n}\left((T-u)+\frac{R}{2}(u^2-T^2)\right) \tag{13}$$

Lost sale cost

$$\text{Lost sale cost} = = \lambda(1-\psi)\int_u^{T}\frac{\tilde{m}}{k^n}\mathrm{e}^{-Rt}dt=\frac{\lambda(1-\psi)\tilde{m}}{k^n}\left((T-u)+\frac{R}{2}(u^2-T^2)\right) \tag{14}$$

Since, the total inventory cost is the sum of the production, set-up, deterioration, holding, shortage *t* and lost sale costs, the joint expected total cost is

$$\widetilde{\Pi}(t_1,u,T)=\frac{\tilde{m}}{Tk^n}\left(a\alpha\left(t_1-\frac{Rt_1^2}{2}\right)+\beta\frac{k^n}{\tilde{m}}+\gamma(a-1)\left(\frac{t_1^2}{2}-\frac{Rt_1^3}{3}-\frac{\zeta t_1^4}{12}-\frac{\zeta Rt_1^5}{30}-\frac{\zeta^2Rt_1^6}{72}\right)\right.$$
$$+\gamma\left(\frac{u^2}{2}+\frac{\zeta u^4}{12}-R\left(\frac{t_1u^2}{2}+\frac{u^3}{6}\right)-\frac{\zeta R}{4}\left(\frac{u^4}{2}t_1+\frac{u^5}{5}\right)-\frac{\zeta^2u^6}{72}+\left(ut_1-\frac{t_1^2}{2}\right)-\frac{\zeta}{6}\left(\frac{u^3t_1}{3}+\frac{t_1^4}{2}\right)\right.$$
$$+\frac{3uRt_1^2}{2}-\frac{5Rt_1^3}{6}+\frac{\zeta Ru^3t_1^5}{4}-\frac{3\zeta Rt_1^5}{40}+\frac{\zeta ut_1^3}{6}+\frac{\zeta u^3t_1^3}{36}-\frac{\zeta^2t_1^6}{72}\Big\}+\delta(at_1-u)(u-\frac{Ru^2}{2}) \tag{15}$$
$$\left.+\left(\eta\psi+\lambda(1-\psi)\right)\left((T-u)+\frac{R}{2}(u^2-T^2)\right)\right)$$

After completing the fuzzification process the total cost function should have to convert into a single crisp value. This process is called the defuzzification method. To defuzzify the result we have used the centroid method for two different fuzzy numbers as defined by Wang et al. [35] and Maheswari et al. in [21]. By using the centroid formula for trapezoidal fuzzy number, the joint expected total cost is

$$\widetilde{\Pi}(t_1,u,T)=\frac{m_3^2+m_4^2+m_3m_4-m_1^2-m_2^2-m_1m_2}{3T(m_3+m_4-m_1-m_2)\,k^n}\left(a\alpha\left(t_1-\frac{Rt_1^2}{2}\right)\right.$$
$$+\frac{3\beta\,(m_3+m_4-m_1-m_2)k^n}{m_3^2+m_4^2+m_3m_4-m_1^2-m_2^2-m_1m_2}+\gamma(a-1)\left(\frac{t_1^2}{2}-\frac{Rt_1^3}{3}-\frac{\zeta t_1^4}{12}-\frac{\zeta Rt_1^5}{30}-\frac{\zeta^2Rt_1^6}{72}\right)$$
$$+\gamma\left(\frac{u^2}{2}+\frac{\zeta u^4}{12}-R\left(\frac{t_1u^2}{2}+\frac{u^3}{6}\right)-\frac{\zeta R}{4}\left(\frac{u^4}{2}t_1+\frac{u^5}{5}\right)-\frac{\zeta^2u^6}{72}+\left(ut_1-\frac{t_1^2}{2}\right)-\frac{\zeta}{6}\left(\frac{u^3t_1}{3}+\frac{t_1^4}{2}\right)\right. \tag{16}$$
$$\left.+\frac{3uRt_1^2}{2}-\frac{5Rt_1^3}{6}+\frac{\zeta Ru^3t_1^5}{4}-\frac{3\zeta Rt_1^5}{40}+\frac{\zeta ut_1^3}{6}+\frac{\zeta u^3t_1^3}{36}-\frac{\zeta^2t_1^6}{72}\right\}+\delta\left(at_1-u\right)\left(u-\frac{Ru^2}{2}\right)$$
$$\left.+\left(\eta\psi+\lambda(1-\psi)\right)\left((T-u)+\frac{R}{2}(u^2-T^2)\right)\right)$$

