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REMARKS ON UNIFORMLY BOUNDED COMPOSITION OPERATOR ACTING BETWEEN BANACH SPACES OF FUNCTIONS OF TWO VARIABLES OF BOUNDED SCHRAMM Φ-VARIATION

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Abstract

In this paper we prove that if the composition operator H of generator $h : I_a^b \times C \to Y$ (X is a real normed space, Y is a real Banach space, C is a convex cone in X and $I_a^b \subset \mathbb{R}^2$) maps $\Phi_1 BV(I_a^b, C)$ into $\Phi_2 BV(I_a^b, Y)$ and is uniformly bounded, then the left-left regularization h^* of h is an affine function in the third variable.

1. Introduction

Let I_a^b be the rectangle $[a_1, b_1] \times [a_2, b_2]$, let $(X, |\cdot|)$, $(Y, |\cdot|)$ be real normed spaces and C be a convex cone of X. For a given function $h : I_a^b \times C \to Y$, the mapping $H : C^{I_a^b} \to Y^{I_a^b}$ defined by

$$(Hf)(t, s) = h(t, s, f(t, s)),\ f \in C^{I_a^b},\ (t, s) \in I_a^b,$$

is called the Nemytskij composition operator of the generator h.

Let $(\Phi BV\left(I_a^b, X\right), \|\cdot\|_\Phi)$ be a Banach space of functions $f \in X^{I_a^b}$ which have bounded Φ-variation in the sense of Schramm, where the norm $\|\cdot\|_\Phi$ is defined with the aid of Luxemburg-Nakano-Orlicz seminorm.

Assume that H maps the set of all functions $f \in \Phi BV\left(I_a^b, X\right)$ such that $f\left(I_a^b\right) \subset C$ into $\Phi BV\left(I_a^b, Y\right)$. The main result of the present paper (Theorem 1) reads as follows: *if H is uniformly bounded, then the left-left regularization of its generator h with respect to the two first variables*

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are affine functions in the third variable. As special cases we obtain the results of [1] and [3], where H is assumed to be uniformly continuous or Lipschitzian, respectively.

2. Preliminaries and auxiliary results

Denote by $\mathbb{R}$, $\mathbb{N}$ the set of real and natural numbers, respectively, and put $\mathbb{R}_+ := [0, \infty)$. Let $\mathcal{F}$ be the set of all increasing convex functions $\varphi : \mathbb{R}_+ \to \mathbb{R}_+$ such that $\varphi(0) = 0$ and $\phi(t) > 0$ for $t > 0$. Let $\Phi = \{\phi_{n,m}\}$ stands for a bidimensional sequence of functions from $\mathcal{F}$ such that for all $n, m, n', m' \in \mathbb{N}$, $t \in \mathbb{R}_+$,

$$\phi_{n',m'}(t) \leq \phi_{n,m}(t) \text{ if } n' \leq n,\ m' \leq m,$$

and

$$\sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \phi_{n,m}(t) \text{ diverges if } t > 0. \tag{1}$$

Let $a = (a_1, a_2) \in \mathbb{R}^2$, $b = (b_1, b_2) \in \mathbb{R}^2$ be such that $a_1 < b_1$ and $a_2 < b_2$ and denote by I_a^b the rectangle generated by the points a and b, i.e. $I_a^b = [a_1, b_1] \times [a_2, b_2]$. Next, assume that $\{I_n\}$ and $\{J_m\}$ are two sequences of closed subintervals of the interval $[a_1, b_1]$ and $[a_2, b_2]$, respectively; i.e. $I_n = [a_1^n, b_1^n] \subset [a_1, b_1]$, $J_m = [a_2^m, b_2^m] \subset [a_2, b_2]$, $b_1^n < a_1^{n+1}$, $b_2^m < a_2^{m+1}$ $n, m \in \mathbb{N}$, and $I_1 := [a_1, b_1]$, $J_1 := [a_2, b_2]$.

