

DOI 10.21008/j.1897-0737.2019.97.0018

Marcin SOWA<sup>\*</sup>, Agnieszka JAKUBOWSKA-CISZEK<sup>\*</sup>

## DAQ-BASED MEASUREMENTS AND STUDY OF SUPERCAPACITOR FREQUENCY CHARACTERISTICS

The paper concerns the beginning of a modeling study for supercapacitors. A data acquisition (DAQ) based system is presented, where an automated procedure has been implemented for the measurement of frequency characteristics. For a typical range of the supply voltage the characteristics are obtained. A model basing on fractional calculus is recalled and parameters for the model are obtained. The frequency characteristics of the model are compared with those obtained from measurements. Later the tested supercapacitor has its characteristics taken for various amplitudes and offsets of the source voltage. A few remarks are given for a possible expansion of the model when nonlinearity should be considered.

**KEYWORDS:** supercapacitor, frequency characteristics, data acquisition device, measurement card, fractional derivative.

### 1. INTRODUCTION

The study of supercapacitors is important due to their role in high energy density storage systems [1]. They also find applications in hybrid electric vehicles [2]. Their presence in circuits requires simulations including models that can closely recreate their behavior. For various types of analyses with complex circuit elements and nonstandard source waveforms it is useful to have models that are both applicable in steady state analyses and in simulations of dynamics. There are various approaches in the modeling of supercapacitors, where charging and discharging waveforms are studied [3] or frequency characteristics [4], or sometimes both in order to allow a model to accurately resemble various dynamics. This paper is a part of a larger project aiming at the modeling of supercapacitors. This particular study focuses on obtaining the frequency characteristics of a supercapacitor for various offsets and amplitudes of the supply voltage. Initial remarks for modeling using fractional derivatives are also made along with a discussion on possible nonlinearities that could be modeled.

---

<sup>\*</sup> Silesian University of Technology

Fractional derivatives are a part of a mathematics branch called fractional calculus. There are various definitions of the fractional derivative [5], where in this paper only the Caputo definition [6] is applied:

$${}_0D_t^\alpha x(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{x^{(1)}(\tau)}{(t-\tau)^\alpha} d\tau. \quad (1)$$

In the equation above  $x^{(1)}(t)$  is the first derivative of the variable  $x(t)$ ,  $\alpha$  is the order of the derivative (where only  $\alpha \in [0, 1]$  is considered) and  $\Gamma(\cdot)$  is the gamma function:

$$\Gamma(z) = \int_0^\infty x^{z-1} e^{-x} dx. \quad (2)$$

Models using fractional derivatives have been applied with much success recently not only for supercapacitors [7–9] but also for coils with ferromagnetic cores [10–13].

## 2. MEASUREMENT SETUP

### 2.1. Considerations for voltage operating range

There are a few factors that need to be taken into account when considering the measurement of frequency characteristics for supercapacitors. One of them is the operating range of the input voltage. Because of their composition – they should operate below their maximum voltage (e.g. 2.7 V [14] or 5.5 V [15]). They should also not be charged with reverse polarity [16, 17], i.e. the user should connect the supercapacitor according to its appropriate marking.

Another factor is the frequency range. For the purpose of this paper the frequency range [0.1, 500] Hz has been selected (reflecting components of slow responses up to those where frequency characteristics still change with some significance). All measurements have been performed in a room temperature – in this paper temperature dependencies are not studied.

### 2.2. The setup

The setup assembled for the frequency characteristics is depicted in Fig. 1, where:

- the voltage source is an arbitrary function generator (AFG) – in the study the GW Instek AFG-2105 [18] has been applied,
- the AFG output waveforms are constructed according to commands sent from a PC through the USB interface,
- the DAQ (data acquisition) device consisted of a NI-9239 voltage input module [19], which was mounted on the cDAQ-9174 chassis [20],

- the measurements have been gathered through the application of the DAQmx C# library [21],
- a resistor with known resistance  $R$  is connected in series (for current measurements), where in this study  $R = 10.2 \Omega$ .