By using the centroid formula for triangular fuzzy number, the joint expected total cost as represent in equation (17)

$$\widetilde{\Pi}(\mathrm{t}_1,u,T)=\frac{m_1+m_2+m_3}{3Tk^n}[a\alpha\left(t_1-\frac{Rt_1^2}{2}\right)+\frac{3\beta k^n}{m_1+m_2+m_3}$$
$$+\gamma(a-1)\left(\frac{t_1^2}{2}-\frac{Rt_1^3}{3}-\frac{\zeta t_1^4}{12}-\frac{\zeta Rt_1^5}{30}-\frac{\zeta^2Rt_1^6}{72}\right)+\gamma\left(\frac{u^2}{2}+\frac{\zeta u^4}{12}-R\left(\frac{t_1u^2}{2}+\frac{u^3}{6}\right)\right.$$
$$-\frac{\zeta R}{4}\left(\frac{u^4}{2}t_1+\frac{u^5}{5}\right)-\frac{\zeta^2u^6}{72}+\left(ut_1-\frac{t_1^2}{2}\right)-\frac{\zeta}{6}\left(\frac{u^3t_1}{3}+\frac{t_1^4}{2}\right)+\frac{3uRt_1^2}{2} \tag{17}$$
$$\left.-\frac{5Rt_1^3}{6}+\frac{\zeta Ru^3t_1^5}{4}-\frac{3\zeta Rt_1^5}{40}+\frac{\zeta ut_1^3}{6}+\frac{\zeta u^3t_1^3}{36}-\frac{\zeta^2t_1^6}{72}\right)+\delta(at_1-u)\left(u-\frac{Ru^2}{2}\right)$$
$$\left.+\left(\eta\psi+\lambda(1-\psi)\right)\left((T-u)+\frac{R}{2}(u^2-T^2)\right)\right\}$$

5. Computational algorithm

To optimize the total cost function, we have to consider the Hessian matrix. If the Hessian matrix is positive definite then the total cost function is the minimum function. Here the total cost function has four decision variables t_1, t_2, t_3, and T. Hessian matrix is formed. For minimizing the fuzzy cost $\widetilde{\Pi}(\mathrm{t}_1, u, T)$ we have used the Hessian matrix and applied the following computational algorithm:

Step 1. Start with differentiating $\widetilde{\widetilde{\Pi}}(\mathrm{t}_1, u, T)$ concerning t_1, u, and T, respectively, equate it to zero, i.e.,

$$\frac{\partial\widetilde{\Pi}(t_1, u, T)}{\partial t_1} = 0, \quad \frac{\partial\widetilde{\Pi}(t_1, u, T)}{\partial u} = 0, \quad \frac{\partial\widetilde{\Pi}(t_1, u, T)}{\partial T} = 0$$

Step 2. By solving the above equations, we can calculate the values of critical points t_1, u and T.

Step 3. Take the second derivative $\widetilde{\Pi}(t_1, u, T)$ concerning t_1, u and T, respectively.

Step 4. Use the values of critical points t_1, t_2, t_3, and T in the Hessian matrix

$$\Delta = \begin{bmatrix} \dfrac{\partial^2\widetilde{\Pi}(t_1, u, T)}{\partial {t_1}^2} & \dfrac{\partial^2\widetilde{\Pi}(t_1, u, T)}{\partial t_1 \partial u} & \dfrac{\partial^2\widetilde{\Pi}(t_1, u, T)}{\partial t_1 \partial T} \\ \dfrac{\partial^2\widetilde{\Pi}(t_1, u, T)}{\partial u \partial t_1} & \dfrac{\partial^2\widetilde{\Pi}(t_1, u, T)}{\partial u^2} & \dfrac{\partial^2\widetilde{\Pi}(t_1, u, T)}{\partial u \partial T} \\ \dfrac{\partial^2\widetilde{\Pi}(t_1, u, T)}{\partial T \partial t_1} & \dfrac{\partial^2\widetilde{\Pi}(t_1, u, T)}{\partial T \partial u} & \dfrac{\partial^2\widetilde{\Pi}(t_1, u, T)}{\partial T^2} \end{bmatrix}$$

Step 5. If all possible minors of Δ are of negative definite at t_1, u and T then the total cost function is the minimum function.