For a given function $f : I_a^b \to \mathbb{R}$, a sequence $\Phi = \{\phi_{n,m}\}$ and $x_2 \in J_1$, the quantity V_{Φ,I_1}^S defined by the formula

$$V_{\Phi,I_1}^S(f) = \sup \sum_{n=1}^{\infty} \phi_{n,m}\left(|f(b_1^n, x_2) - f(a_1^n, x_2)|\right),$$

where the supremum is taken over all closed subintervals I_n, $n \in \mathbb{N}$, of the interval $[a_1, b_1]$, is said to be the Φ-*variation in the Schramm sense of the function* $f(\cdot, x_2)$. When $V_{\Phi,I_1}^S(f) < \infty$ we will say that f has *bounded* Φ-*variation in the sense of Schramm with respect to the first variable* (with fixed the second one). In the same way we can define the concept of the Φ-variation of the function $f(x_1, \cdot)$ in the Schramm sense. We denote it by V_{Φ,J_1}^S. If $V_{\Phi,J_1}^S(f) < \infty$ then we say that f *has bounded* Φ-*variation in the sense of Schramm with respect to second variable* (with fixed the first one).

Finally, the quantity $V^S_{\Phi,I^b_a}(f)$ defined by the formula

$$V^S_{\Phi,I^b_a}(f) =$$

$$\sup_{\pi_1,\pi_2} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \phi_{n,m} \left(\left| f\left(a_1^n, a_2^m\right) + f\left(b_1^n, b_2^m\right) - f\left(a_1^n, b_2^m\right) - f\left(b_1^n, a_2^m\right) \right| \right)$$

is said to be *the bidimensional variation in the sense of Schramm of the function* f (the supremum, is considering on all collections of closed and bounded subintervals $\{I_n\}$, $\{J_m\}$ of intervals I_1 and J_1, respectively).

Definition 1. *The quantity* $TV^S_\Phi(f)$ *defined as*

$$TV^S_\Phi(f) = V^S_{\Phi,I_1}(f) + V^S_{\Phi,J_1}(f) + V^S_{\Phi,I^b_a}(f)$$

is said to be the total Φ*-variation in the Schramm sense of the function* f*, and a function* f *is referred to a function with bounded total* Φ*-variation in the Schramm sense if* $TV^S_\Phi(f) < \infty$.

In what follows we denote by $\Phi BV\left(I^b_a, X\right)$ ([8]) the set of all functions $f : I^b_a \to X$ having bounded total Φ-variation in the Schramm sense. It is known ([7], [5]) that the space $\Phi BV\left(I^b_a, X\right)$ endowed with the norm

$$\|f\|_\Phi = |f(a)| + P_\Phi(f),$$

where $P_\Phi : \Phi BV\left(I^b_a, X\right) \to \mathbb{R}_+$ is defined in the following way

$$P_\Phi(f) = \inf \left\{ \epsilon > 0 : TV^S_\Phi\left(\frac{f}{\epsilon}\right) \leq 1 \right\}$$

forms a Banach algebra.

We will need the following:

Lemma 1. ([2]) *Let* $f \in \Phi BV\left(I^b_a; X\right)$ *and* $r > 0$*. Then* $TV^S_\Phi\left(\frac{f}{r}\right) \leq 1$ *if and only if* $P_\Phi(f) \leq r$.

