A COMPARISON OF NUMERICAL SOLUTIONS OF DEAD RECKONING NAVIGATION

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Abstract

Calculations of position coordinates in dead reckoning navigation essentially comes down to the integration of ship movements assuming an initial condition (position) of the ship. This corresponds to Cauchy's problem. However, in this case the ship's velocity vector as a derivative of its track (trajectory) is not a given function, but comes from navigational measurements performed in discrete time instants. Due to the discrete character of velocity vector or acceleration measurements, ship's movement equations particularly qualify for numerical calculations. In this case the equation nodes are the time instants of measurements and navigational parameter values read out at those instants. This article presents the applications of numerical integration of differential equations (movement) for measurements of velocity vectors and acceleration vector (inertial navigation systems). The considerations are illustrated with navigational measurements recorded during sea trials of the rescue ship integrated system.

1. Introduction

Dead reckoning navigation is a basic and essentially simple method for the determination of ship's current position. In dead reckoning (DR) navigation systems position coordinates of ship's present position is obtained by integrating the velocity vector or by integrating twice the acceleration vector in inertial navigational systems (INS) [2]. Both cases differ in measuring methods, while the principle of calculations is similar. In each case we adopt a discrete or analogue model of measurements. Corresponding to them are, respectively: difference equations or ordinary differential equations, although we can regard the former case as a series of values of the differential equation at preset time instants (nodes) and bring it down to the problem of numerical integration of an ordinary differential equation [4], [6]. Further in the article this issue will be dealt with.

2. Taylor's formula

We can approximate a certain class of continuous functions by using Taylor's formula. It defines the expansion of a function f of C^n class (functions with continuous derivatives to n – order inclusive) on a closed interval $[a,b], a,b \in R$.
The formula is often used in numerical problems: calculation of a function value at a point, linearization of non-linear equations and equation systems, differentiation and integration of functions and others. Its form is set forth by the following theorem.

THEOREM

If $f \in C^n[a,b]$ and if $f^{(n+1)}$ exist/s in an open interval (a,b), then for any points $c,x \in [a,b]$ we get this equation

$$f(x) = \sum_{k=0}^{n} \frac{1}{k!} f^k(c)(x-c)^k + R_n(x), \tag{1}$$

Where for a certain point ξ lying between c and x (i.e. $x < \xi < c$ or $c < \xi < x$)

$$R_n(x) = \frac{1}{(n+1)!} f^{(n+1)}(\xi)(x-c)^{n+1}. \tag{2}$$

The term $R_n(x)$ is called *Lagrange remainder* of the Taylor's polynomial.

EXAMPLE 1

For a function with one variable we will obtain its expansion in an environment of point x_0 in the following form:

$$f(x) = f(x_0) + f'(x_0)(x-x_0) + \tfrac{1}{2!} f''(x_0)(x-x_0)^2 + + \tfrac{1}{3!} f'''(x_0)(x-x_0)^3 + \ \dots \tag{3}$$

You will see that in the above formula the first and second derivatives of the function occur. Neglecting the other terms of formula (3), as small values of higher orders, we get this approximation:

$$f(x) \approx f(x_0) + f'(x_0)(x-x_0) + \tfrac{1}{2!} f''(x_0)(x-x_0)^2, \tag{4}$$

that is

$$f(x) - f(x_0) \approx f'(x_0)(x-x_0) + \frac{1}{2!} f''(x_0)(x-x_0)^2. \tag{5}$$

A question arises whether in dead reckoning navigation we can assume that we have a function n times differentiable continuously, i.e. whether we can use the Taylor's formula. The answer is affirmative since we deal with objects whose velocities (first derivatives of distance) and accelerations (second derivatives of distance) are well defined. Thus equation (5) represents an approximation of distance covered by a ship:

$$S(t) - S(t_0) \approx v(t_0)(t - t_0) + \frac{1}{2}a(t_0)(t - t_0)^2, \quad (6)$$

where: S – distance covered, v – ship velocity, a – acceleration, t, t_0 – time instants.

If we take into account both velocity and acceleration components (longitudinal and transverse), then we obtain a model of dead reckoning navigation on an appropriate reference area [1], [2], [4].