The whole process of obtaining the frequency characteristics is controlled through an algorithm implemented in an original program written in C#. For each selected frequency from a desired range – the algorithm depicted in Fig. 2 is used (it is similar to the one used in [13] for coils with ferromagnetic cores).

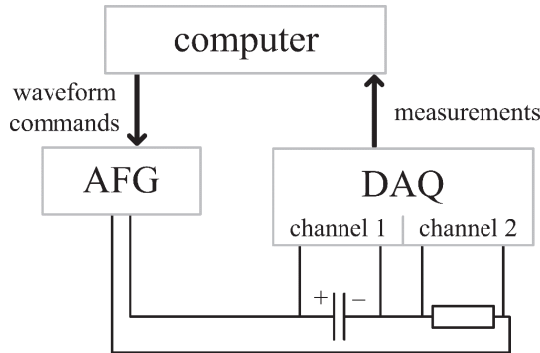


Fig. 1. Measurement setup for obtaining frequency characteristics of the supercapacitor

The block highlighted in gray in Fig. 2 is realized through the following steps (also depicted in Fig. 3):

- from the function  $f(t)$  (the voltage or current waveform – actually built from measured samples) the first time harmonic parameters  $A_1$  and  $\phi_1$  are determined along with the constant term  $A_0$ ,
- the following waveform is constructed (also actually it is built from time samples)

$$g(t) = A_0 + A_1 \sin(\omega t + \phi_1), \quad (3)$$

so that the expected form of the function in the steady state is produced,

- the difference between  $f(t)$  and  $g(t)$  is computed – this new waveform is called  $h(t)$  (obviously – it is also constructed from time samples),
- the following integral is computed numerically:

$$H = \int_0^T |h(t)| dt, \quad (4)$$

- to determine if the steady state has been reached, the following criterion is checked:

$$\frac{H}{A_1 T} \cdot 100\% \leq e_{\text{tol}}, \quad (5)$$

where  $e_{\text{tol}}$  is a given tolerance value (in percentage) for the steady state.

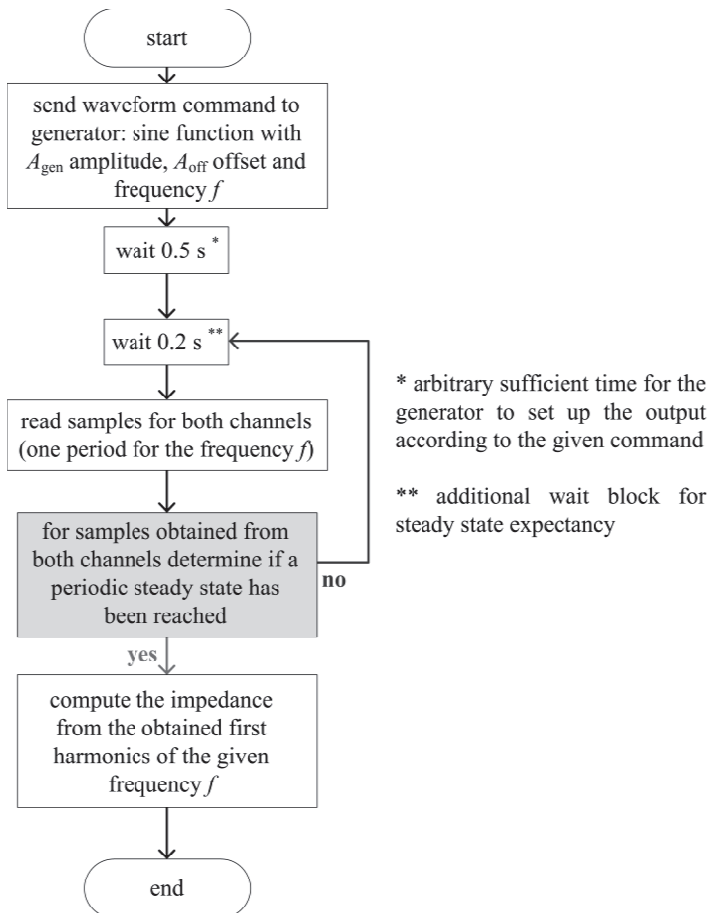


Fig. 2. Simple algorithm for obtaining the impedance of the supercapacitor for a selected frequency

### 3. INITIALLY ACQUIRED CHARACTERISTICS

The test object in this study has been a simple supercapacitor [22] with the nominal capacitance 0.22 F and nominal voltage 5.5 V.

