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Particle filtering for the estimation of system mode of operation

Keywords

particle filtering, Monte Carlo, hybrid system, discrete modes of operation

Abstract

Efficient diagnosis and prognosis of system faults depend on the ability to estimate the system state on the basis of noisy measurements of the system dynamic variables and parameters. The system dynamics is typically characterized by transitions among discrete modes of operation, each one giving rise to a specific continuous dynamics of evolution. The estimation of the state of these hybrid dynamic systems is a particularly challenging task because it requires keeping track of the transitions among the multiple modes of system dynamics corresponding to the different modes of operation. In this paper a Monte Carlo estimation method is illustrated with an application to a case study of literature which consists of a tank filled with liquid, whose level is autonomously maintained between two thresholds. The system behavior is controlled by discrete mode actuators, whose states are estimated by a Monte Carlo-based particle filter on the basis of noisy level and temperature measurements.

1. Introduction

Fault detection, isolation and prognosis are crucial tasks for the safe and economic operation of modern plants. Based on the estimation of the system dynamic state, these tasks can provide advanced warning and lead time for preparing the necessary corrective actions to maintain the system in safe operation.

In real systems, often the dynamic states cannot be directly observed; on the other hand, measurements of parameters or variables related to the system states are available, albeit usually affected by noise and disturbances. Then, the problem becomes that of inferring the system state from the measured parameters. Two general approaches exist: i) the model-based techniques, which make use of a quantitative analytical model of the component behavior [1] and ii) the knowledge-based or model-free methods, which rely on empirical models built on available data of the component behavior [2]-[3].

This work falls within the first category for which analytical descriptions of the system dynamics and its relation with the measured parameters are assumed to be available.

The soundest model-based approaches to the estimation of the state of a dynamic system or component build a posterior probability distribution of the unknown states by combining the probability distribution assigned a priori to the possible states with the likelihood of the observations of the measurements actually collected [4]-[5]. In this Bayesian setting, the estimation method most frequently used in practice is the Kalman filter, which is optimal for linear state space models and independent, additive Gaussian noises. In this case, the posterior distributions are also Gaussian and can be computed exactly, without approximations.

In practice, however, the dynamic evolution of many systems and components is non-linear and the associated noises are non-Gaussian [6]. For these cases, approximate methods, e.g. analytical approximations of extended Kalman (EKF) and Gaussian-sum filters and numerical approximations of the grid-based filters [7] can be used, usually at large computational expenses. Alternatively, one may resort to Monte Carlo sampling methods also known as particle filtering methods, which are capable of approximating the continuous distributions of interest by a discrete set of weighed 'particles' representing random trajectories of system evolution in the state space and whose weights are estimates of the probabilities of the trajectories [8]-[11].

The state estimation task becomes quite challenging for systems with a hybrid dynamic behavior characterized by continuous states and discrete modes. Sudden transitions of the discrete modes, often autonomously triggered by the continuous dynamics, affect the system evolution and a large computational effort is required to keep track of the multiple models of the discrete system modes and the autonomous transitions between them [12].

In this paper, particle filtering is applied for the estimation of the state of a hybrid system of literature often taken as a benchmark for dynamic reliability estimation and fault diagnosis/prognosis methods [13]-[16]. The system consists of a tank filled with a liquid whose level is autonomously maintained between two thresholds by actuators driving three fillings and emptying flows triggered by the actual liquid level. The actuators discrete mode is estimated by the particle filter on the basis of noisy level and temperature measurements.

2. Model-based state estimation by Monte Carlo sampling

2.1. General framework

Let us consider a continuous system whose evolution can be described by:

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}, \boldsymbol{\omega}) \tag{1}$$

where $\mathbf{x}$ is the system state vector, $\mathbf{f}: R^{n_x} \times R^{n_\omega} \to R^{n_x}$ is possibly non-linear and $\boldsymbol{\omega}$ is an independent identically distributed (i.i.d.) state noise vector of known distribution.

The state $\mathbf{x}$ cannot in general be directly observed; rather, information about $\mathbf{x}$ can be inferred from the observation of a related variable $\mathbf{z}$ whose relation to the state $\mathbf{x}$ is described in general terms by the equation:

$$\mathbf{z} = \mathbf{h}(\mathbf{x}, \boldsymbol{\upsilon}) \tag{2}$$

where $\mathbf{h}: R^{n_x} \times R^{n_\omega} \to R^{n_x}$ is possibly non-linear and $\boldsymbol{\upsilon}$ is an i.i.d. measurement noise vector sequence of known distribution. The measurements $\mathbf{z}$ are, thus, assumed to be conditionally independent given the state process $\mathbf{x}$.

