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NUMERICAL SIMULATION OF SUBSIDENCE CAUSED BY ROADWAY SYSTEM

SYMULACJA NUMERYCZNA DEFORMACJI POWIERZCHNI SPOWODOWANYCH EKSPLOATACJĄ CHODNIKOWĄ

Exploitation of hard coal seams by roadway system is applied by two coal mines in southern Poland in Upper Silesian Basin. It is a secondary mining exploitation carries out in safety pillars of urban areas and shafts within mining areas of closed coal mines. Roadway system is the excavation process of gateways which are made in parallel order leaving coal pillars between them. An optimal width of coal pillar makes roadway stable and reduces subsidence of terrain surface. The article presents results of subsidence simulation caused by partial extraction using empirical and numerical methods on the example of one exploitation field of "Siltech" coal mine. The asymptotic state of subsidence was considered after mining ceased in the study area. In order to simulate of subsidence, numerical model of rock mass and model of Knothe-Budryk theory were calibrated. Simulation of vertical displacements in numerical method was carried out using RS3 program by Rocscience based on finite element method. The assumption was made that model of rock mass is transversely isotropic medium, in which panels were designed according to order of extraction of coal seams. The results of empirical and numerical methods were compared with measured values of subsidence at benchmarks along drawn lines (subsidence profiles).

Keywords: mining, roadway system, numerical modeling, back analysis

Eksploatacja złoża węgla kamiennego systemem chodnikowym jest stosowana w Polsce w Górnośląskim Zagłębiu Węglowym przez dwie kopalnie. Jest to eksploatacja wtórna prowadzona na obszarach górniczych zlikwidowanych kopalń w filarach ochronnych szybów i zabudowy powierzchni. Proces technologiczny polega na drążeniu równoległych do siebie chodników w granicach pól eksploatacyjnych. Pomiędzy wyrobiskami pozostawiane są filary węglowe o odpowiedniej szerokości w celu zapewnienia ich stateczność i minimalizacji deformacji powierzchni terenu. W artykule przedstawiono wyniki symulacji obniżeń terenu spowodowanych eksploatacją częściową przy wykorzystaniu metody empirycznej i modelowania numerycznego na przykładzie jednego pola eksploatacyjnego kopalni „Siltech". Analiza

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obejmowała asymptotyczny stan deformacji terenu po zakończonej eksploatacji w tym rejonie. W celu odwzorowania obniżeń terenu wykonano kalibrację numerycznego modelu górotworu i teorii Knothego--Budryka. Symulację obniżeń powierzchni terenu metodą numeryczną (MES) wykonano programem RS3 firmy Rocscience. Założono, że model górotworu jest ośrodkiem transwersalnie izotropowym, w którym odtworzono eksploatację systemem chodnikowym zgodnie z kolejnością wybierania pokładów. Dla wytypowanych punktów obserwacyjnych tworzących ciąg linii porównano obliczone obniżenia metodą empiryczna i numeryczną z wartościami zmierzonymi w terenie.

Słowa kluczowe: eksploatacja górnicza, system chodnikowy, modelowanie numeryczne, analiza wsteczna

1. Introduction

Widespread way of extraction of hard coal seams in Poland is longwall system with caving. The basic advantages of mentioned system are effectiveness, safety and high level of coal extraction ratio. The concentration of extraction of coal deposit using longwalls induces high level of surface deformation in urban areas and brings high risk of damage of structures and infrastructure.

There are alternative systems in order to minimize mining influence in urban areas. The subsidence prediction of partial extraction and its impact on structures and ground surface were presented among others by Knothe (1958), Missavage (1986), Saxena (1991), Singh & Yadav (1995), Gandhe et al. (2005), Tajduś et al. (2012), Shaojie (2016) and Zhang et al., (2016). The discussed roadway system with coal pillars between workings can be classified as one of them. The system is applied to extract coal in remains of coal beds and in safety pillars designed for shafts and sensitive structures withing mining areas. Since 2004 two coal mines "Siltech" and "Eko-Plus" extract coal seams in the areas of closed old coal mines.

The article presents results of subsidence simulation caused by partial extraction using empirical and numerical methods on the example of one exploitation field called "Rokitnica". "Rokitnica" field is located in northern part of "GIGANT I" mining area where "Siltech" coal mine operates in safety pillars of three shafts: "Mieczysław" (liquidated), "Kościuszko" (liquidated), "Staszic". "Rokitnica" field is also situated north of nearby "Gigant" shaft. The area is undergoing liquidation.

The assymptotic state of subsidence were considered after mining ceased in the study area. The conclusions from stages of roadway system in "Rokitnica" field were presented in previous papers (Kowalski & Walentek, 2013; Gruchlik et al., 2014; Kowalski et al., 2017).