The characteristics for various source voltage outputs have been obtained. First the characteristics for a 2.75 V offset and 2.75 V amplitude on the AFG output are observed. This is an appropriate range when referring to the operating voltage of the supercapacitor; however, because of other elements in the circuit – the supercapacitor will operate in a different voltage range (i.e. smaller values). The characteristics are presented in Fig. 4.

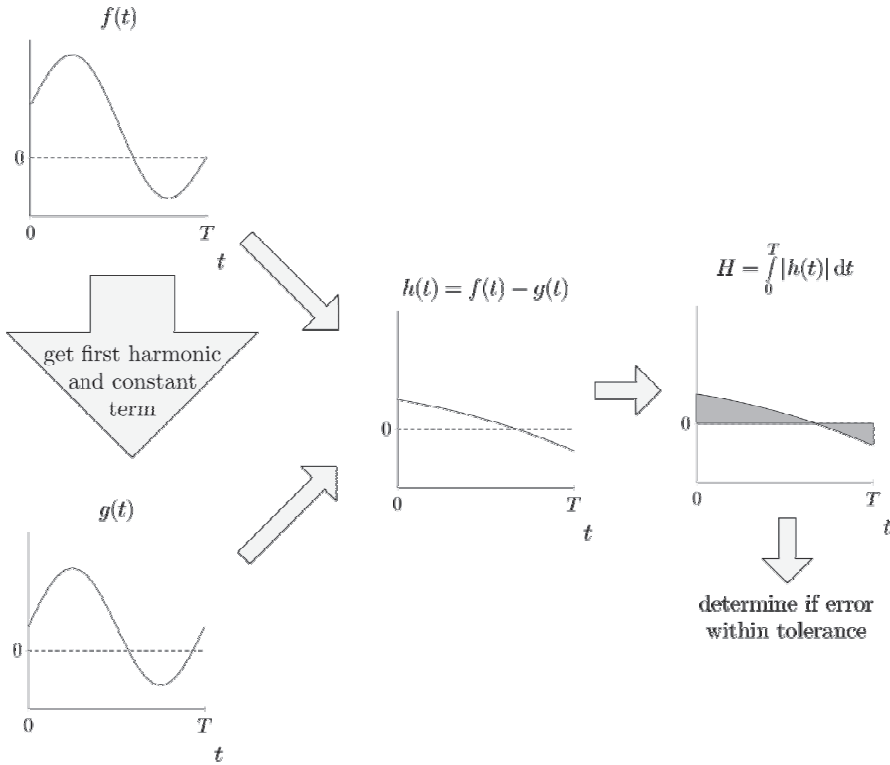


Fig. 3. Applied process of determining the steady state

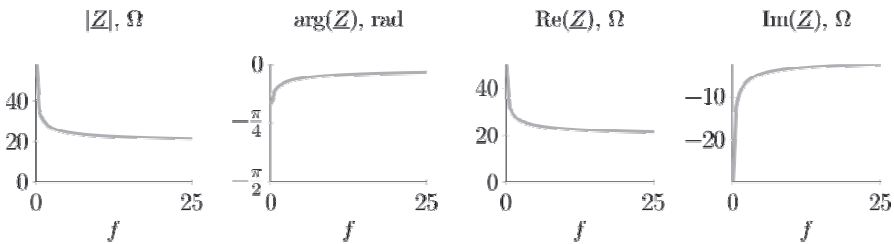


Fig. 4. Frequency characteristics of the studied supercapacitor (only frequencies up to  $f=25$  Hz are shown as for higher frequencies the characteristics do not change significantly)

