Scientific Journal of Silesian University of Technology. Series Transport
Zeszyty Naukowe Politechniki Śląskiej. Seria Transport

Volume 106 **2020**

p-ISSN: 0209-3324

e-ISSN: 2450-1549

DOI: https://doi.org/10.20858/sjsutst.2020.106.2

Journal homepage: http://sjsutst.polsl.pl

Article citation information:
Constantinescu, C.G., Şandru, V.I., Strimbu, C. Comparative analysis regarding surface-to-air missiles guidance in terms of using the three points and proportional approach methods. *Scientific Journal of Silesian University of Technology. Series Transport*. 2020, **106**, 29-39. ISSN: 0209-3324. DOI: https://doi.org/10.20858/sjsutst.2020.106.2.

Cristian George CONSTANTINESCU[1], Vasile Ioan ŞANDRU[2], Constantin STRIMBU[3]

COMPARATIVE ANALYSIS REGARDING SURFACE-TO-AIR MISSILES GUIDANCE IN TERMS OF USING THE THREE POINTS AND PROPORTIONAL APPROACH METHODS

Summary. Surface-Based Air Defence systems are the main actions on the ground means of response in fighting with aerial threats. Effectiveness of response is given by its basic elements as defended area size, type and number of missiles systems or command and control and logistical support. Guiding method means a well-defined law ruling the missile trajectory near the target, depending on target coordinates and motion parameters, to ensure a successful mission. The influence of target routing law on the missile guidance should not be confused with some guidance methods. Therefore, for missile guidance, it is enough to determine (regardless of the guidance method), the target motion equations related only to its coordinate elevation angle, kinematics, azimuth, and weather conditions.

Keywords: surface-based air defence systems, guidance methods by three points, proportional approach

---

[1] "Henri Coandă" Air Force Academy, Mihai Viteazul 160 Street, Braşov, Romania.
Email: ccg_cristi@yahoo.com
[2] "Henri Coandă" Air Force Academy, Mihai Viteazul 160 Street, Braşov, Romania.
Email: svasile1966@yahoo.com
[3] "Henri Coandă" Air Force Academy, Mihai Viteazul 160 Street, Braşov, Romania.
Email: strimbu_c@yahoo.com

1. INTRODUCTION

The direct guidance method by three points (T.P.M.) states that missile approaching to target law is that which keeps throughout the guidance process, the orientation of the missile's longitudinal axis to the target's current position, so that the points O_D, O_R, O_T are collinear. This law is simplified and shown in Figure 1. The guidance method by three points is called the "after radius" method or the method of "target covering" in technical literature [1].

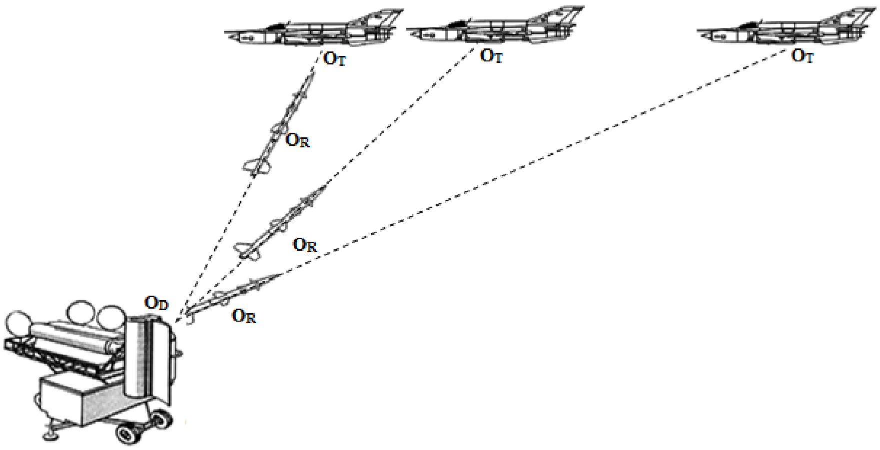

Fig. 1. The O_D, O_R and O_T points collinearity, the main feature of the (direct) three points guidance method

In the missile guidance process using the proportional approach method (P.A.M.), two mobile points are involved, the missile and the target. This method assumes keeping constant the ratio between the missile angular velocity (ω_R) and the missile-target line angular velocity (ω_{RT}). The principle of this guidance method is synthetically outlined in Figure 2. The proportional approach method may be also called the proportional navigation method [1].