Step 6. Obtain the minimum join expected total cost $\widetilde{\widetilde{\Pi}}(\mathrm{t}_1, u, T)$ by putting the value of t_1, u and T in equations (16) and (17). Thus, the minimum fuzzy cost $\widetilde{\widetilde{\Pi}}(\mathrm{t}_1, u, T)$ is obtained for trapezoidal fuzzy number and triangular fuzzy number respectively.

6. Numerical example

The manufacturer of the pencil box must decide to set up the production on some quantity with a selling price Rs 25/unit and the set-up cost is Rs 160 for a small period. In addition, the store accountants estimate that there is a cost of Rs 0.20/unit for each cycle. The approximate estimation of demand for the pencil boxes is considered as 550 units/cycle. To fulfil the demand in the market he decides to start the production with a rate of 1.8 and the deterioration rate is 0.2. In this cycle, time the shortage cost is Rs 0.6/unit and the lost sale cost is Rs 0.8/unit. Due to an increase in demand, the prices will also increase, and inflation starts. Because of the high rate of inflation, it is critical to explore how different inventory policies are influenced by the time value of money. So, he considers the rate of inflation 6% for his inventory system. The fuzzy set theory explains a convenient platform to deal with inconclusive parameters. Thus, two different fuzzy numbers trapezoidal fuzzy number and triangular fuzzy number are considered in this inventory framework. The parameter with the appropriate unit is as follows:

$\tilde{m} = (m_1, m_2, m_3, m_4) = (530, 535, 580, 585)$ for trapezoidal fuzzy number

$\tilde{m} = (m_1, m_2, m_3) = (548, 555, 562)$ for triangular fuzzy number

$k =$ Rs25/unit, $n = 0.5$, $\alpha =$ Rs1.5/unit/cycle, $\beta =$ Rs160/set-up, $\gamma = Rs0.2$/unit/cycle

$\delta =$ Rs0.05/unit/cycle, $\eta =$ Rs0.6/unit/cycle, $\lambda =$ Rs0.8/unit/cycle, $\zeta = 0.2$, $a = 1.8$, $\psi = 0.5$, $R = 6$

By using these values in equation (16) for trapezoidal fuzzy number (TrFN) and systematically implementing the computational algorithm, we obtain the optimum value of t_1 equal to 0.011167 years, the value of u 0.839841 years and the value of T 0.840431 years. Then the optimum joint expected total cost $\widetilde{\Pi}(t_1, u, T)$ is Rs191.251607 and the production quantity is 160.901487units.

By using these values in equation (17) for triangular fuzzy number (TFN) and implementing the computational algorithm systematically, we obtain the optimum value of t_1 0.0110694 years, the value of u 0.84701 2years and the value of T 0.847602 years. Then the optimum joint expected total cost $\widetilde{\Pi}(t_1, u, T)$ is Rs189.445789 and the production quantity is 160.179956 units.

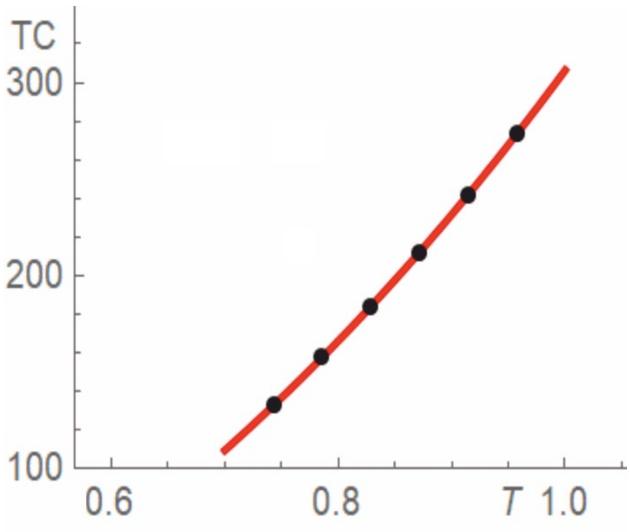

Figure 2. Variation of the joint expected total cost for trapezoidal fuzzy number over time T