The commonly applied model [9] for supercapacitors is recalled (Fig. 5). The fractional capacitor is described by the equation:

$$C_{\alpha} D_t^{\alpha} u(t) = i(t). \tag{6}$$

The impedance for the fractional capacitor is then described by:

$$Z_{\alpha} = \frac{1}{j_{\alpha} \omega^{\alpha} C_{\alpha}}, \tag{7}$$

where:

$$j_\alpha = \exp\left(j\alpha \frac{\pi}{2}\right). \quad (8)$$

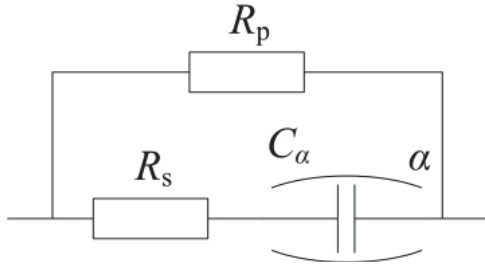


Fig. 5. Studied supercapacitor model (the fractional capacitor is depicted as a capacitor in curly brackets along with the order of the element – a symbol introduced first in [23]).

An estimation procedure has been completed in order to obtain the parameters of the model. The parameter estimation has been done through an original program written in C#, applying the Accord.NET Framework [24] where the COBYLA [25] method has been applied for constrained optimization. The obtained parameters are (values rounded to four significant digits):

$$R_p = 672.6\Omega,$$

$$R_s = 18.85\Omega,$$

$$\alpha = 0.4447,$$

$$C_\alpha = 26.77\text{mF} \cdot \text{s}^{\alpha-1}.$$

A comparison between the measurement results and the frequency response of the model is depicted in Fig. 6 (for two different frequency ranges for better readability).

A very good reflection of the measured characteristics has been observed in the model response.

#### 4. CHARACTERISTICS FOR VARIOUS AFG OUTPUTS

In the previous section the frequency characteristics have been obtained for the same amplitude and offset of the AFG for each frequency. This approach is valid if the model is linear and the object is also assumed to mostly exhibit linear behavior. This section puts in question if the linearity can be assumed and if not then what are the conditions for the model to be valid.

No evident differences can be observed for various amplitudes when comparing the characteristics for common source offsets.

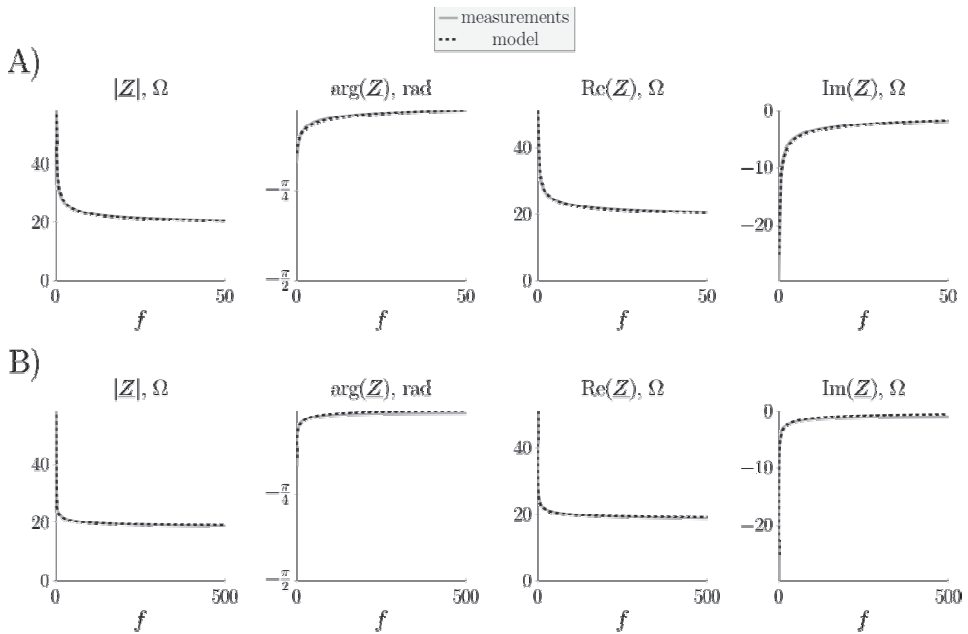


Fig. 6. Comparison of frequency characteristics: A) frequency up to 50 Hz, B) frequency range up to 500 Hz