3. Euler's method

The Taylor's formula is directly or indirectly used in many numerical methods of solving ordinary differential equations. One of these, *Euler's method*, is a first order method and is represented by the following formula:

$$x(t + h) \approx x(t) + hx'(t) = x(t) + hf(t, x), \quad (7)$$

where h is an integration step, $x'(t) = f(t, x)$.

Advantages of the method are its simplicity and the fact that we need not differentiate the function f at all. A disadvantage is that a very small h must be chosen (for the sake of accuracy). The method also has a theoretical importance as based on it is one of the proofs for the existence of a solution to Cauchy's problem. An application of this formula in dead reckoning navigation is presented in the example below.

EXAMPLE 2

An object movement on a plane (ship, airplane) is described by this equation

$$\mathrm{x}'(t) = \mathrm{v}(t), \quad (8)$$

where x – vector of position (coordinates), v – velocity vector, known from measurements.

Solutions are provided by:

- dead reckoning (DR)

$$\mathrm{x}(t + h) = \mathrm{x}(t) + h\mathrm{v}(t), \quad (9)$$

- inertial navigation system (INS)

$$\mathrm{x}(t + h) = \mathrm{x}(t) + h\mathrm{v}(t) + \frac{h^2}{2}\mathrm{a}(t). \quad (10)$$

A comparison of both solutions (formulas (9) and (10)) using real navigational measurements performed on a rescue vessel during sea trials [1] is given in Figures 1 and 2 below.

Figure 1 illustrates differences between DR navigation that makes use of the log-compass system and INS in which measurements of acceleration are also used.

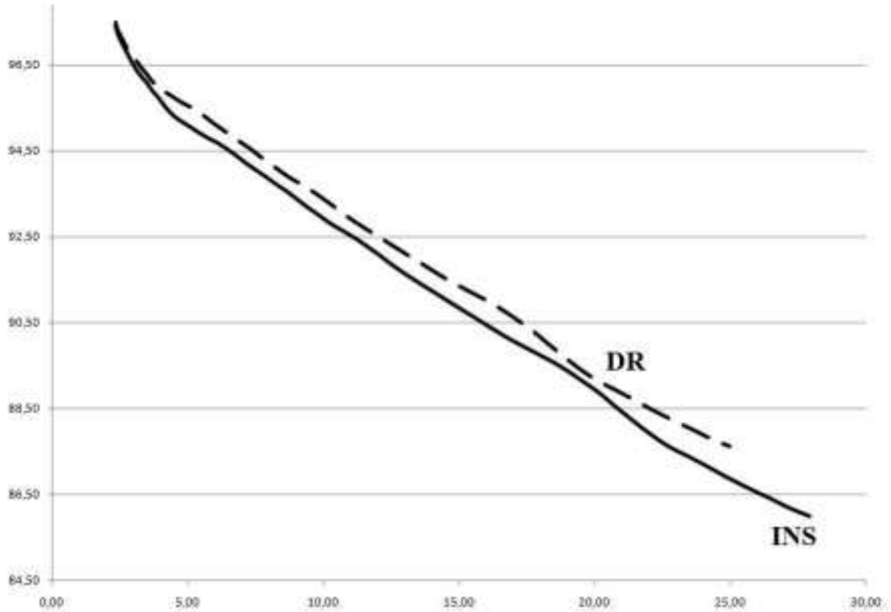

Fig. 1. Comparison of DR and INS

Figure 2 additionally presents a trajectory obtained from DGPS measurements. As one can see, due to systematic errors of logs and accelerometers, DR and INS trajectories are shorter than the DGPS trajectory. However, the DGPS trajectory is characterized by large disturbances of high frequency (wavy line) and short-time large blunders (a few milimeter shift of the trajectory in the central part of the DGPS graph).