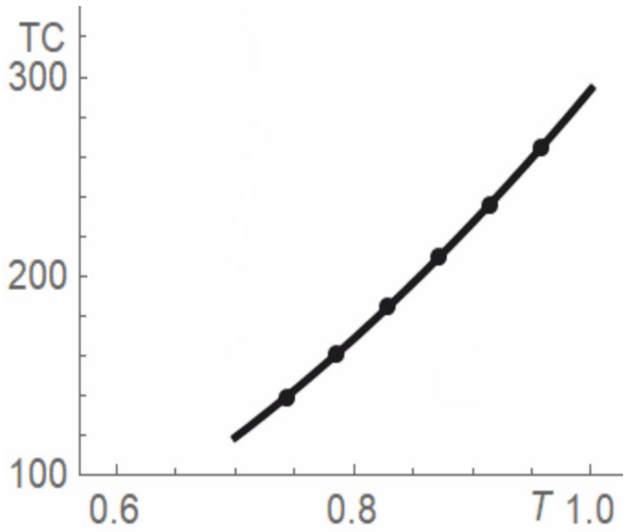

Figure 3. Variation of the joint expected total cost for triangular fuzzy number over time T

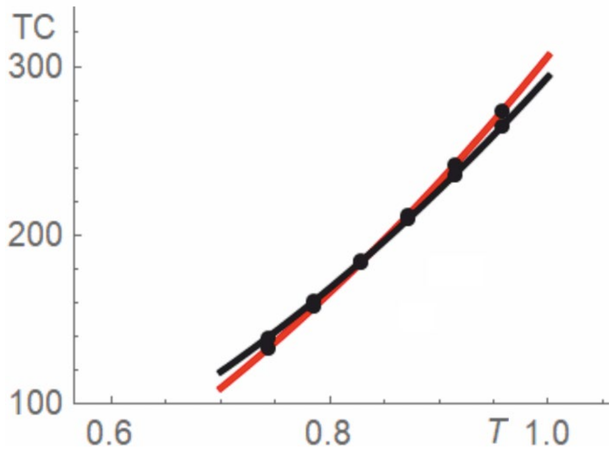

Figure 4. Comparative analysis of total overtime T for trapezoidal fuzzy number and triangular fuzzy number

We analysed each of the fuzzy numbers to obtain a comparative study within the model for uncertain demand. Figures 2 and 3 show considerable variations in annual joint expected total cost on defuzzification of different fuzzy numbers by the centroid method. Figure 2 illustrates that if the consideration periods are changed, the values of the annual joint expected total cost for trapezoidal fuzzy number is changed by a big amount. Figure 3 shows that if the consideration periods are changed, the values of the annual joint expected total cost for triangular fuzzy numbers also change by a big amount. We conclude that the joint expected total cost has just a few deviations for triangular fuzzy numbers and trapezoidal fuzzy numbers (shown in Figure 4).

7. Discussion of the results

The optimum production quantity of 160.901487 units has been obtained at 0.840431 years for TrFN. The optimum production quantity of 160.179956 units has been achieved at 0.847602 years for TFN, which means that the system's production quantity will be lower for TFN. The values of the annual joint expected total cost for TrFN were considerably changed over time as shown in Figure 2. The expected increase of the total cost over time can be easily seen in Figure 2. The minimum expected cost of Rs 191.251607 has been achieved as time varied from 0 to 1.5.

- The value of the annual joint expected total cost varies as TFN has been applied as an increment in time variable shown in Figure 3. The optimum joint expected total cost of Rs 189.445789 has been observed as an increase in the time of the system from 0 to 1.5.
- The comparison of the joint expected total cost has been shown in Figure 4. There has been observed a slight deviation for TFN and TrFN. This can be concluded that the expected total cost is minimum for TFN has been applied in this production model with the defuzzification technique centroid method.

Figure 4 represents that the annual joint expected total cost for the manufacturer deviates just slightly for TrFN and TFN as time increases. The joint expected total cost is better for TFN as compared to TrFN and extra production quantity is obtained for TrFN as compared to TFN.