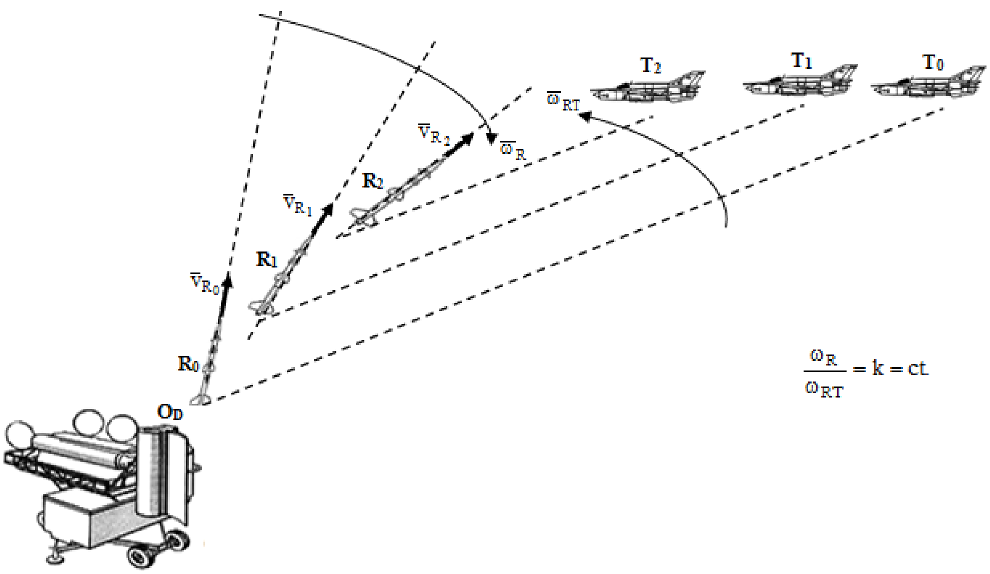

Fig. 2. Constancy of the ratio $= \frac{\omega_R}{\omega_{RT}}$, characteristic of the proportional approach guidance method

It may be noticed that the particular case $k = 1$ is the one previously discussed, that is, the three points (collinearity of) method. Thus, it comes naturally to the conclusion that there are no more missile guidance methods, but only a set of possible values for k – the proportionality constant.

2. THE GRAPHICAL CONSTRUCTION OF THE MISSILE'S KINEMATIC TRAJECTORY

2.1. The three points guidance method

The R_0 and T_0 points marks respectively, the missile and target initial positions (in the moment when the controlled guidance starts). On the target's path represented by the L_T line, there are drawn the upcoming positions of the target T_0, ...,T_i, T_{i+1}, ... which link the R_0, ..., R_i, R_{i+1}, ... missile trajectory points. According to the T.P.M substance, the missile must continuously be on the target's sightline, namely in the t_1, t_2, t_3, ..., t_i moments it is necessary that its centre of gravity to be on the R_0T_1, R_0T_2,...., R_0T_i lines, that is, the points R_0, R_i, T_i are colinear. Knowing the targets motion law, it may compute the distances between T_0 and T_1, T_1 and T_2, ..., T_i and T_{i+1} and so on. Knowing these segments, the sectors R_0R_1, ..., R_iR_{i+1}, ... of the missile trajectory are determined.

To find the position of the missile in the moment t_1, it is necessary to draw a circular arc from point R_0 which intersects R_0T_1 line, with a radius equal to $v_{R_m}\Delta t$, where Δt is the time step.

This is the method to calculate the missile following positions and by joining these points are the missile kinematic trajectory using the three points guidance method as obtained. In Figure 3 is presented the algorithm of trajectory construction.