For a given function $f \in \Phi BV\left(I^b_a\right)$, the function $f^* : I^b_a \to X$ defined by the formula

$$f^*(x_1, x_2) = \begin{cases} \lim\limits_{(y_1,y_2)\to(x_1-0,x_2-0)} f(y_1, y_2), & (x_1, x_2) \in (a_1, b_1] \times (a_2, b_2], \\ \lim\limits_{(y_1,y_2)\to(x_1-0,a_2+0)} f(y_1, y_2), & x_1 \in (a_1, b_1] \text{ and } x_2 = a_2, \\ \lim\limits_{(y_1,y_2)\to(a_1+0,x_2-0)} f(y_1, y_2), & x_1 = a_1 \text{ and } x_2 \in (a_2, , b_2], \\ \lim\limits_{(y_1,y_2)\to(a_1+0,a_2+0)} f(y_1, y_2), & x_1 = a_1 \text{ and } x_2 = a_2 \end{cases}$$

is called the *left-left regularization of the function* f. The existence of all one-sided limits used above was proved in [2].

Definition 2. *A function* $f : I_a^b \to \mathbb{R}$ *is said to be left-left continuous if*

$$\lim_{y_1 \to x_1-,\, y_2 \to x_2-} f(y_1, y_2) = f(x_1, x_2) \quad \textit{for all} \quad (x_1, x_2) \in (a_1, b_1] \times (a_2, b_2].$$

From now on, we denote by $\Phi BV^*\big(I_a^b, X\big)$ the subspace of $\Phi BV\left(I_a^b, X\right)$ of those functions which are left-left continuous on $(a_1, b_1] \times (a_2, b_2]$ and by $\mathcal{L}(X, Y) = \big\{f \in Y^X : \ f \text{ is linear}\big\}$.

Lemma 2. ([2]) *If* $f \in \Phi BV\left(I_a^b, X\right)$, *then* $f^* \in \Phi BV^*\left(I_a^b, X\right)$.

The following result may be proved in much the same way as Theorem 1 of [1].

Proposition 1. *Let* $I_a^b \subset R^2$ *be a rectangle,* $(X, |\cdot|_X)$ *be a real normed space,* $(Y, |\cdot|_Y)$ *be a real Banach space,* C *be a convex cone in* X *and assume* Φ_1, Φ_2 *are two bidimensional sequences of functions from* $\mathcal{F}$. *Suppose that a function* $h : I_a^b \times C \to Y$ *is such that, for any* $(t, s) \in I_a^b$ *the function* $h(t, s, \cdot) : C \to Y$ *is continuous with respect to the third variable. If composition operator* H *generated by* h *maps* $\Phi_1 BV(I_a^b, C)$ *into* $\Phi_2 BV\left(I_a^b, Y\right)$ *and satisfies the inequality*

$$\|H(f_1) - H(f_2)\|_{\Phi_2} \le \gamma\big(\|f_1 - f_2\|_{\Phi_1}\big), \quad \textit{if} \quad f_1, f_2 \in \Phi_1 BV\left(I_a^b, C\right), \quad (2)$$

for some function $\gamma : [0, \infty) \to [0, \infty)$, *then there exist two functions* $A : I_a^b \to \mathcal{L}(C, Y)$ *and* $B \in \Phi BV^*\left(I_a^b, Y\right)$ *such that*

$$h^*(t, s, y) = A(t, s)y + B(t, s), \quad (t, s) \in I_a^b, \quad y \in C,$$

where h^* *is the left-left regularization of* h.

Proof. For every $y \in C$, the constant function $f(t, s) = y$ with $(t, s) \in I_a^b$ belongs to $\Phi_1 BV\left(I_a^b, C\right)$. Since H maps $\Phi_1 BV\left(I_a^b, C\right)$ into $\Phi_2 BV\left(I_a^b, Y\right)$, it follows that function $(t, s) \mapsto h(t, s, y)$ $\big((t, s) \in I_a^b\big)$ belongs to $\Phi_2 BV\left(I_a^b, Y\right)$. Now, by Lemma 2, we get the existence of the left-left regularization h^* of h.