The practical implementation of computational tools for state estimation requires that the continuous system dynamics be discretized appropriately. Regardless of the discretisation method adopted, the system state dynamics can be represented by an unobserved (hidden) Markov process of order one:

$$\mathbf{x}_k = \mathbf{f}(\mathbf{x}_{k-1}, \boldsymbol{\omega}_{k-1}) \tag{3}$$

where $\mathbf{f}_k: R^{n_x} \times R^{n_\omega} \to R^{n_x}$ is possibly non-linear and $\{\boldsymbol{\omega}_k, k \in \mathrm{N}\}$ is an independent identically distributed (i.i.d.) state noise vector sequence of known distribution.

The transition probability distribution $p(\mathbf{x}_k|\mathbf{x}_{k-1})$ is defined by the system equation (3) and the known distribution of the noise vector $\boldsymbol{\omega}_k$. The initial distribution of the system state $p(\mathbf{x}_0)$ is assumed known.

A sequence of measurements $\{\mathbf{z}_k, k \in \mathrm{N}\}$ is assumed to be collected at the successive time steps t_k. The sequence of measurement values is described by the measurement (observation) equation:

$$\mathbf{z}_k = \mathbf{h}_k(\mathbf{x}_k, \boldsymbol{\upsilon}_k) \tag{4}$$

where $\mathbf{h}_k: R^{n_x} \times R^{n_\omega} \to R^{n_x}$ is possibly non-linear and $\{\boldsymbol{\upsilon}_k, k \in \mathrm{N}\}$ is an i.i.d. measurement noise vector sequence of known distribution. The measurements $\{\mathbf{z}_k, k \in \mathrm{N}\}$ are, thus, assumed to be conditionally independent given the state process $\{\mathbf{x}_k, k \in \mathrm{N}\}$.

Within a Bayesian framework, the filtered posterior distribution $p(\mathbf{x}_k|\mathbf{z}_{0:k})$ can be recursively computed in two stages: *prediction* and *update* [4], [17].

Given the probability distribution $p(\mathbf{x}_{k-1}|\mathbf{z}_{0:k-1})$ at time k–1, the prediction stage involves using the system model (3) to obtain the prior probability distribution of the system state $\mathbf{x}_k$ at time k via the Chapman-Kolmogorov equation:

$$\begin{aligned} p(\mathbf{x}_k|\mathbf{z}_{0:k-1}) &= \int p(\mathbf{x}_k|\mathbf{x}_{k-1}|\mathbf{z}_{0:k-1}) p(\mathbf{x}_{k-1}|\mathbf{z}_{0:k-1}) d\mathbf{x}_{k-1} = \\ &= \int p(\mathbf{x}_k|\mathbf{x}_{k-1}) p(\mathbf{x}_{k-1}|\mathbf{z}_{0:k-1}) d\mathbf{x}_{k-1} \end{aligned} \tag{5}$$

where the Markovian assumption underpinning the system model (3) has been used.

At time k, a new measurement $\mathbf{z}_k$ is collected and used to update the prior distribution via Bayes rule, so as to obtain the required posterior distribution of the current state $\mathbf{x}_k$ [17]:

$$p(\mathbf{x}_k|\mathbf{z}_{0:k}) = \frac{p(\mathbf{x}_k|\mathbf{z}_{0:k-1})p(\mathbf{z}_k|\mathbf{x}_k)}{p(\mathbf{z}_k|\mathbf{z}_{0:k-1})} \qquad (6)$$

where the normalizing constant is

$$p(z_k|z_{0:k-1}) = \int p(x_k|z_{0:k-1})p(z_k|x_k)dx_k \qquad (7)$$

The recurrence relations (5) and (6) form the basis for the exact Bayesian solution. Unfortunately, except for a few cases, including linear Gaussian state space models (Kalman filter) and hidden finite-state space Markov chains (Wohnam filter), it is not possible to evaluate analytically these distributions, since they require the evaluation of complex high-dimensional integrals.
This problem can be circumvented by resorting to Monte Carlo sampling methods [5], [9], [18]-[19].
Writing the posterior probability $p(\mathbf{x}_{0:k}|\mathbf{z}_{0:k})$ of the entire state sequence $\mathbf{x}_{0:k}$ given the measurement vector $\mathbf{z}_{0:k}$ as:

$$p(\mathbf{x}_{0:k}|\mathbf{z}_{0:k}) = \int p(\xi_{0:k}|\mathbf{z}_{0:k})\delta(\xi_{0:k} - \mathbf{x}_{0:k})d\xi_{0:k} \qquad (8)$$

and assuming that the true posterior probability $p(\mathbf{x}_{0:k}|\mathbf{z}_{0:k})$ is known and can be sampled, an estimate of (8) is given by [20]:

$$\hat{p}(\mathbf{x}_{0:k}|\mathbf{z}_{0:k}) = \frac{1}{N_s}\sum_{i=1}^{N_s}\delta(\mathbf{x}_{0:k} - \mathbf{x}^i_{0:k}) \qquad (9)$$

where $\mathbf{x}^i_{0:k}$, $i = 1, 2,\ldots, N_s$ is a set of independent random samples drawn from $p(\mathbf{x}_{0:k}|\mathbf{z}_{0:k})$.
Since, in practice, it is usually not possible to sample efficiently from the true posterior distribution $p(\mathbf{x}_{0:k}|\mathbf{z}_{0:k})$, importance sampling is used, i.e. the state sequences $\mathbf{x}^i_{0:k}$ are drawn from an arbitrarily chosen distribution $\pi(x_{0:k}|z_{0:k})$, called importance function [20]. The probability $p(x_{0:k}|z_{0:k})$ is written as:

$$p(\mathbf{x}_{0:k}|\mathbf{z}_{0:k}) = \int \pi(\xi_{0:k}|\mathbf{z}_{0:k})\frac{p(\xi_{0:k}|\mathbf{z}_{0:k})}{\pi(\xi_{0:k}|\mathbf{z}_{0:k})}\delta(\xi_{0:k} - \mathbf{x}_{0:k})d\xi_{0:k} \qquad (10)$$

and an unbiased estimate is obtained by [5], [17]:

$$\hat{p}^*(\mathbf{x}_{0:k}|\mathbf{z}_{0:k}) = \frac{1}{N_s}\sum_{i=1}^{N_s} w_k^{*i}\delta(\mathbf{x}_{0:k} - \mathbf{x}^i_{0:k}) \qquad (11)$$

where:

$$w_k^{*i} = \frac{p(\mathbf{z}_{0:k}|\mathbf{x}^i_{0:k})p(\mathbf{x}^i_{0:k})}{p(\mathbf{z}_{0:k})\pi(\mathbf{x}^i_{0:k}|\mathbf{z}_{0:k})} \qquad (12)$$

is the importance weight associated to the state sequence $\mathbf{x}^i_{0:k}$, $i = 1,2,\ldots,N_s$, sampled from $\pi(\mathbf{x}_{0:k}|\mathbf{z}_{0:k})$ and $p(\mathbf{z}_{0:k}|\mathbf{x}^i_{0:k})$ is the likelihood of the observation sequence.
Computing the weight requires knowledge of the $p(\mathbf{z}_{0:k}) = \int p(\mathbf{z}_{0:k}|\mathbf{x}_{0:k})p(\mathbf{x}_{0:k})d\mathbf{x}_{0:k}$, normalizing constant, which cannot typically be expressed in closed form. It can be shown that, to overcome this problem, an estimate of the posterior probability distribution $p(\mathbf{x}_{0:k}|\mathbf{z}_{0:k})$ may be computed as [5], [17]:

$$\hat{p}(x_{0:k}|z_{0:k}) = \sum_{i=1}^{N_s}\tilde{w}_k^i\delta(x_{0:k} - x^i_{0:k}) \qquad (13)$$

where the "Bayesian" importance weights $\tilde{w}_k^i$ are given by:

$$\tilde{w}_k^i = \frac{w_k^i}{\sum_{j=1}^{N_s} w_k^j} \qquad (14)$$

$$w_k^i = \frac{p(\mathbf{z}_{0:k}|\mathbf{x}^i_{0:k})p(\mathbf{x}^i_{0:k})}{\pi(\mathbf{x}^i_{0:k}|\mathbf{z}_{0:k})} = w_k^{*i}\,p(\mathbf{z}_{0:k}) \qquad (15)$$