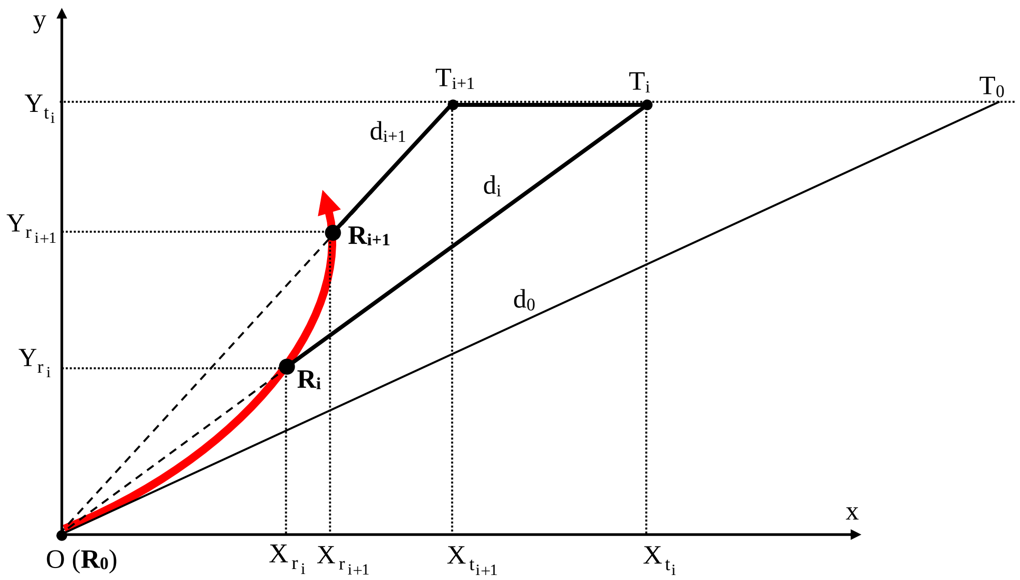

Fig. 3. The algorithm of computing the missile trajectory by the T.P.M. method

2.2. The proportional approach guidance method

In the case of the proportional approach routing method, the points R_0 and T_0 (Figure 4) signify the initial positions of the two air mobiles at the start of self-guidance. These points are joined together by a straight line, and the time step Δt, required to divide the target path into equal segments, is established. The distance between the points thus obtained is calculated using the relation (1):

$$d_{T_i T_{i+1}} = V_{T_m} \Delta t \quad (1)$$

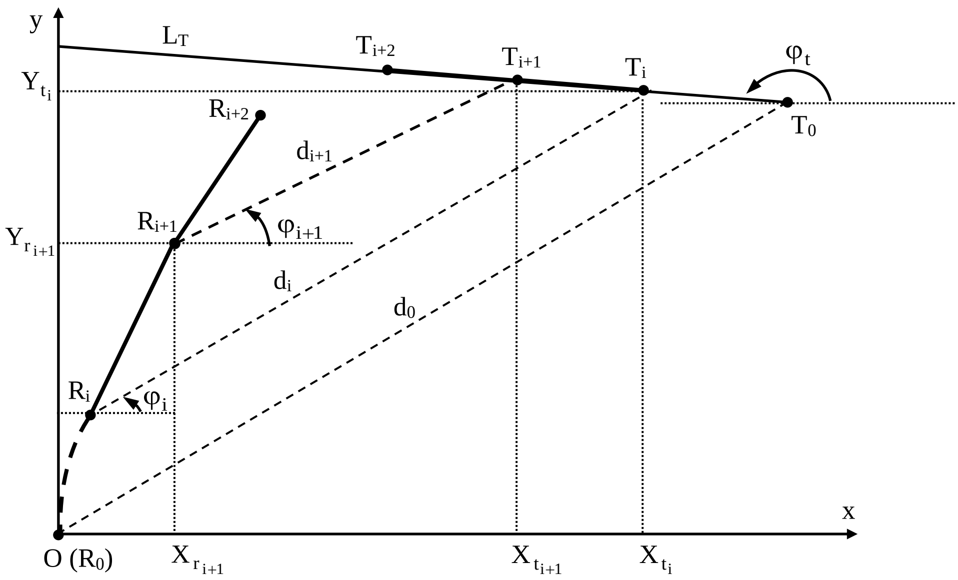

Fig. 4. The trajectory construction algorithm through the proportional approach guidance method