By assumption, H satisfies inequality (2) and by the definition of the norm $|\cdot||_{\Phi_2}$, we have

$$P_{\Phi_2}\big(H(f_1) - H(f_2)\big) \le \|H(f_1) - H(f_2)\|_{\Phi_2}, \text{ if } f_1, f_2 \in \Phi_1 BV\big(I_a^b, C\big). \quad (3)$$

Taking into account (3), Lemma 1 and $\gamma\big(\|f_1 - f_2\|_{\Phi}\big) > 0$, we get

$$V^S_{\Phi_2}\left(\frac{(H(f_1) - H(f_2))(\cdot, a_2)}{\gamma\big(\|f_1 - f_2\|_{\Phi_1}\big)}\right) \le TV^S_{\Phi_2}\left(\frac{Hf_1 - Hf_2}{\gamma\Big(\|f_1 - f_2\|_{\Phi_1}\Big)}\right) \le 1. \tag{4}$$

Therefore, for any $a_1 \le \alpha_1 < \beta_1 < \alpha_2 < \beta_2 < \ldots < \alpha_n < \beta_n \le b_1$, $a_2 \le \overline{\alpha}_1 < \overline{\beta}_1 < \overline{\alpha}_2 < \overline{\beta}_2 < \ldots < \overline{\alpha}_m < \overline{\beta}_m \le b_2$, $m, n \in \mathbb{N}$, the definitions of the operator H and the functional $V^S_{\Phi_2}$, we have

$$\sum_{i=1}^{n}\sum_{j=1}^{m}\phi_{i,j}\left(\frac{|h(\alpha_i,\overline{\alpha}_j,f_1(\alpha_i,\overline{\alpha}_j))-h(\alpha_i,\overline{\alpha}_j,f_2(\alpha_i,\overline{\alpha}_j))-h(\alpha_i,\overline{\beta}_j,f_1(\alpha_i,\overline{\beta}_j))+h(\alpha_i,\overline{\beta}_j,f_2(\alpha_i,\overline{\beta}_j))}{\gamma\big(\|f_1-f_2\|_{\Phi_1}\big)}\right.$$
$$\left.+\frac{-h(\beta_i,\overline{\alpha}_j,f_1(\beta_i,\overline{\alpha}_j))+h(\beta_i,\overline{\alpha}_j,f_2(\beta_i,\overline{\alpha}_j))+h(\beta_i,\overline{\beta}_j,f_1(\beta_i,\overline{\beta}_j))-h(\beta_i,\overline{\beta}_j,f_2(\beta_i,\overline{\beta}_j))|}{\gamma(\|f_1-f_2\|_{\Phi_1})}\right) \le 1. \tag{5}$$

For $\alpha, \beta \in \mathbb{R},\ \alpha < \beta$, we define functions $\eta_{\alpha,\beta} : \mathbb{R} \to [0, 1]$ by

$$\eta_{\alpha,\beta}(t) := \begin{cases} 0 & \text{if} \quad t \le \alpha \\ \dfrac{t-\alpha}{\beta-\alpha} & \text{if} \quad \alpha \le t \le \beta \\ 1 & \text{if} \quad \beta \le t\,. \end{cases}$$

Take $t \in (a_1, b_1]$, $s \in (a_2, b_2]$, $n, m \in \mathbb{N}$. For arbitrary finite sequences

$$a_1 \le \alpha_1 < \beta_1 < \alpha_2 < \beta_2 < \ldots < \alpha_n < \beta_n \le t$$

$$a_2 \le \overline{\alpha}_1 < \overline{\beta}_1 < \overline{\alpha}_2 < \overline{\beta}_2 < \ldots < \overline{\alpha}_m < \overline{\beta}_m \le s$$

and $y_1, y_2 \in C$, $y_1 \ne y_2$, the functions $f_1, f_2 : I^b_a \to X$ defined by

$$f_\ell(\tau, \gamma) := \frac{1}{2}\Big[(\eta_{\alpha_i,\beta_i}(\tau) + \eta_{\overline{\alpha}_j,\overline{\beta}_j}(\gamma) - 1)(y_1 - y_2) + y_\ell + y_2\Big], \tag{6}$$