In order to simulate of subsidence, model of Knothe-Budryk theory and numerical model of rock mass were calibrated according to levelling surveys and geomechanical parameters of rocks. Simulation of vertical displacements in numerical method was carried out using RS3 program by Rocscience based on finite element method. The assumption was made that model of rock mass is transversely isotropic medium, in which panels were designed according to order of extraction of coal seams.

The results of empirical and numerical methods were compared with measured values of subsidence at benchmarks along drawn lines (subsidence profiles).

The article has two objectives presenting results of vertical displacements caused by partial extraction and testing simulation of subsidence applying numerical modelling of rock mass.

2. Overview of study area

The study area is situated in southern Poland in Upper Silesian Coal Basin within boundaries of cities of Zabrze and Bytom. „Siltech“ coal mine carries out extraction of coal seams in safety pillars within boundaries of „GIANT I“ mining area. It‘s constitutes about 3% of mining area of abondened coal mine called „Pstrowski“, which was closed in 1977. Land development of northern part of observation field is diverse. There are industrial structures of „Siltech“ coal mine in the central part of the area, structures of service companies in eastern part of the area and wasteland in the western part of observation field. One- and five-floor residential buildings are placed in the northern part of the area (Fig. 1).

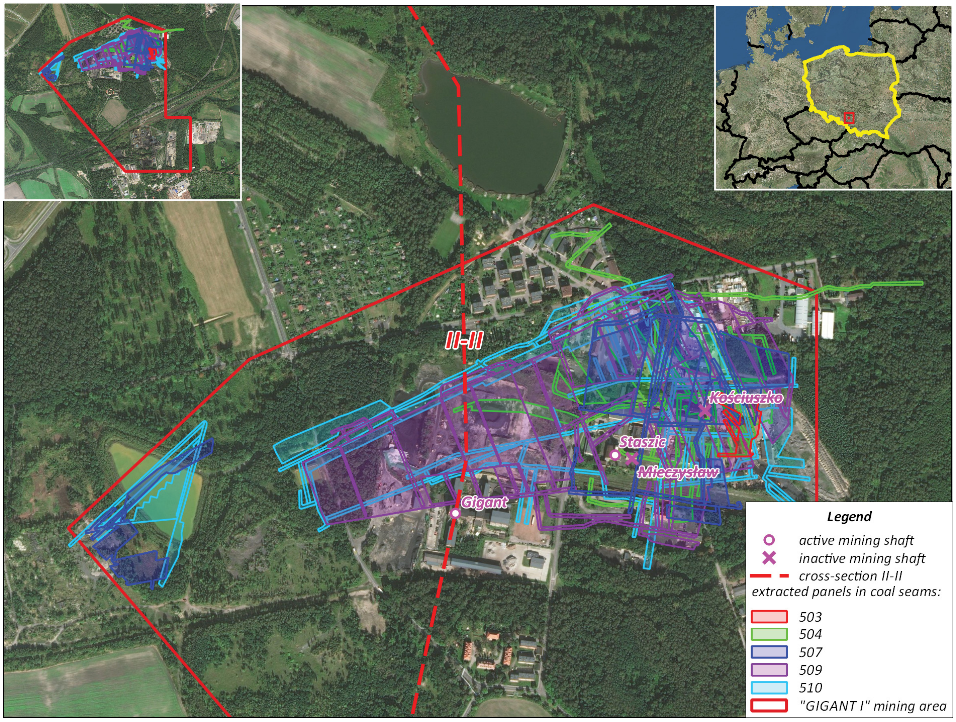

Fig. 1. Site of study area

3. Mining and geological condition

„Jadwiga 2 “hard coal deposit is located in southeastern side of western part of “Bytom” basin which gradually changes into “Zabrska” dome. The Fig. 2 shows north-south cross-section acrossing “Gigant” shaft which is situated southwest of the mined out area.

The overburden consists of Quaternary and Triassic layers with thickness from 100 m to 130 m. Quaternary layers contain silts which form regular layers with total thickness of about 30 m. Their thickness increases up to 55 m in eastern part of the area. Triassic layers composes shell limestone with thickness from 50 m to 80 m. Below lie layers of red sandstone which thickness ranges from 3 m in central to 16 m in western part of study area (Fig. 2).

Carboniferous layers were explored during excavation of shafts and extension of working system in rock and in coal. Thickness of Carboniferous layers ranges from 750 m to 920 m. Major rocks in rock mass are shales which constitute 51% of all layers. Sandstones and coal seams constitute respectively 43% and 6% of all Carboniferous layers. An average slope of rock layers is about 13° in the area of extracted panels. The angle increases up to 30° to the north of the field towards a bottom of „Bytom“ basin (Fig. 2).