For on-line applications, the estimate of the distribution $p(\mathbf{x}_{0:k}|\mathbf{z}_{0:k})$ at the k-th time step can be obtained from the distribution $p(\mathbf{x}_{0:k-1}|\mathbf{z}_{0:k-1})$ at the previous time step by the following recursive formula obtained by extension of equation (6) for the Bayesian filter $p(\mathbf{x}_k|\mathbf{z}_{0:k})$ [5], [17]:

$$\begin{aligned}
p(\mathbf{x}_{0:k}|\mathbf{z}_{0:k}) &= \frac{p(\mathbf{x}_{0:k}|\mathbf{z}_{0:k-1})p(\mathbf{z}_k|\mathbf{x}_{0:k}|\mathbf{z}_{0:k-1})}{p(\mathbf{z}_k|\mathbf{z}_{0:k-1})} = \\
&= \frac{p(\mathbf{x}_k|\mathbf{x}_{0:k-1}|\mathbf{z}_{0:k-1})p(\mathbf{x}_{0:k-1}|\mathbf{z}_{0:k-1})p(\mathbf{z}_k|\mathbf{x}_{0:k}|\mathbf{z}_{0:k-1})}{p(\mathbf{z}_k|\mathbf{z}_{0:k-1})} = \\
&= \frac{p(\mathbf{x}_k|\mathbf{x}_{0:k-1})p(\mathbf{x}_{0:k-1}|\mathbf{z}_{0:k-1})p(\mathbf{z}_k|\mathbf{x}_{0:k})}{p(\mathbf{z}_k|\mathbf{z}_{0:k-1})} = \\
&= p(\mathbf{x}_{0:k-1}|\mathbf{z}_{0:k-1})\frac{p(\mathbf{z}_k|\mathbf{x}_{0:k})p(\mathbf{x}_k|\mathbf{x}_{0:k-1})}{p(\mathbf{z}_k|\mathbf{z}_{0:k-1})}
\end{aligned} \qquad (16)$$

Again, use has been made of the fact that the system model (1) is Markovian of order one and that the observations governed by the measurement equation (2) are conditionally independent given the system state sequence.

Furthermore, if the importance function is chosen such that:

$$\pi(\mathbf{x}_{0:k}|\mathbf{z}_{0:k})=\pi(\mathbf{x}_0|\mathbf{z}_0)\prod_{j=1}^{k}\pi(\mathbf{x}_j|\mathbf{x}_{0:j-1}|\mathbf{z}_{0:j})= \\ =\pi(\mathbf{x}_k|\mathbf{x}_{0:k-1}|\mathbf{z}_{0:k})\pi(\mathbf{x}_{0:k-1}|\mathbf{z}_{0:k-1}) \tag{17}$$

the following recursive formulas for the non-normalized weights w_k^{*i} and w_k^i can be obtained:

$$w_k^{*i}=\frac{p(\mathbf{x}_{0:k}^i|\mathbf{z}_{0:k})}{\pi(\mathbf{x}_{0:k}^i|\mathbf{z}_{0:k})}= \\ =\frac{p(\mathbf{x}_{0:k-1}^i|\mathbf{z}_{0:k-1})\dfrac{p(\mathbf{z}_k|\mathbf{x}_k^i)p(\mathbf{x}_k^i|\mathbf{x}_{k-1}^i)}{p(\mathbf{z}_k|\mathbf{z}_{k-1})}}{\pi(\mathbf{x}_k^i|\mathbf{x}_{0:k-1}^i|\mathbf{z}_{0:k})\pi(\mathbf{x}_{0:k-1}^i|\mathbf{z}_{0:k-1})}= \\ =w_{k-1}^{*i}\frac{p(\mathbf{z}_k|\mathbf{x}_k^i)p(\mathbf{x}_k^i|\mathbf{x}_{k-1}^i)}{\pi(\mathbf{x}_k^i|\mathbf{x}_{0:k-1}^i|\mathbf{z}_{0:k})}\frac{1}{p(\mathbf{z}_k|\mathbf{z}_{k-1})} \tag{18}$$

$$w_k^i=w_k^{*i}p(\mathbf{z}_{0:k})=w_{k-1}^i\frac{p(\mathbf{z}_k|\mathbf{x}_k^i)p(\mathbf{x}_k^i|\mathbf{x}_{k-1}^i)}{\pi(\mathbf{x}_k^i|\mathbf{x}_{0:k-1}^i|\mathbf{z}_{0:k})}$$