for every $(\tau, \gamma) \in I^b_a,\ \ell = 1, 2$; belong to the space $\Phi_1 BV\left(I^b_a, C\right)$. From (6), we have

$$f_1(\cdot, \cdot) - f_2(\cdot, \cdot) = \frac{y_1 - y_2}{2},$$

therefore

$$\|f_1 - f_2\|_{\Phi} = \left|\frac{y_1 - y_2}{2}\right|;$$

moreover
$f_1(\alpha_i, \overline{\alpha}_j) = y_2;\ f_2(\alpha_i, \overline{\alpha}_j) = \frac{-y_1+3y_2}{2};\ f_1(\alpha_i, \overline{\beta}_j) = \frac{y_1+y_2}{2};\ \ f_2(\alpha_i, \overline{\beta}_j) = y_2,$
$f_1(\beta_i, \overline{\alpha}_j) = y_2;\ f_2(\beta_i, \overline{\alpha}_j) = \frac{-y_1+3y_2}{2};\ f_1(\beta_i, \overline{\beta}_j) = y_1;\ \ f_2(\beta_i, \overline{\beta}_j) = \frac{y_1+y_2}{2}.$
Applying (5), we hence get

$$\sum_{i=1}^{n}\sum_{j=1}^{m}\phi_{i,j}\left(\frac{\left|h(\alpha_i,\overline{\alpha}_j,y_2)-h\left(\alpha_i,\overline{\alpha}_j,\frac{-y_1+3y_2}{2}\right)-h\left(\alpha_i,\overline{\beta}_j,\frac{y_1+y_2}{2}\right)+h(\alpha_i,\overline{\beta}_j,y_2)\right.}{\gamma\left(\|f_1-f_2\|_{\Phi_1}\right)}\right.$$
$$\left.+\frac{\left.-h(\beta_i,\overline{\alpha}_j,y_2)+h\left(\beta_i,\overline{\alpha}_j,\frac{-y_1+3y_2}{2}\right)+h(\beta_i,\overline{\beta}_j,y_1)-h\left(\beta_i,\overline{\beta}_j\,\frac{y_1+y_2}{2}\right)\right|}{\gamma(\|f_1-f_2\|_{\Phi_1})}\right)\leq 1. \tag{7}$$

From the continuity of $\phi_{i,j}$, and the left-left continuity of h^* passing to the limit in (7) when $\alpha_1\uparrow t$ and $\overline{\alpha}_j\uparrow s$, we obtain:

$$\sum_{i=1}^{n}\sum_{j=1}^{m}\phi_{i,j}(x)\leq 1 \text{ if } n,m=1,2,\ldots, \tag{8}$$

where

$$x=\frac{|h^*(t,s,y_1)-2h^*\left(t,s,\frac{y_1+y_2}{2}\right)+h^*(t,s,y_2)|}{\gamma\left(|\frac{y_1-y_2}{2}|\right)}.$$

Since $n,m\in\mathbb{N}$ are arbitrary, condition (8) implies that

$$\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\phi_{i,j}(x)\leq 1$$

and, by (1), $x=0$, i.e.,

$$h^*\Big(t,s,\frac{y_1+y_2}{2}\Big)=\frac{h^*(t,s,y_1)+h^*(t,s,y_2)}{2}, \tag{9}$$

for all $(t,s)\in(a_1,b_1]\times(a_2,b_2]$ and $y_1,\ y_2\in C$. For $t\in(a_1,b_1]$ and $s=b_2$, let us fix $a_1<\alpha_1<\beta_1<\alpha_2<\beta_2<\cdots<\alpha_n<\beta_n<t$ and $a_2<\overline{\alpha}_1<\overline{\beta}_1<\overline{\alpha}_2<\overline{\beta}_2<\cdots<\overline{\alpha}_m<\overline{\beta}_m<b_2$. Proceeding as above we get (7). Taking the limit when $\alpha_1\uparrow t$ and $\beta_m\downarrow s$ in (7), we get, again (9). The cases when $t=a_1$ and $s\in(a_2,b_2]$ or $t=a_1$ and $s=a_2$ can be treated similarly. Consequently

$$h^*\Big(t,s,\frac{y_1+y_2}{2}\Big)=\frac{h^*(t,s,y_1)+h^*(t,s,y_2)}{2}$$

is valid for all $(t,s)\in I_a^b$ and all $y_1,y_2\in C$.