Extraction of 12 coal seams were carried out continuously from 1910 to 1979 with caving, stowing and backfilling nearby „Mieczysław“, „Kościuszko“, „Staszic“ and „Giant“ shafts. The thickness of extracted coal ranged from 0.6 m to 7.0 m (extraction of both 508 and 509 seams). Old mine workings lie at a depth of 100 m to 860 m. The thickness of extracted coal is between 23 m and 26 m. The last exploitation were carried out with single longwall panel with caving in 620 seam in 2000 at an average depth of 645 m before application of roadway system in the study area. The height of longwall was 2.0 m.

The underground part of the study area were disturbed by many old workings in shaft pillar and it surroundings, especially in “anticlinal” carboniferous layers in the main levels at the depth of 248 m and 380 m.

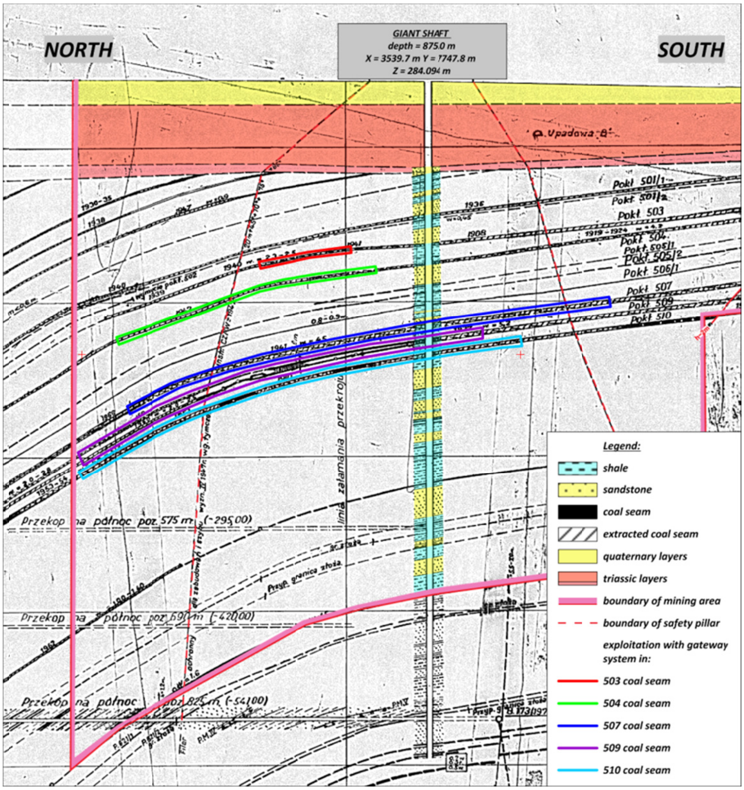

Fig. 2. Cross-section along study area from south to north

4. Overview of mining system

Mining system applied by "Siltech" coal mine is the excavation process of gateways driven in parallel order to each other leaving coal pillars between them. An optimal width of coal pillar makes roadway stable and reduces subsidence of terrain surface. The roadways are protected by steel arch support and they are excavated in perpendicular order from main entry using continous miners. Backfilling materials are transported by system of pipes to extracted panels. Backfilling body consists water and ashes in right proportion to fill excavated voids and prevent self-ignition of coal pillars. Long-term strength of the filler was obtained after about 28 days. The process of mining technology can be divided by two stages. The first stage is excavation of roadway along the entire lenght and the second stage is filling up of mine out area with backfilling materials (Fig. 3).

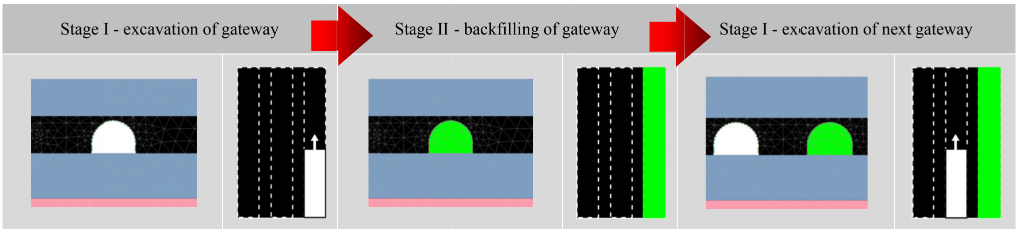

Fig. 3. Scheme of mining process

In the study area mining operations were carried out continously in 509, 507, 510, 504 and 503 seams during october of 2004 and april of 2014 (Fig. 1, Table 1). The depth of extraction and technology gateways ranges from 230 m to 495 m. The width and height of extraction gateways were respectively 5.6 m and 3.8 m. The heigth of extraction gateways had been reduced up to 2.8 m because of thinning of west part of 509 seam. An average width of coal pillars between gateways was 4 m which consists 58% of coal extraction ratio.