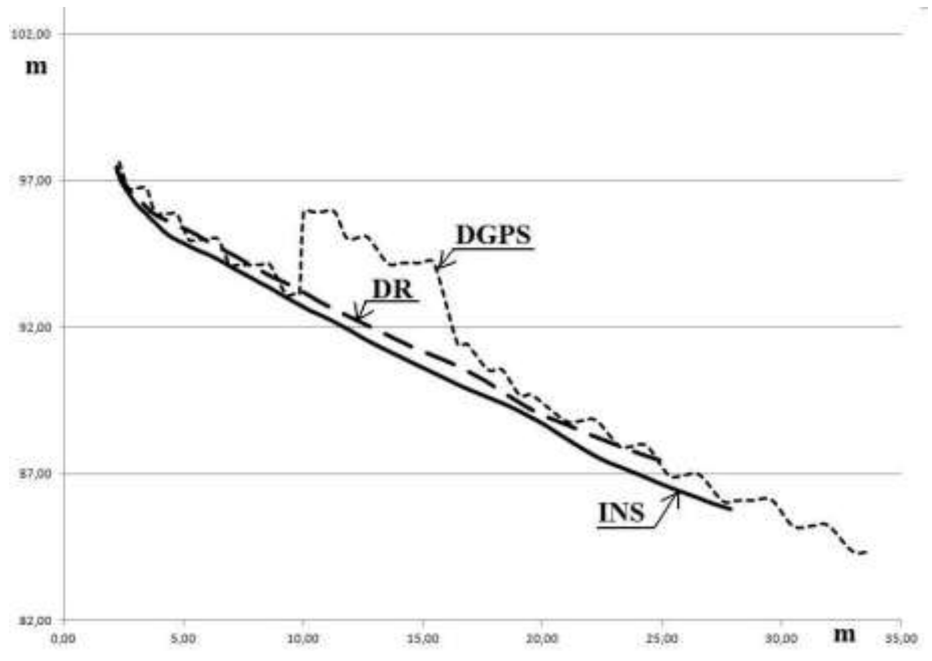

Fig. 2. Comparison of DR, INS and DGPS trajectories

Both charts clearly show an advantage of INS over traditional DR navigarion. Figure 2 suggests that DR and INS should be integrated with position measurements (GPS) [4]. This need results from the separate character of disturbances spectra of different navigational measurements methods.

Multi-step methods of integrating differential equations with approximating properties can also be used. Mean square approximation of speed vector components is discussed further in the article.

4. Approximation of velocity vector and acceleration vector

A valuable feature of DR is that position coordinates are obtained at any moment (this of course refers to automated systems and without taking into account time delays resulting from the measurement and computing cycles). In some cases we may want to reproduce a trajectory history, e.g. for the purpose of examining a marine accident, in hydrographic and geophysical surveys and others.

Let us then check to what extent a DR trajectory from on–line measurements differs from a corresponding trajectory obtained by avereging the measurements. In this case a polynomial mean square approximation was used [5], [6], which resulted in these polynomials:

$$\begin{gathered} v_x(t) = -1{,}68572 \cdot 10^{-11} t^6 + 5{,}94347 \cdot 10^{-10} t^5 - 8{,}03415139 \cdot 10^{-7} t^4 + \\ +5{,}08742174 \cdot 10^{-5} t^3 - 1{,}384972991 \cdot 10^{-3} t^2 + \\ +1{,}5693 \cdot 10^{-2} t - 0{,}07693262577, \\ \\ v_y(t) = -0{,}4 \cdot 10^{-11} t^6 + 1{,}23 \cdot 10^{-10} t^5 + 1{,}18019 \cdot 10^{-7} t^4 - \\ -1{,}331996 \cdot 10^{-5} t^3 + 4{,}5182904 \cdot 10^{-4} t^2 - \\ -3{,}78853 \cdot 10^{-3} t + 0{,}08356441728, \end{gathered} \tag{11}$$

where:

v_x – longitudinal component of ship velocity,

v_y – transverse component of ship velocity,

t – time (measurement number).

Figure 3 shows deviations of the approximating function from the set of speed log measurements [1], [3]. These differences are contained in the interval up to 0.12 m/s (absolute value). This corresponds to a speed of about 0.2 knot.