At the moment t_i, the target reaches the future position marked with T_i. Concurrently with the target, the missile moves too and based on the calculation of the product between v_{R_m} and Δt, the lengths of the trajectory segments (R_0R_1, …, $R_{i-1}R_i$ as they are marked on the graph) are determined. In order to observe the characteristic of the guidance method, particularly keeping constant the ratio $= \frac{\omega_R}{\omega_{RT}}$, the successive positions of the missile and the target are joined and the angular speed of these lines is computed (approximated) as:

$$\omega_{RT} = \dot{\varphi}_i = \frac{\varphi_{i+1} - \varphi_i}{\Delta t} \quad (2)$$

The angular velocity ω_R will be approximated by:

$$\omega_R = k \cdot \omega_{RT} \quad (3)$$

Finally, the kinematic trajectory of the rocket guided by the proportional approach method materialises through a curve that passes through all the future points of the rocket, as in Figure 4 (showing the algorithm for building the kinematic trajectory of the missile described above).

3. THE S.W.O.T. ANALYSIS OF GUIDANCE METHODS

The S.W.O.T analysis of guidance methods is presented in Table 1.

Tab. 1

S.W.O.T. analysis of T.P.M and PA.M guidance methods [1]

<table>
<tr><th colspan="2">The guidance method</th><th>T.P.M</th><th>P.A.M</th></tr>
<tr><td rowspan="2">Internal factors</td><td>Strengths</td><td>Knowing the distance to the target is not necessary;
It is used in shootings against targets which manoeuvre under active jamming protection;
The computing equipment is simple so that there appear economic advantages, safety in functioning and convenience in exploitation.</td><td>Increased precision, specially in shooting by following;
The standard missile's accelerations in the impact points area are small;
Reduced gauge of the missile's board equipment;
The trajectory's bend is small.</td></tr>
<tr><td>Weaknesses</td><td>The missile's accelerations are high in the meeting point area;
At high target velocities, the missile's guidance errors are considerably increasing;
At missile's high velocities, the trajectory's bend is pronounced.</td><td>The possibility to combine with other guidance methods in order to increase the precision;
The guidance addiction to the transmitter work;
The necessity of sensitive reception equipment while using the missile in long-range shootings.</td></tr>
<tr><td rowspan="2">External factors</td><td>Opportunities</td><td colspan="2">The existence of a shooting range to evaluate the shooting facts which use these methods;
The existence of equipment designed to prepare the personnel, which serve the system.</td></tr>
<tr><td>Threats</td><td colspan="2">They depend on the financial instability and the direct effects of the depression;
Reduced decision power according to the revival of the national defence industry.</td></tr>
</table>

The description of the two guidance methods according to the performances and drawbacks were realised using the comparative S.W.O.T analysis. In this analysis (Table 1), there are presented the Strengths, Weaknesses, Opportunities and Threats for T.P.M and P.A.M.

Its purpose is to emphasize the conditions of using the guidance methods because using one or another mainly depends on the target's movement laws and parameters.

In conclusion, realising this type of analysis adapted to this paper's scientific field shows the next step in the scheme of things:

- In order to choose a guidance method, the next elements must be contained in the decisions by the operator:
  - to be based on the strengths,
  - to reduce weaknesses to the minimum,
  - to exploit the external opportunities,
  - to counteract the external threats [4, 5].

4. THE SOFTWARE ANALYSIS OF THE KINEMATIC TRAJECTORY [3, 10, 12]

We made a simulation of the two types of trajectories analysed, taking into account the above illustrations, and the wind velocity, w.

The latter was considered horizontal, as well as it was assumed a tilted target trajectory towards the horizontal direction, with an angle φ_t.

In order to get a proper simulation, it was necessary to create a different computing algorithm for every method, where we show the main steps that make possible the functioning of the modelling and simulation programme.