TABLE 1

Basic geological and mining data of gateway system in Rokitnica field

Exploited seam	Heigth of exploitation [m]	Depth of exploitation [m]	Duration of exploitation	Average vertical distance between seams [m]
503	2.8	230 – 255	01.03.2011 – 31.05.2011	-
504	3.8	230 – 375	01.10.2008 – 30.11.2011	25
507	3.8	310 – 440	01.04.2007 – 30.04.2014	85
509	2.8/3.8	330 – 420	01.10.2004 – 30.04.2009	20
510	3.8	340 – 495	01.04.2010 – 31.12.2013	20

5. Subsidence observation in Rokitnica field

Subsidence-monitoring network was designed and established in the half of 2004. The observation network includes benchmarks set in structures and in the ground forming measure-

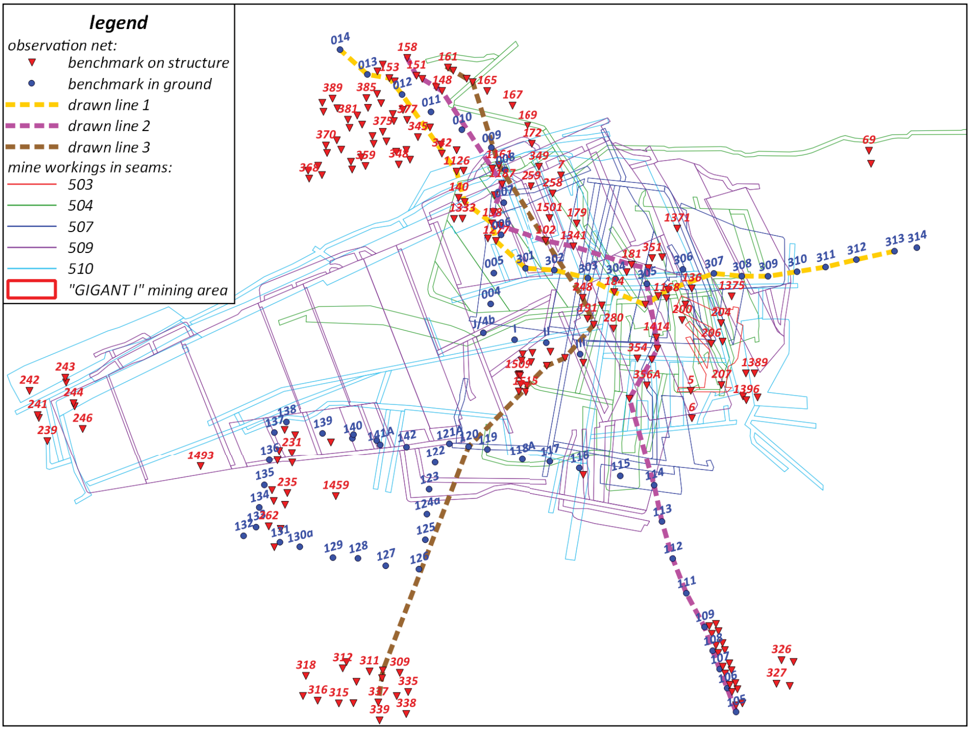

Fig. 4. Observation net and drawn lines to determine parameters of Knothe-Budryk theory

ment lines (Fig. 4). 21 observation cycles had been carried out at semi-annual intervals during mining operations. Direct leveling was used to measure an elevation of benchmarks. The distance between observation points was surveyed by tape 50 m long.

In the years 2004-2015 the large and extensive subsidence trough was forming over extracted panels. The maximum vertical displacements were about 1.98 m (Fig. 5). An average slope of subsidence trough was 6 mm/m while locally up to 10 mm/m. The values of horizontal deformations ranged from –3.2 mm/m to +2.7 mm/m. The height increments of observation points were up to ±12 mm in the last two surveys. The results were within survey error. Figure 6 shows vertical displacements of observation point with identification number 305 in terms of time during mining operations and after finish of coal seams extraction with roadway system (asymptotic values).

6. Simulation of subsidence using empirical and numerical methods

6.1. Simulation of subsidence using empirical method

Simulation of subsidence was carried out through "back-analysis" using empirical theory of Knothe and Budryk (Knothe, 1984; Whittaker & Reddish, 1989).