Figure 8 presents a set of frequency characteristics, where for various offsets (1.1 V, 2.2 V, 3.3 V and 4.5 V) a comparison is performed for different amplitudes (0.1 V, 0.5 V and 1 V). In the figure  $-A_{\text{off}}$  is the offset and  $A_1$  is the amplitude of the sine wave.

A different observation can be made when looking at the characteristics while they are presented for common amplitudes and the cases for different offsets are compared (Fig. 8).

A slight difference is observed in the characteristics for every amplitude. This, however, only concerns the lowest offset, i.e. 1.1 V. This suggests that the supercapacitor has a response close to a linear one for greater voltages, while in fact having a slight nonlinearity. For a model attempting to capture behaviors for the typical voltage range the supercapacitor was designed for (i.e. up to 5.5 V) a linear model is sufficient enough and can be applied reliably. For a more accurate model (when the voltage is at lower levels) one could consider adding a slight nonlinearity. Note that this would be for very accurate and demanding simulation tasks since the lower voltage characteristic does not vary by that much (only the real part, which one can notice in the modulus, while the differences in the argument and imaginary part are barely visible).

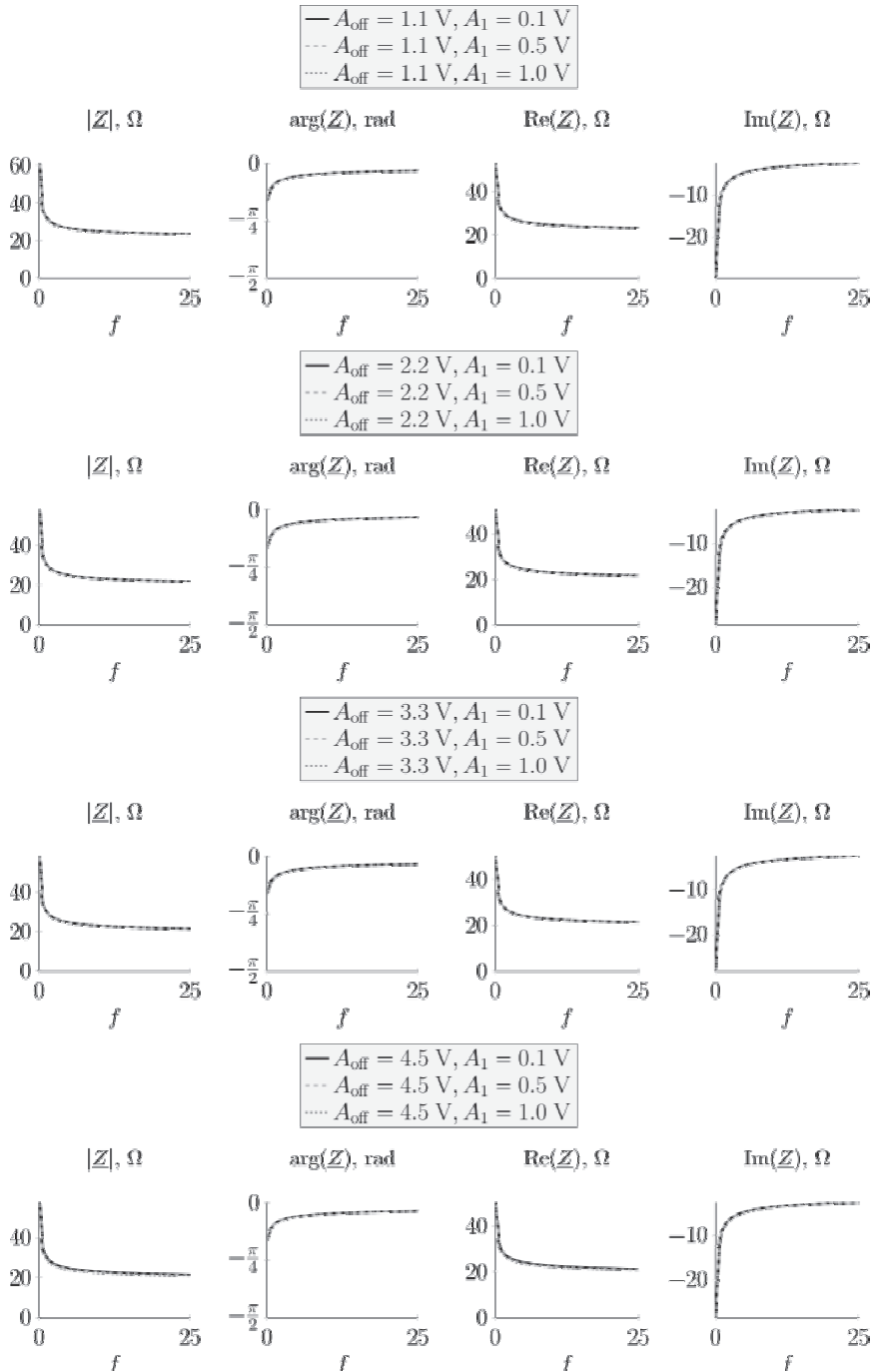


Fig. 7. Frequency characteristics for different offsets and comparisons for various amplitudes (characteristics overlap)