Table 2 presents the steps sequence used to build the trajectories of the two aerial objects. We mention that in relationship included in Table 2 "pas" means the time step, Δt, which was discussed above.

This algorithm can be traced synthesised in flowcharts. They have the same structure, differences intervening only in input and calculation blocks (updates) as input quantities and equations that describe each of the two methods are different.

As a result, in Figure 5 is shown the logical diagram that was built for computing the trajectory points of each guidance method.

The input and calculation blocks (updates), except generalities (missile and target characteristics, their positions – original and current, the distance between them, the accuracy of calculations) consist in the quantities and equations presented in Table 2.

Tab. 2

The computing algorithm of the two guidance methods

The Method's Name / The Algorithm Steps	The direct guidance method by three points	The guidance method by proportional approach
1.	Computing the missile's pathway in the \`pas\` period	
	$d_r = v_r \cdot pas$	$d_r = v_r \cdot pas$

2.	Computing the hade	
	$\varphi = arctg\left(\dfrac{Y_{t_{i+1}} - Y_{r_i}}{X_{t_{i+1}} - X_{r_i}}\right)$	$\varphi := \varphi + \dot{\varphi} \cdot pas$
3.	Computing the target's coordinates in the T_{i+1} position	
	$X_{t_{i+1}} = X_{t_i} + \left(w + v_t \cos\varphi_t\right) \cdot pas$	$X_{t_{i+1}} = X_{t_i} + \left(w + v_t \cos\varphi_t\right) \cdot pas$
	$Y_{t_{i+1}} = Y_{t_i} + v_t \sin\varphi_t \cdot pas$	$Y_{t_{i+1}} = Y_{t_i} + v_t \sin\varphi_t \cdot pas$
4.	Computing the R_{i+1} missile's position coordinates, $X_{r_{i+1}}$ and $Y_{r_{i+1}}$	
	$X_{r_{i+1}} = X_{r_i} + d_r cos(\varphi_i)$	$X_{r_{i+1}} = X_{r_i} + d_i cos(\dot{\varphi} \cdot pas)$
	$Y_{r_{i+1}} = Y_{r_i} + d_r sin(\varphi_i)$	$Y_{r_{i+1}} = Y_{r_i} + d_i sin(\dot{\varphi} \cdot pas)$
5.	The distance between the new positions R_{i+1} and T_{i+1}:	
	$d = \sqrt{\left(Y_{t_{i+1}} - Y_{r_{i+1}}\right)^2 + \left(X_{t_{i+1}} - X_{r_{i+1}}\right)^2}$	$d_{i+1} = d_i + \dot{d}_i \cdot pas$