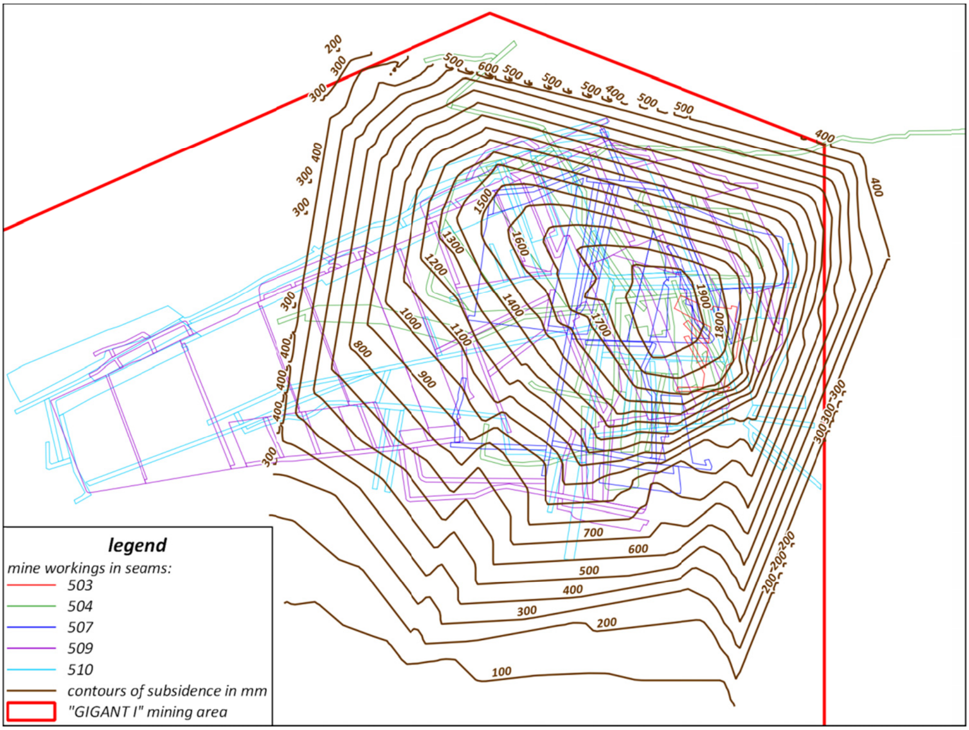

Fig. 5. Subsidence contours of observation points in the years 2004-2015 caused by roadway system

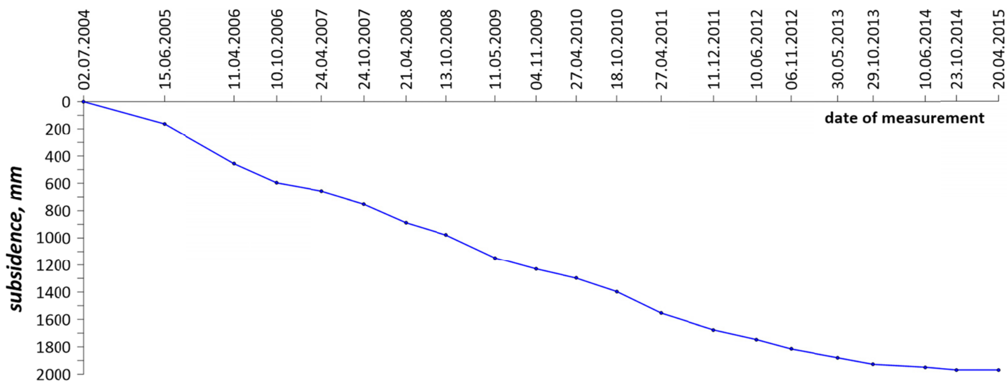

Fig. 6. Vertical displacements of observation point no 305 in terms of time in the years 2004-2015

Before determining parameters of the theory, three lines were drawn crossing observation points in order to capture the range of mining influences and maximum value of vertical displacements over extracted panels in respective seams. Benchmarks in ground and in structuers were taken into consideration.

Matching of profile of subsidence trough along drawn lines were carried out by determining the value of subsidence factor in further iterations for defined range of tangent β values. The method of least squares was used to calculate residuals between measured and calculated values of subsidence. The standard deviation was determined to evaluate the accuracy of approximation.

Parameters of Knothe-Budryk theory were calculated for each stage of exploitation with roadway system for Rokitnica field and the results. Table 2 presents range of parameters values and extreme values of vertical displacements and horizontal deformations for three stages of the exploitation.