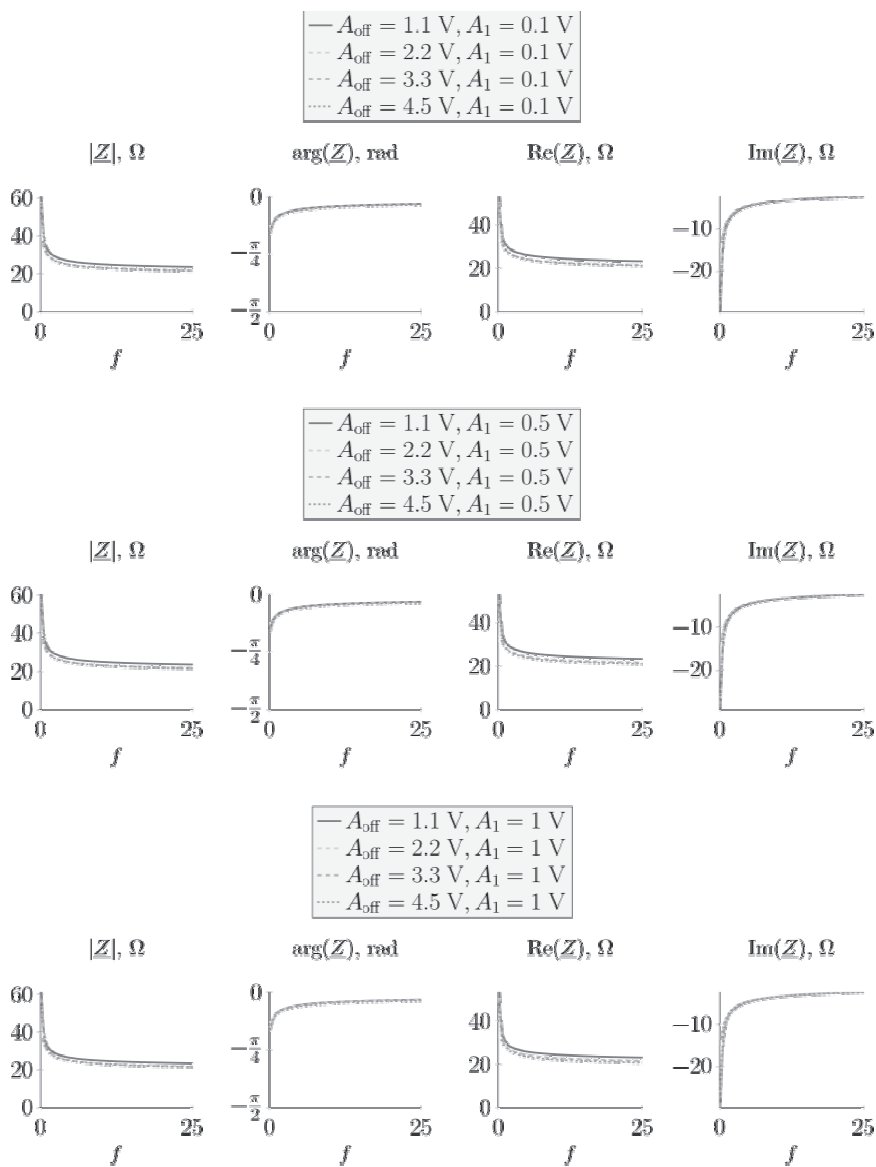


Fig. 8. Frequency characteristics for different amplitudes and comparisons for various offsets (all characteristics except for  $A_{\text{off}} = 1.1 \text{ V}$  are overlapping)

When considering a nonlinear model the nonlinear resistance should be tested first as it does not introduce elements, which are tougher to handle in computations. However, when that is not enough then one could consider adding nonlinearity to the fractional capacitor. This would introduce an element, which is not only fractional but also nonlinear – it would be described by the differential equation:

$${}_0D_t^\alpha q(t) = i(t). \quad (9)$$

In the above equation  $q$  is not the charge but an artificial variable with the unit  $C \cdot s^{\alpha-1}$ . As is the case with the fractional (linear) capacitor itself – the element should be considered as a means to phenomenologically capture certain features of real objects and not as an element, which can be derived from physics principles. The nonlinear equation of the element would then be either a  $q(u)$  function or a  $u(q)$  function. The modeling attempts including nonlinear elements are planned for future studies.

## 5. CONCLUSIONS

A DAQ-based measurement system has been introduced and applied on a tested supercapacitor in order to obtain its frequency characteristics. For a typical source voltage range the modeling of the fractional capacitor has been performed, where the fractional-order model very accurately allows to resemble the frequency characteristics of the real object.