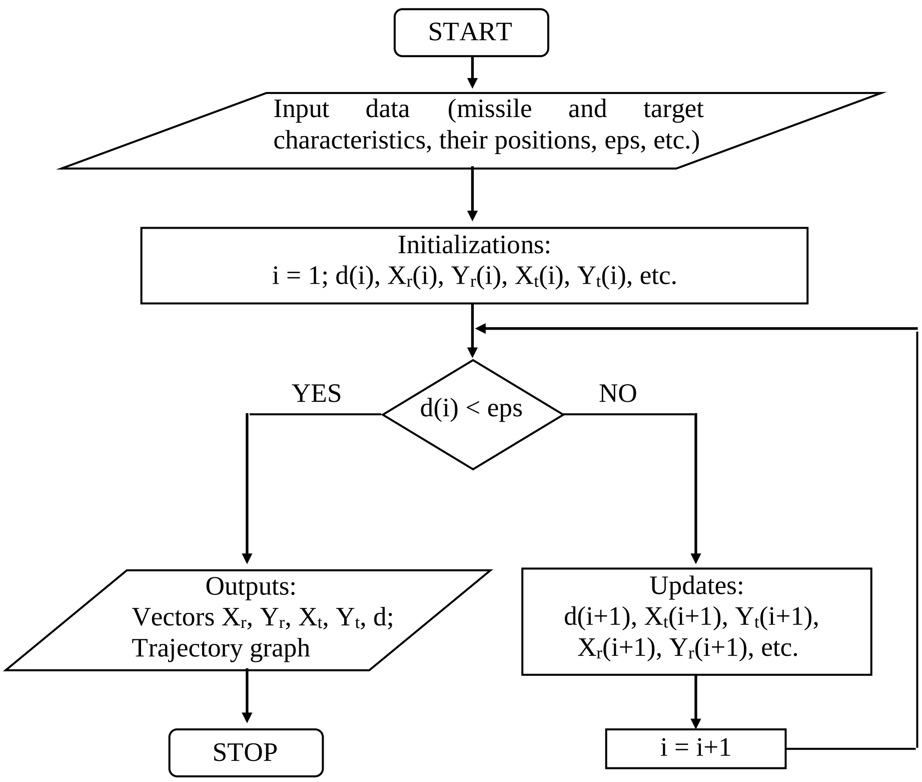

Fig. 5. The logical diagram of building the missile's trajectory

5. MATLAB SIMULATIONS OF THE MISSILE'S AND TARGET'S TRAJECTORIES [3, 6, 10, 13]

MATLAB is one of the most used programme for scientific and numeric calculus. It offers the possibilities of graphical plotting, its basic element being the matrix. SIMULINK is the graphical operating medium, based on MATLAB through which complex systems can be defined and simulated. [5]

Using this software, we carried out the simulation of the missile's and target's trajectory for the two guidance methods previously presented in the S.W.O.T analysis.

The following preliminary conditions were established:

- the target develops in cases:
  - case I: Uniform motion (UM), with the angle of cabrage 20^0,
  - case II: Horizontal rectilinear uniform motion (HRUM),
- the target velocity is 700 m/s,
- the maximum slant range is 25 km.

In order to observe the differences between the two guidance methods, according to the trajectory's bend, the missile interception velocity and the M-T distance at the moment of the fight load explosion, we inputted to the missile for both cases the following velocities:

a. 300 m/s,
b. 600 m/s,
c. 1000 m/s (maximum initial speed).

If the missile's velocity is under 300 m/s and the target moves with 700 m/s, according to the previously highlighted cases (UM with angle of cabrage 20^0 or HRUM), then the missile will miss the target regardless of the guidance method used (T.P.M. or PA.M.).

In this paper, we considered the subject to the previous conditions for target and attributing for missile the initial speed equal to 300 m/s. In the future, we will study cases for initial missile speed equal to 600 m/s and 1000 m/s. In this situation, two cases are possible:

Case I (U.M. with angle of cabrage 20^0):

- The missile guidance using the direct three points method is inefficient because the missile's velocity is too low so that the procedure will not end with the target's destruction.
- Using the guidance method by proportional approach is efficient in this case, because the target is brought down (Figure 6b). This initial velocity of the missile (300 m/s) represents the inferior limit which we can insert in the programme so that the guidance by P.A.M. will be productive.

Case II (HRUM):

- If the target ideally moves, with 300 m/s missile's initial velocity, it cannot be combated by the surface-to-air missile system, which uses the controlled guidance methods (ex. T.P.), in comparison with a missile system that uses the homing guidance methods (ex. P.A). Inserting into the programme, the data stated in the demand from point 2, in figure 7a and b, it can be observed how the missile misses the target using T.P.M and how it destroys the target using P.A.M