The choice of the importance function is obviously crucial for the efficiency of the estimation. In this work, the prior distribution of the hidden Markov model is taken as importance function, i.e. $\pi(\mathbf{x}_k|\mathbf{x}^i_{0:k-1}|\mathbf{z}_{0:k}) = p(\mathbf{x}_k|\mathbf{x}^i_{k-1})$, and the resampling is applied at each time step. Many efficient resampling techniques are available in literature [21]; in order to enhance readability and focus on tackling the hybrid nature of the system, in what follows we shall refer to the basic resampling algorithm [22]-[23]. Prior to resampling, the non-normalized weights (18), would then be:

$$w_k^i=w_{k-1}^i p(\mathbf{z}_k|\mathbf{x}_k^i) \tag{19}$$

However, since the resampling is performed at every iteration, $w_{k-1}^i=1/N_s$ and after normalization the updated weight simply becomes equal to the likelihood of the measurement $\mathbf{z}_k$, viz.

$$\tilde{w}_k^i=p(\mathbf{z}_k|\mathbf{x}_k^i) \tag{20}$$

2.2. Hybrid system

Let us consider a hybrid system whose dynamic evolution can be described by:

$$\begin{cases}\mathbf{x}_k=\mathbf{f}_{\beta_k}(\mathbf{x}_{k-1},\boldsymbol{\omega}_{k-1})\\ \beta_k=\mathbf{g}_k(\beta_{k-1},\mathbf{x}_{k-1})\end{cases} \tag{21}$$

β_k = {1,2,3,…,M} is the discrete state which indicates the mode in which the system is evolving at time k, $\mathbf{f}_{\beta_k}$ is the non-linear function describing the (discretized) continuous evolution of system state x when the system is in mode β_k at time k, $\mathbf{g}_k$ is the discrete mode transition function. In what follows, we shall consider only autonomous transitions between the system modes, i.e. those triggered by the control of the continuous state **x** which demands transitions among the system modes when reaching specified thresholds.

Let $s_k^i=(\beta_k^i,\mathbf{x}_k^i)$ indicate the i^{th} sample of the extended hybrid system state, where $\mathbf{x}_k^i$ is the random sample drawn from the importance function $p(\mathbf{x}_k|\mathbf{x}_{k-1}^i)$ and β_k^i is the corresponding discrete mode of system behavior. Then, the posterior probability density of the continuous and discrete states can be represented by the random measure $\{s_k^i,w_k^i,i=1...N_s\}$, where w_k^i is the particle weight of the i^{th} sample of the hybrid state at time k after resampling.

The particle filtering algorithm for the hybrid state estimation may be summarized as follows:

- Predict Cycle (Importance sampling)
  - Importance-sample the system continuous states

$$\mathbf{x}_k^i\approx\pi(\mathbf{x}_{0:k}|\mathbf{z}_{0:k}) \tag{22}$$

- Filter or update cycle
  - Compute the weights

$$w_k^i=w_{k-1}^i\frac{p(\mathbf{z}_k|\mathbf{x}_k^i)p(\mathbf{x}_k^i|\mathbf{x}_{k-1}^i)}{\pi(\mathbf{x}_k^i|\mathbf{x}_{0:k-1}^i|\mathbf{z}_{0:k})} \tag{23}$$

  - Resampling which includes normalizing the weights, bootstrap-sample the system states with replacement and update the weights, respectively:

$$\tilde{w}_k^i=w_k^i\Big/\sum_{j=1}^{N_s}w_k^j \tag{24}$$

$$\mathbf{x}_k^i\approx\hat{p}(\mathbf{x}_k|\mathbf{z}_{0:k}) \tag{25}$$

$$w_k^i=1/N_s \tag{26}$$

- Estimate the system mode as the most likely one:

$$\hat{\beta}_k = \arg\max \sum_{i \in \hat{G}_j} w_k^i \tag{27}$$

where $\hat{G}_j = \{i | \beta_k^i = j\}$.

- Compute the posterior estimate mean of the continuous state $\mathbf{x}_k$ and its variance $\hat{\sigma}_k^2$:

$$\hat{\mathbf{x}}_k = \frac{\sum_{i \in \hat{G}} w_k^i \mathbf{x}_k^i}{\sum_{i \in \hat{G}} w_k^i} \tag{28}$$

$$\hat{\sigma}_k^2 = \frac{\sum_{i \in \hat{G}}^{N_s} w_k^i (\mathbf{x}_k^i - \hat{\mathbf{x}}_k)}{\sum_{i \in \hat{G}} w_k^i} \tag{29}$$

where only the particles belonging to the most likely mode $\hat{\beta}_k^i$ are considered $\hat{G}_j = \{i | \beta_k^i = \hat{\beta}_k^i\}$.