The frequency characteristics for various source voltage amplitudes and offsets have also been compared. It has been observed that for source voltages around 2 V and above the linear model can very accurately describe the behavior of the supercapacitor. For lower voltages the frequency characteristics are slightly different and a nonlinear model could be considered in the future.

## REFERENCES

- [1] González A., Goikolea E., Barrena J.A., Mysyk R., Review on supercapacitors: Technologies and materials. *Renewable and Sustainable Energy Reviews*, Volume 58, 2016, pp. 1189–1206.
- [2] Chu A., Braatz P., Comparison of commercial supercapacitors and high-power lithium-ion batteries for power-assist applications in hybrid electric vehicles: I. Initial characterization, Volume 112, Issue 1, pp. 236–246.
- [2] Pinkiewicz I., Kaźmierski M., Olech W., Malinowski J., Sopoćki R., On-site Processing of Insulation System of Large Power Transformers and Hot-spot Computer Determination, CIGRE, Session 2004, A2-208.
- [3] Kumar M.R., Ghosh S., Das S., Charge-discharge energy efficiency analysis of ultracapacitor with fractional-order dynamics using hybrid optimization and its experimental validation, *International Journal of Electronics and Communications (AEÜ)*, Volume 78, 2017, pp. 274–280.
- [4] Zhang L., Hu X., Wang Z., Sun F., Dorrell D.G., Fractional-order modeling and State-of-Charge estimation for ultracapacitors, *Journal of Power Sources*, Volume 314, 2016, pp. 28–34.
- [5] de Oliveira E.C., Machado J.A.T., A Review of Definitions for Fractional Derivatives and Integral, *Mathematical Problems in Engineering*, Volume 2014, ID 238459, 6 pages, 2014.