TABLE 2

Values of vertical displacements, horizontal deformations and determined parameters of Knothe-Budryk theory in Rokitnica field [8]

Stage of exploitation	Extreme values of measured indicators		Determined parameters of Knothe-Budryk theory		Extracted seams
	Subsidence [m]	Horizontal deformation [mm/m]	*a*	tg β	
1	0.67	–1.8÷+1.0	0.25÷0.30	0.9÷1.3	509
2	1.01	–2.2÷+1.6	0.25÷0.40	1.0÷1.3	509, 507
3	1.98	–3.2÷+2.7	0.40	1.0	509, 507, 510, 504, 503

Subsidence of observation points were much higher than expected for applied mining system. An assumed value of subsidence factor was 0,19. The values from 0,25 to 0,4 indicates that designed average width of coal pillars was to narrow (table 2). It can be interpreted as effect of a greater deflection of the roof of exploited seams as result of partial crushing of coal pillars. Reactivation of old workings in safety pillars was also the cause of higher values of the factor.

Less value of rock mass parameter (tg β) than an average, indicates that range of main mining influence is wide, which is confirmed by large subsidence trough (Fig. 5). A high degree of disturbance of the rock mass caused by mining exploitation out of safety pillar and nearby it, could have a significant impact on the value of this parameter.

6.2. Simulation of subsidence using numerical modelling

The three-dimensional numerical model of rock mass was made in the RS3 program by Rocscience, in which the calculation algorithms are based on the finite element method (MES). The model has the cuboid form with the base dimensions 2000×1500 m. The height of the model equals 500 m and it was set according to depth of lower extracted panel measured from terrain surface (Fig. 8). Rock layers in the model were designed acoording to a profile of "Giant" shaft. In the next step, reconstruction of mining operations was made in order of seams extracion (Fig. 7).

Model of rock mass was defined as transversely isotropic elastic medium,which has different mechanical properties along horizontal and vertical plane. The model enables to reconstruct vector components of vertical and horizontal displacements of ground surface caused by underground mining and right profile of observed subsidence trough in comaprison to isotropic models (Wesołowski, 2013). Application results of this medium for simulation of subsidence was published among others by Berry (1962), Hazine (1977), McNabb (1987), Tajduś (2007, 2009, 2013) and Wesołowski (2013, 2014).

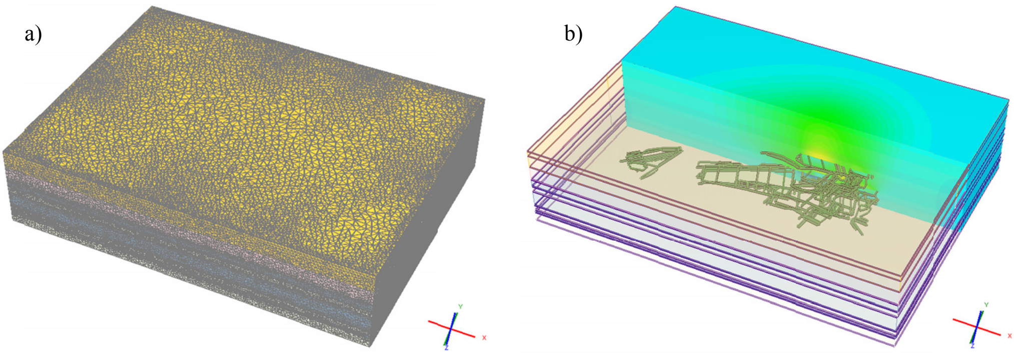

Fig. 7. Scheme of numerical model. a) discrete model; b) extracted panels in coal seams and vertical displacements in rock mass

Transversely isotropic medium is described by five elastic parameters:

- E_1 – Young's Modulus at load in pararell direction to isotropic plane,
- E_2 – Young's Modulus at load in perpendicular direction to isotropic plane,
- υ_{12} – Poisson's ratio at load in pararell direction to isotropic plane,
- υ_{13} – Poisson's ratio at load in perpendicular direction to isotropic plane,
- G_{13} – shear modulus perpendicular to isotropic planes.

The following assumptions were made in the model:

- boundary nodes along a plane of the model base are allowed to move only in horizontal direction,
- boundary nodes along lateral planes of the model are allowed to move only in vertical direction,
- other nodes are allowed to move in XYZ directions,
- initial stress field depends on the depth of gateways and average unit of overburden (vertical stress at the depth of 500 m equals 12 MPa).