3. Application to a tank control system

The particle filter estimation algorithm is applied to a hybrid system of literature [13]-[16]. The system consists of a tank containing a fluid whose level is controlled by three control units which open or close depending on the fluid level crossing of predefined thresholds (*HLV* and *HLP*) (*Figure 1*). The fluid in the tank is uniformly heated, under adiabatic conditions, by a thermal power source W.

The control aims at maintaining the fluid level x_1 in the range ($x_{1,\min}$ = *HLV*, $x_{1,\max}$ = *HLP*), while also monitoring the fluid temperature x_2 which may become relevant from a safety point of view.

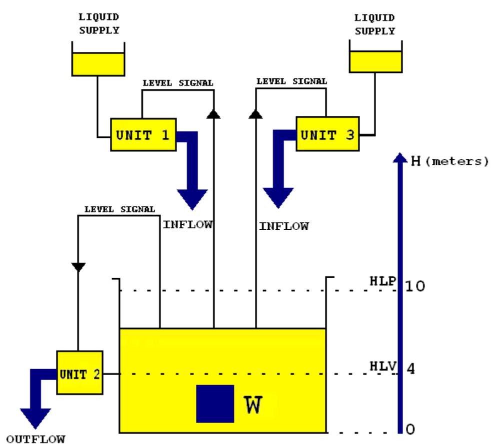

Figure 1. Tank control system [13]-[16]

The operational states of the control units at time k are described by the Boolean indicator $\alpha_{l,k}$, l = 1,2,3, where $\alpha_{l,k}$ assumes the value 1 or 0 according to whether the unit is on ($\alpha_{l,k}$ = 1) or off ($\alpha_{l,k}$ = 0). The autonomous control actions modify the states $\alpha_{l,k}$ of the units according to the following rules:

$$\alpha_{1,k} = \begin{cases} 1 & if\ x_1 < HLP \\ 0 & if\ x_1 > HLP \\ 0\ or\ 1 & depending\ on\ previous\ switching \end{cases}$$

$$\alpha_{2,k} = \begin{cases} 1 & if\ x_1 > HLV \\ 0 & otherwise \end{cases}$$

$$\alpha_{1,k} = \begin{cases} 1 & if\ x_1 < HLV \\ 0 & if\ x_1 > HLP \\ 0\ or\ 1 & depending\ on\ previous\ switching \end{cases}$$

Thus, the following four modes of system dynamic evolution may be identified:

$$\beta_k = \begin{cases} 1 & if\ x_{1,k} < HLV \\ 2 & if\ HLV < x_{1,k} < HLP \text{ and } \alpha_{1,k-1} = \alpha_{3,k-1} = 1 \\ 3 & if\ HLV < x_{1,k} < HLP \text{ and } \alpha_{1,k-1} = \alpha_{3,k-1} = 0 \\ 4 & if\ x_{1,k} > HLP \end{cases} \tag{30}$$

With the additional simplifying physical assumptions:

- the fluid input in the tank by units 1 and 3 mixes instantaneously
- the flow rate through the outlet unit 2 is independent of the fluid level

and the discretisation of the system dynamics, the time evolution of the states and can be described by two first-order, decoupled, non-linear difference equations determined by the mass and energy conservation laws [13]:

$$\begin{aligned} x_{1,k} &= x_{1,k-1} + \Delta t[\alpha_{1,k-1}Q_{1,k-1} + \alpha_{3,k-1}Q_{3,k-1} - \\ &\quad - \alpha_{2,k-1}Q_{2,k-1}] + \omega_{1,k} \\ x_{2,k} &= x_{2,k-1} + \frac{\Delta t}{x_{1,k-1}}\{[\alpha_{1,k-1}Q_{1,k-1} + \alpha_{3,k-1}Q_{3,k-1}]\cdot \\ &\quad (\vartheta_m - x_{2,k-1}) + 23.88915\} + \omega_{2,k} \end{aligned} \tag{31}$$

where $Q_{j,k}$, j = 1,2,3, are the fluid flow rates (m/h) at time k for units 1, 2 and 3, respectively, ϑ_m is the assigned inlet fluid temperature, Δt is the time step, $\omega_{1,k}$ and $\omega_{2,k}$ are the process noises accounting for the unmodeled dynamics.

In spite of its simple structure, the system considered is representative of the operation of non-linear control systems and possesses mathematical features which pose difficulties to the application of conventional model-based estimation techniques. For instance, the linearization of the original differential equations required by the extended Kalman filter approach is not applicable because of the stepwise dependence of the parameters α_l, l = 1,2,3, on the system variable x_1.