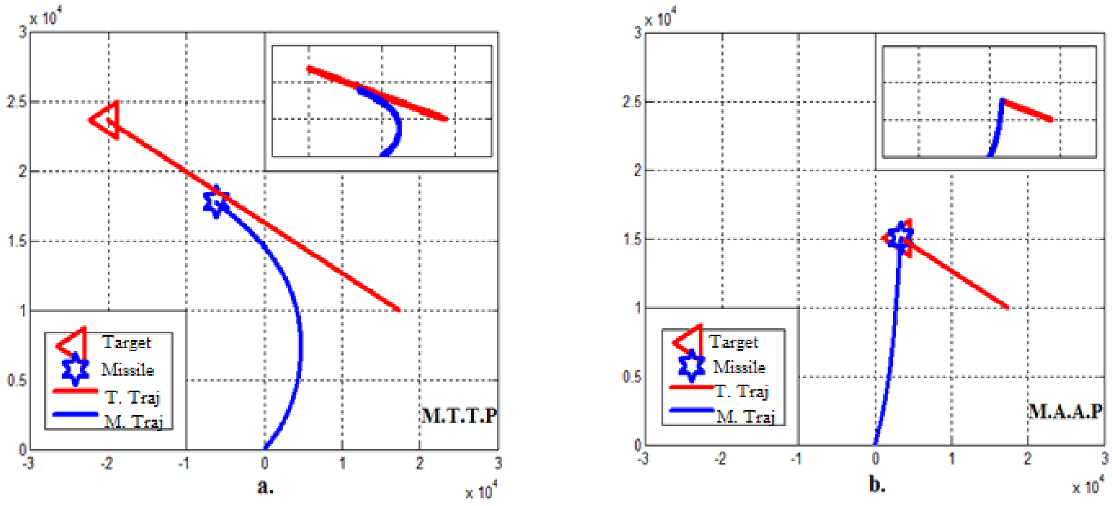

Fig. 6a and b. The graphical aspect of the missile's and target's trajectories
U.M. / V_R = 300 m/s: a) miss; b) success

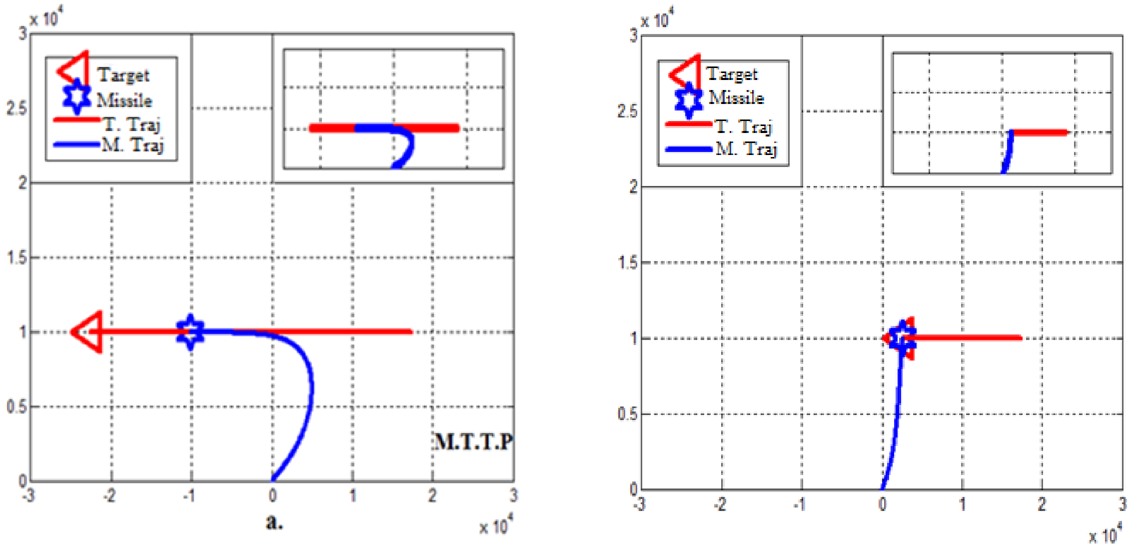

Fig. 7a and b. The graphical aspect of the missile's and target's trajectories
(H.R.U.M. / V_R = 300 m/s: a) miss; b) success

In comparison, for the missile's initial velocity of 300 m/s, the difference between the two guidance methods is obvious, because using P.A.M., in contrast to T.P.M., leads to accomplishing the SAM system mission. In Table 3 are highlighted the main parameters which help to acknowledge the previously stated conclusion.