Therefore, the function $h^*(t,s,\cdot)$ satisfies the Jensen functional equation in C for $(t,s)\in I_a^b$. Modifying a little the standard argument of Kuczma [4], we conclude that, for each $(t,s)\in I_a^b$, there exists an additive function $A(t,s):C\to Y$ and $B(t,s)\in Y$ such that

$$h^*(\cdot,y)=A(\cdot,\cdot)y+B(\cdot,\cdot),\quad y\in C. \tag{10}$$

The continuity of h with respect to the third variable implies continuity of function $A(t,s)$. Consequently $A(t,s) \in \mathcal{L}(X,Y)$. Finally, notice that $A(t,s)(0) = \{0\}$ for every $(t,s) \in I_a^b$. Therefore, putting $y = 0$ in (10), we get

$$h^*(t,s,0) = B(t,s), \quad (t,s) \in I_a^b,$$

which implies that $B \in \Phi_2 BV^* \left(I_a^b, Y\right)$. □

3. Bounded composition operator

In this section the crucial role plays the following

Definition 3. ([6]). *Let Y and Z be two metric (or normed) spaces. We say that a mapping $H : Y \to Z$ is uniformly bounded if for any $t > 0$ there is a nonnegative real number $\gamma(t) \geq 0$ such that for any set $B \subset Y$ we have*

$$\operatorname{diam} B \leq t \Rightarrow \operatorname{diam} H(B) \leq \gamma(t).$$

Remark 1. Obviously, every uniformly continuous operator or Lipschitzian operator is uniformly bounded. Note that, under the assumptions of this definition, every bounded operator is uniformly bounded and the converse is not true.

The main result of this paper reads as follows

Theorem 1. *Let $I_a^b \subset R^2$ be a rectangle, $(X, |\cdot|_X)$ be a real normed space, $(Y, |\cdot|_Y)$ be a real Banach space, C be a convex cone in X and assume that Φ_1, Φ_2 are two bidimensional sequences of functions from $\mathcal{F}$. If the composition operator H generated by $h : I_a^b \times C \to Y$ maps $\Phi_1 BV \left(I_a^b, C\right)$ into $\Phi_2 BV(I_a^b, Y)$ and is uniformly bounded, then there exist two functions $A : I_a^b \to L(X,Y)$ and $B \in \Phi BV^* \left(I_a^b, Y\right)$ such that*

$$h^*(t,s,y) = A(t,s)y + B(t,s), \quad (t,s) \in I_a^b, \quad y \in C,$$

where h^* is the left-left regularization of h.

Proof. Take any $t \geq 0$ and $f_1, f_2 \in \Phi_1 BV(I_a^b; C)$ such that $||f_1 - f_2||_{\Phi_1} \leq t$. Since $\operatorname{diam}\{f_1, f_2\} \leq t$, by the uniform boundedness of H, we have

$$\operatorname{diam} H\left(\{f_1, f_2\}\right) \leq \gamma(t),$$

that is,

$$||H(f_1) - H(f_2)||_{\Phi_2} = \operatorname{diam} H(\{f_1, f_2\}) \leq \gamma(||f_1 - f_2||_{\Phi_1}),$$

and the result follows from Proposition 1. $\square$

Remark 2. The counterpart of Proposition 1 and Theorem1 for the right-right, right-left and left-right regularizations of h are also valid.

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