The aim of the analysis is that of estimating the discrete mode of the system, i.e. the operational states of the three control units on the basis of N_s trajectories drawn from the system model (31) and a sequence of noisy measurements of the level $x_{1,k}$ and the temperature $x_{2,k}$:

$$z_{1,k} = x_{1,k} + \upsilon_{1,k}$$
$$z_{2,k} = x_{2,k} + \upsilon_{2,k} \qquad (32)$$

where $\upsilon_{1,k}$ and $\upsilon_{2,k}$ are the measurement noises. Knowledge of the system mode of operation allows the proper control and maintenance of its components.

The dynamic evolution of the fluid level and temperature has been simulated resorting to (31). Let us suppose that the control system starts from $x_{1,0} = 6\,\text{m}$ and $x_{2,0} = 10\,\text{m}$. The time horizon considered for the evolution of the system dynamics is $N_t = 40\text{h}$, with level and temperature observations at discrete time steps of $\Delta t = 30\,\text{min}$ ($N_k = 80$). As in the application of reference [13], the inlet fluid temperature is $\vartheta_m = 15\,°\text{C}$, the level thresholds are set at *HLV* = 4m and *HLP* = 10m and the fluid flow rates are $Q_1 = 1\,\text{m/h}$, $Q_2 = 4\,\text{m/h}$ and $Q_3 = 4.5\,\text{m/h}$. A zero – mean Gaussian noise with variance $\sigma_Q^2 = 0.0025$ is added to the flow rates, for closer adherence to reality.

The process and the measurement noises are assumed Gaussian with zero mean and variances $\sigma_\omega^2 = [0.02\;\;0.01]$ and $\sigma_\upsilon^2 = [0.16\;\;0.05]$ respectively.

Assuming independence of the level and temperature measurements, the observation likelihood in (20) can be written as:

$$p(\mathbf{z}_k|\mathbf{x}_k^i) = \prod_{h=1,2} p(z_{h,k}|\mathbf{x}_k^i) =$$
$$= \frac{1}{\sqrt{2\pi}\sigma_{\upsilon_1}} e^{-\frac{1}{2}\left(\frac{z_{1,k}-\mu_{1,k}}{\sigma_{\upsilon_1}}\right)^2} \frac{1}{\sqrt{2\pi}\sigma_{\upsilon_1}} e^{-\frac{1}{2}\left(\frac{z_{2,k}-\mu_{2,k}}{\sigma_{\upsilon_2}}\right)^2} \qquad (33)$$

First, a crude, measurement-based, empirical algorithm is proposed for the estimation of the mode β_k at time k:

$$\hat{\beta}_k = \begin{cases} 1 & if\;\; z_{1,k} < HLV \\ 2 & if\;\; HLV < z_{1,k} < HLP \text{ and } \alpha_{1,k-1} = \alpha_{3,k-1} = 1 \\ 3 & if\;\; HLV < z_{1,k} < HLP \text{ and } \alpha_{1,k-1} = \alpha_{3,k-1} = 0 \\ 4 & if\;\; z_{1,k} > HLP \end{cases} \qquad (34)$$

where $z_{1,k}$ is the level measurement at time k.

Figure 2 shows the estimated mode $\hat{\beta}$ (dot-dashed line) and the model simulated one β (solid line). The performance is not satisfactory because the noise υ_1 generates spurious oscillations in the level measurement z_1 with respect to the model-simulated x_1 actually driving the mode transitions.

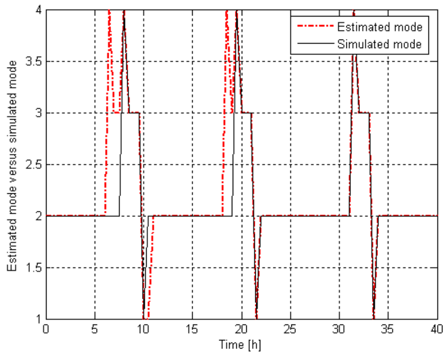

Figure 2. Measurement-based estimated system modes (dotted line) and model-based simulated system modes (solid line).