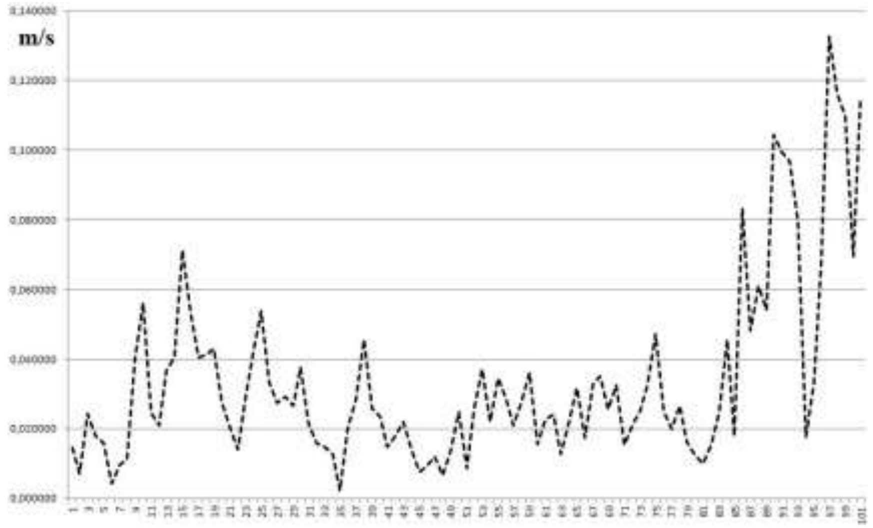

Fig. 3. Deviations of velocity approximation

The deviations of the DR trajectory and the trajectory calculated on the basis of a polynomial approximating a measured ship's velocity are shown in Figure 4. It follows that these differences are insignificant (from nearly 2 m to less than 20 cm).

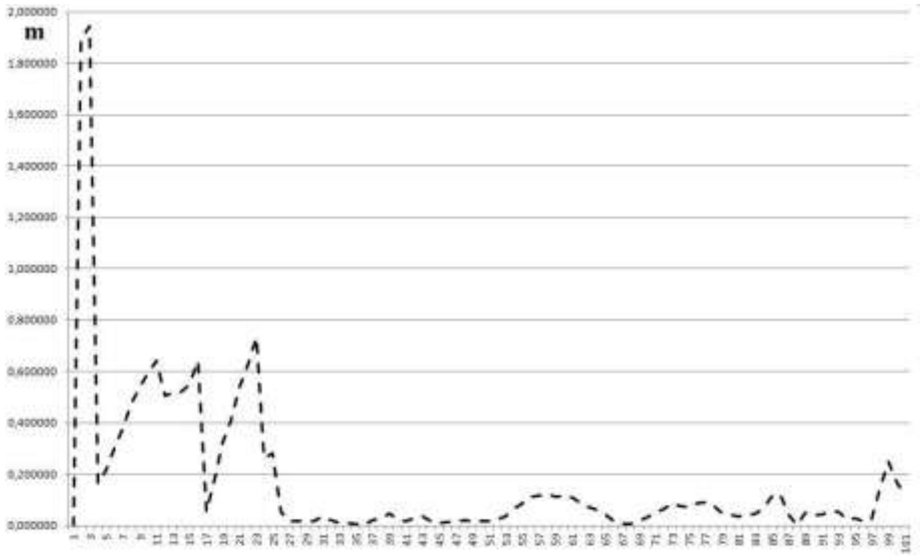

Fig. 4. Deviations of DR and DR from approximated velocity

An advantage of using ship's velocity calculated from approximations of navigational measurements is that blunders and high frequency disturbances can be eliminated. Unfortunately, there is no possibility to identify and eliminate systematic errors.

5. Conclusions

The presented analysis of dead reckoning navigation shows that from the viewpoint of mathematical formalism the considered problem is not a complex issue. The remaining determinants related to measurements and calculations have been discussed in previous works of these authors, e.g. in [1] and [2]. The publication [4] widely presents the issue of estimation in navigational systems.

Additional navigational equipment, e.g. INS, increases accuracy and reliability of navigational information. It goes without saying that positioning systems (GPS, Galileo, GNSS) will not exclude completely other navigation methods. Various navigational technologies and methods should therefore be complementary and used as emergency facilities.
The application of historical trajectory approximation and performed navigational measurements allows to broaden archived recordings of navigational parameters (e.g. in a VDR – *Voyage Data Recorder*, ECDIS – *Electronic Chart Display Information System*) and to identify the navigational process – influence of external disturbances on ship movement, elimination of systematic errors of DR and INS by comparison with position measurements (GPS), and elimination of position errors by their comparison with DR / INS data (Fig. 2).

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