Tab. 3

The variation of the missile's flight parameters

Nr. crt.	Quantities	UM	The target's law of movement			
			U.M. (angle of cabrage 20^0)		H.R.U.M.	
			T.P.M	P.A.M	T.P.M	P.A.M
1.	V_{R_i}	[m/s]	300	300	300	300
2.	V_T	[m/s]	700	700	700	700
3.	V_{R_p}	[m/s]	–	602	–	602
4.	φ[]	[^{0}]	–	39	–	39
5.	D_p	[m]	15000	41	13000	41
6.	T_{zb_r}	[s]	50	21.1	50	21.1

V_T – target's velocity; V_{R_i} – missile's initial velocity; V_{R_p} – missile's velocity while meeting the target; φ – missile's inclination in the meeting point area; D_p – M-T distance at the moment of the fight load explosion; T_{zb_r} – the missile flight duration (from launching till meeting the target).

6. CONCLUSIONS

The description of the two guidance methods according to the performances and drawbacks were realised by using S.W.O.T analysis (Table 1) wherein are presented the Strengths, Weaknesses, Opportunities and Threats for both guidance methods (T.P.M. and P.A.M.)

Its purpose is to emphasize the conditions of using the guidance methods because both guidance methods mainly depend on the target's movement laws and parameters.

Realising this type of analysis adapted to this paper's scientific field shows that to choose a guidance method, the operator must contain the next elements in the decisions: to be based on the strengths, reducing weaknesses to the minimum, to exploit the external opportunities, and to counteract the external threats.

Last but not least: the only present case in theory is when the missile speed is superior to that of the target. Our simulation suggests that by using the method of proportional approach (PAM in Table 3), this condition is not necessarily mandatory. It seems that a more in-depth study of this observation is required and our intention is to perform it in the future.

References

1. Nicula Aurelian. 2003. *Sisteme de autodirijare a rachetelor*. Bucureşti. Editura Academiei Tehnice Militare ISBN: 973-8290-76-7.
2. Stoica Adriam. 2000. *Eficacitatea tragerilor antiaeriene*. Bucureşti. Editura Academiei Tehnice Militare ISBN 973-640-015-8.

3. Otto S.R., J.P. Denier. 2005. *An Introduction to Programming and Numerical Methods in MATLAB*. Springer-Verlag London Limited, ISBN 1852339195.
4. Șandru Vasile, Boșcoianu Mircea. 2015. *Extension life processes of aerial vectors.* Sarbruken. Germany. Lambert Academic Publishing ISBN 978-3-659-68500-2.
5. Țurcanu Ioan. 1993. *Metode de dirijare a rachetelor antiaeriene*. Brașov. Institutul Militar de Artilerie şi Rachete Antiaeriene "General Bungescu".
6. Șandru Vasile. 2015. „Contribuții la studiul proceselor de extindere a duratei de utilizare a vectorilor aerieni din cadrul sistemelor integrate de apărare aeriană". Teza de doctorat, PhD thesis,. Brașov, Romania: „Transilvania" University. [In Romanian: „Contributions to the study of extension life processes of the aerial vectors that belong to air defense integrated systems"].
7. Moșoiu Ovidiu, Bălăceanu Ion, Moldovan Marius. 2017. „New operational requirements of the integrated defense systems in the hibrid war conditions". *Rewiew of the Air Force Academy* 1(33). ISSN 1842-9238.
8. *Lucrarea 1: Introducere în MATLAB 7.0*. [In Romanian: *Work 1: "Introduction to MATLAB" 7.0*]. Available at: http://www.shiva.pub.ro/PDF/PI/Laborator_1.pdf.
9. SWOT analysis. Available at: http://www.strategvest.ro/media/dms/file/Ghiduri%20metodologice/Prezentare_analiza_SWOT.pdf.
10. Shampine L.F., S. Gladwell, S. Thompson. 2003. *Solving ODEs with MATLAB*. Cambridge University Press. ISBN: 0-521-53094-6.
11. Siouris George M. *Missile And Control Systems*. Springer 2004. ISBN: 978-0-387-00726-7.
12. Zarchan Paul. 2012. *Tactical and Strategic Missile Guidance*. American Institute Of Aeronautics And Astronautics. ISBN: 978-1-60086-894-8.
13. Balakrishnan S.N., A. Tsourdos, B.A. White. 2012. *Advances In Missile Guidance, Control And Estimation*. CRC Press. ISBN: 978-1-4200-8313-2.

Received 25.10.2019; accepted in revised form 20.12.2019