The values of parameters of transversely isotropic model (columns No. 1-5) and parameters of Hoek-Brown failure criterion (columns No. 6-8) were set in table 3. Stages of roadway ex-

TABLE 3

Values of parameters and constants for rock layers in the numerical model

Rock layers	**$E_1 = E_2$ [GPa]**	**E_3 [GPa]**	**$\upsilon_{12} = \upsilon_{31}$**	**G_{12} [GPa]**	**G_{13} [GPa]**	**Compressive strength R_c [MPa]**	**Parameter m_b**	**Constant s**
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
quaternary layers	0.02	0.02	0.35	0.007	0.007	0.1	0.100	0.0001
triassic layers	0.165	3.3	0.20	0.069	0.156	34.0	1.016	0.0014
clay shale	0.165	3.3	0.20	0.068	0.154	35.0	0.821	0.0013
mudstone	0.255	5.1	0.20	0.106	0.238	33.0	1.249	0.0031
sandstone	0.420	8.4	0.20	0.175	0.392	45.0	2.708	0.0060
coal	0.060	1.3	0.20	0.027	0.061	14.0	0.529	0.0005

ploitation are divided into panels depending on the depth of workings and extraction order. The total number of panels in the model is 140. The panel included 2 to 5 roadways with coal pillars depending on extracted region.

7. Results of subsidence simulation using empirical and numerical methods

In order to assess the correct calibration of the rock mass model, the results of calculations of numerical method were compared with values of measured subsidence along the lines (Fig. 4). The analysis also included vertical displacements according to the Knothe-Budryk theory, which mathematical model has been used since 1950. Calculations were carried out by Damage program written by Eligiusz Jędrzejec at Central Mining Institute. The parameters of empirical method were taken from table 2 for the third stage of exploitation. Subsidence profiles along lines according to survey, Knothe-Budryk theory and numerical modeling are presented in figures 8-10.

TABLE 4

Maximum values of subsidence according to survey, Knothe-Budryk theory, numerical modeling and deviation values of calculated subsidence in relation to measured values

No of line	Survey	Knothe-Budryk theory		Transversely isotropic model	
	Subsidence [mm]	Subsidence [mm]	θ [%]	Subsidence [mm]	θ [%]
1	1980	2058	3.9	2031	2.6
2	1968	1994	1.3	1987	1.0
3	1798	2043	13.6	1987	10.5

where:

$$\theta = \frac{S_c - S_m}{S_m} \cdot 100\% \tag{5}$$

θ_D — deviation of calculated subsidence in relation to measured values,
S_c — maximum value of calculated subsidence,
S_m — maximum value of measured subsidence.

Measured and calculated vertical displacements along lines determined by two methods are similar (Fig. 8-10, Table 4), despite the high level of fracturing of rock layers in rock mass caused by past mining exploitation carried out outside and inside safety pillar designed for shafts and structures in the study area. According to maps of coal seams, there are a lot of old mine workings at the level of 248 m deep (in seams 503 and 504) and at the level of 380 m deep (in seams 507, 509 and 510). Most of these old workings are located at the same depth as panels of roadway system.

Differences between maximum values of measured and calculated subsidence according to the Knothe-Budryk theory range from –245 mm to –26 mm, which is up to 14% of measured values, and according to the numerical model from –189 mm to –19 mm, which is up to 11% of measured one. The aproximation degree of the numerical model in terms of accuracy is the same order as the Knothe-Budryk model.

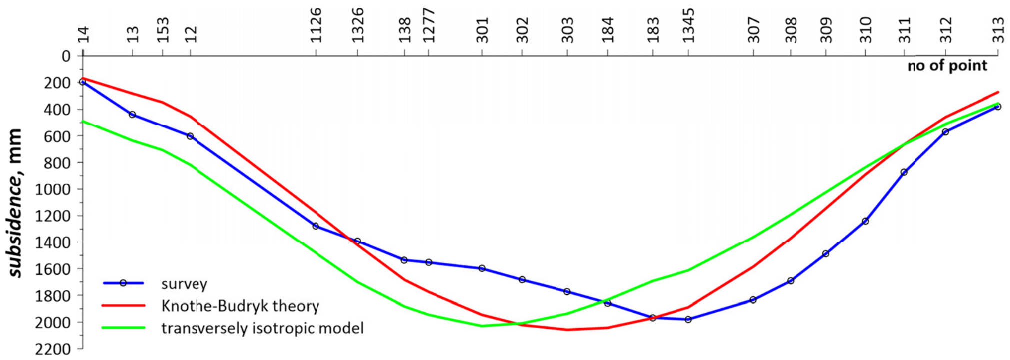

Fig. 8. Subsidence profile along drawn line 1 according to survey, Knothe-Budryk theory and numerical modeling