8. Conclusions

The applicability of the model is investigated through a real problem of a manufacturer to fulfil the demand for the product in the market. In real-life problems, the demand for newly launched items is not identified in advance. Due to specific conditions, vagueness can be noticed in demand and the production rate is directly proportional to the demand rate therefore, the production rate also fluctuates. Therefore, these parameters are not determined, and usually, it is not logical to decide based on the crisp values while the situation is uncertain. Thus, the demand parameter is taken with different fuzzy numbers as Triangular fuzzy number (TFN) and Trapezoidal fuzzy number (TrFN) to deal with inconclusive parameters. Numerical analysis for each of the fuzzy numbers to obtain a comparative study within the model for uncertain demand is conducted for the applicability of the system. The considerable variations in annual joint expected total cost concerning time on defuzzification of different fuzzy numbers by centroid method is shown in Figures 3 and 4.

Due to an increase in demand, the prices will also be increased, and inflation starts. Because of the high rate of inflation, it is critical to explore how different inventory policies are influenced by the time value of money. So, he considers the rate of inflation for his inventory system which is the average price change over time of goods and services. Considering the effect of inflation can give businesses pricing power and boost their profit margins. Thus, he is concerned to minimize the overall cost and increase his benefits. He wants to determine the optimal number of units of the product to order so that he minimizes the total cost associated with the purchase, delivery and storage of the product.

The present study determines the manufacturer's production quantity to obtain a minimum total cost for uncertain demand and production which allows the flexibility to choose desired decisions. The nature of the net fuzzy cost function for shortages under fuzzy unit selling price has been addressed. In this study, the impact of inflation on the production inventory model for deteriorating items has been introduced. In a practical environment considering inflation on cost is a good policy specifically for long-term investment and forecasting. The optimality is reached at the highest unit selling price, which is an unfavourable condition. We analysed each of the fuzzy numbers to obtain a comparative study within the models for two separate fuzzy numbers. We discovered that making a managerial decision using the fuzzy strategy is a better option. From this study, we conclude that the annual joint expected total cost for manufacturer deviate just slightly. In addition, we also conclude as the inflation rate increases, the overall cost of the system also increases. The model presented here reflects a real-life situation. Based on our findings, we concluded that the selling price would promote the model's maximum value. To minimize the joint expected total cost, a computational algorithm with the Hessian matrix is used. A numerical illustration has explained the applicability of this fuzzy production model.

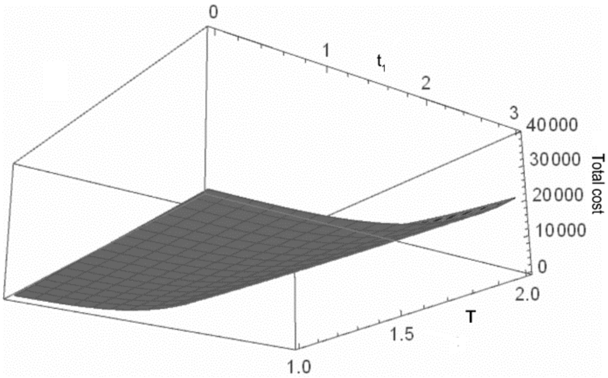

Figure 5. Convexity behaviour of joint expected total cost for a trapezoidal fuzzy number

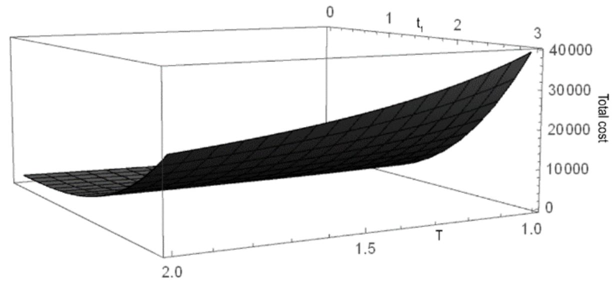

Figure 6. Convexity behaviour of joint expected total cost for a triangular fuzzy number

The convexity behaviour of the total cost function is represented in Figures 5 and 6. For further research, this model has the potential to be expanded in a variety of ways, such as by making the demand factor quadratic, cubic, and stock-based or advertisement dependent etc. or by changing the rate of deterioration as Weibull deterioration rate with two or three parameters or by using rework, remanufacturing etc. It is possible to use the proposed model further by consisting the time value of money into it. In addition, his model can be extended for different fuzzy numbers by using a dense fuzzy approach.

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