- 
- [6] Caputo M., Linear models of dissipation whose Q is almost frequency independent – II, *Geophysical Journal International*, Volume 13, Number 5, 1967, pp. 529–539.
- [7] Allagui A., Freeborn T.J., Elwakil A.S., Fouda M.E., Maundy B.J., Radwan A.G., Said Z., Abdelkareem M.A., Review of fractional-order electrical characterization of supercapacitors, *Journal of Power Sources* 400, 2018, pp. 457–467.
- [8] Freeborn T.J., Maundy B., Elwakil A.S., Fractional-order models of supercapacitors, batteries and fuel cells: a survey, *Mater Renew Sustain Energy*, 2015, 4:9, 7 pages.
- [9] Mitkowski W., Skruch P., Fractional-order models of the supercapacitors in the form of RC ladder networks, *Bull. Pol. Ac.: Tech.*, Volume 61, Number 3, 2013, pp. 581–587.
- [10] Majka Ł., Applying a fractional coil model for power system ferroresonance analysis, *Bull. Pol. Ac.: Tech.*, Volume 66, Number 4, pp. 467–474.
- [11] Majka Ł., Fractional derivative approach in modeling of a nonlinear coil for ferroresonance analyses, *Non-integer order calculus and its applications*, Springer, 2019, pp. 135–147.
- [12] Schäfer I., Krüger K., Modelling of lossy coils using fractional derivatives, *Journal of Physics D: Applied Physics*, Volume 41, Number 4, 2008, pp. 1–8.
- [13] Sowa M., DAQ-based measurements for ferromagnetic coil modeling using fractional derivatives, 2018 International Interdisciplinary PhD Workshop (IIPhDW), Świnoujście, 2018, pp. 91–95.
- [14] Chen Z., Augustyn V., Wen J., Zhang Y., Shen M., Dunn B., Lu Y., High-Performance Supercapacitors Based on Intertwined CNT/V<sub>2</sub>O<sub>5</sub> Nanowire Nanocomposites, *Advanced Materials*, Volume 23, Number 6, 2011, pp. 791–795.
- [15] Hu A.P., You Y.W., Chen F.-Y.B., McCormick D., Budgett D.M., Wireless Power Supply for ICP Devices With Hybrid Supercapacitor and Battery Storage, *IEEE Journal of Emerging and Selected Topics in Power Electronics*, Volume 4, Number 1, March 2016, pp. 273–279.
- [16] <https://www.murata.com/support/faqs/products/capacitor/edlc/conf/0002>.
- [17] <http://www.cooperindustries.com/content/dam/public/bussmann/Electronics/Resources/product-datasheets/bus-elx-ds-10510-xlr-supercapacitor-module.pdf>. Resources
- [18] [http://www.gwinstek.com/en-global/products/Signal\\_Sources/Arbitrary\\_Function\\_Generators/AFG-2100\\_AFG-2000](http://www.gwinstek.com/en-global/products/Signal_Sources/Arbitrary_Function_Generators/AFG-2100_AFG-2000).
- [19] <http://www.ni.com/pl-pl/support/model.ni-9239.html>.
- [20] <http://www.ni.com/pl-pl/support/model.cdaq-9174.html>.
- [21] <http://www.ni.com/product-documentation/54391/en/>.
- [22] <http://www.bigcap.net/en/article/389.html>.
- [23] Sowa M., A Harmonic Balance Methodology for Circuits with Fractional and Nonlinear Elements, *Circuits, Systems and Signal Processing*, Volume 37, Number 11, 2018, pp. 4695–4727.
- [24] <http://accord-framework.net/index.html>.
- [25] Powell M.J.D., A view of algorithms for optimization without derivatives. Cambridge University Technical Report DAMTP 2007.

*(Received: 07.02.2019, revised: 08.03.2019)*