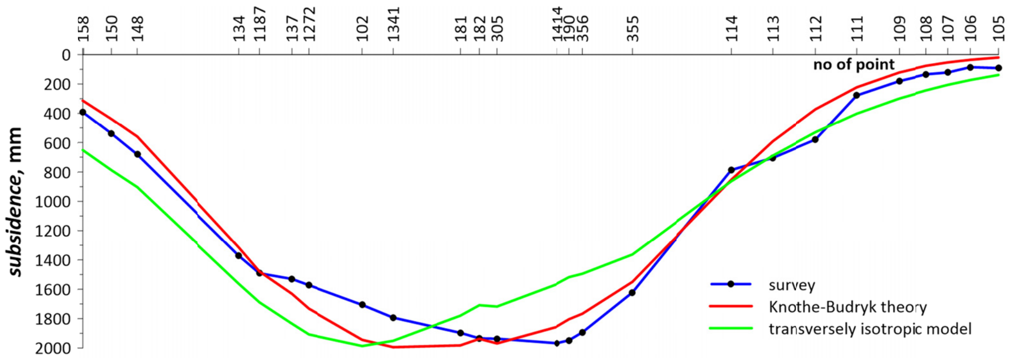

Fig. 9. Subsidence profile along drawn line 2 according to survey, Knothe-Budryk theory and numerical modeling

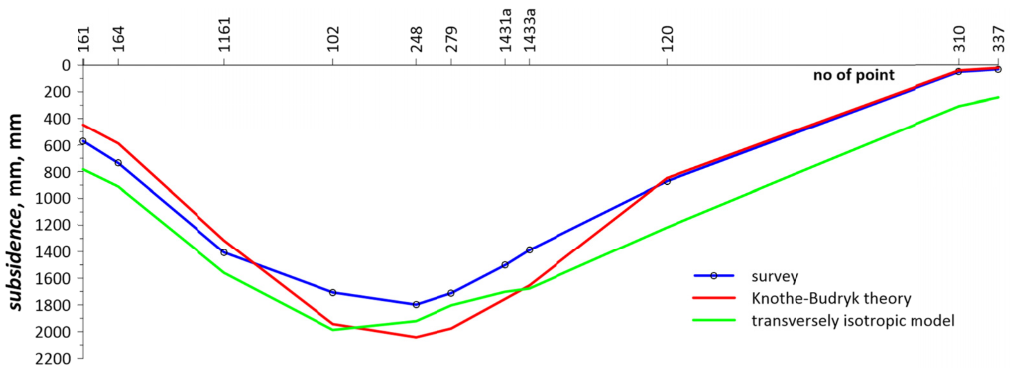

Fig. 10. Subsidence profile along drawn line 3 according to survey, Knothe-Budryk theory and numerical modeling

The occurring quantitative differences result from two reasons. The first reason is the simplification of the description of the rock mass in the theoretical models in relation to real rock mass condition in which there are remains of unexploited seams. The smaller measured subsidence in sections of the line 1 (points 1326-183, Fig. 9) and line 2 (points 1187-182, Fig. 10) can be interpreted as result of unexploited parts of coal seams. The second reason is the reactivation of short longwalls and gateways gobs in several seams. Larger measured subsidence can be explained as additional displacements because of weak immediate roofin the upper layer of 509 seam during exploitation with short longwalls.

8. Conclusion

1. Maximum values of calculated vertical displacements are fall within the limits of the dispersion error of the Knothe-Budryk theory model, the value of which according to Popiołek and Ostrowski (1981) is ±6%, and according to Kowalski (2015) at 14% (standard deviation value of relative differences between maximum values of measured and calculated subsidence).

2. Presented case of coal exploitation with gateway system is not typical, taking into account observed subsidence and determined parameters of Knothe-Budryk theory. The values of subsidence factor are much higher than for other cases of partial extraction (Knothe, 1958). The determined value of tangent β is much less than the average value for Upper Silesian Coal Basin (tgβ = 2,0).

3. The high values of subsidence factor are results of too narrow coal pillars left between gateways and reactivation of old workings in the safety pillar. Much less value of rock mass parameter indicates that observed large subsidence through is the result of high level disturbance of rock mass caused by coal exploitation out of safety pillar.

4. The obtained results should be treated as the effect of initial calibration. The attempts of next simulations will be made to improve the description of all deformation indices such as vertical and horizontal displacements, tilts, curvatures and horizontal deformations.

5. Numerical modeling in addition to simulation of vertical displacements was applied to design the width of coal pillars for another region of "GIGANT I" mining area in order to minimize subsidence on surface.

6. The significant case to evaluate of the model is testing it in other areas for different geological and mining conditions.

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