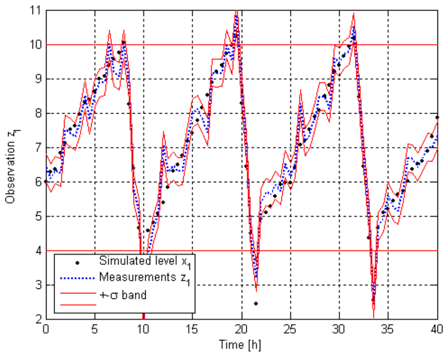

Figure 3. Fluid level measurements (dotted line), with measurement noise uncertainty $\pm 1\sigma_{\upsilon_1}$ bands (solid line); model-simulated fluid level (dots).

To overcome this problem, the particle filter of Section 2.2 (*Figure 3*) is implemented with a number of particles $N_s = 1000$. *Figure 4* shows the particle filter-estimated mode $\hat{\beta}$ (dot-dashed line) and the model simulated one β (solid line). The agreement is satisfactory, with the only exception at the first time when the system enters mode $\beta = 4$, i.e. the fluid level is higher than *HLP*.

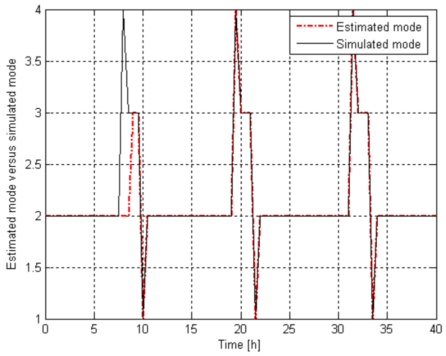

Figure 4. Particle filter-estimated (dotted line) and model-simulated (solid line) modes.

This is due to the fact that the first few observations of the fluid level higher than *HLP* do not provide the filter with enough information for properly performing the mode estimation. This is confirmed in *Figure 5*, where the estimated level $\hat{x}_1$ (dotted) is affected by a larger uncertainty $\hat{\sigma}_{1,k}$ when approaching the threshold HLP for the first time.
For completeness, *Figure 6* shows the good particle filtering capability of estimating also the second state variable, i.e. the temperature x_2.
The mode estimation capability of the algorithm in (34) is clearly affected by the level measurement noise variance $\sigma^2_{\upsilon_1}$. Indeed, for $\sigma^2_{\upsilon_1} = 0$ the algorithm would always yield the correct mode estimate.

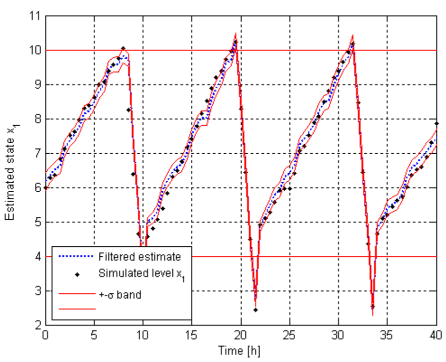

Figure 5. Particle filter-estimated mean fluid level (dotted line), with $\pm 1\hat{\sigma}_k$ uncertainty bands (solid line) and model-simulated fluid level (dots).

In this regard, a sensitivity analysis has been performed in which the performances of the crude, measurement-based algorithm (34) and of the particle filter are compared for different values of the level measurement noise. For the comparison, the following figure of merit has been introduced:

$$I = \frac{M_\varepsilon}{M} \tag{35}$$

where M_ε is the number of time steps for which the model-simulated fluid level falls outside the $\pm 1\hat{\sigma}_k$ uncertainty and around the estimated mean, and *M*=80 is the total number of time steps in the observation period.
Table 1 shows the values of *I* for the crude algorithm (34) (second column) and for the particle filter (third column). When $\sigma^2_{\upsilon_1} = 0.10$, the crude algorithm achieves a perfect performance, i.e. correctly identifies the evolution mode of the system at every time step, since the fluid level measurement z_1 is very similar to the actual fluid level x_1. As the variance of the mean on the fluid level measurement increases, the performance of the crude algorithm rapidly degenerates with respect to the particle filtering. The large values of *I* for the smallest two values of $\sigma^2_{\upsilon_1}$ are due to the well – known degeneracy of the likelihoods (33) as $\sigma^2_{\upsilon_1} \to 0$, whereas the value of *I* for $\sigma^2_{\upsilon_1} = 0.22$ is indicating that, above certain noise levels, no useful information can be extracted from the measurement.

Table 1. Sensitivity test results: the case presented at the beginning of the Section is highlighted.

$\sigma^2_{\upsilon_1}$	I_{z_1}	I_{x_1}
0.10	**0**	**0.125**
0.13	**0.012**	**0.050**
0.16	**0.075**	**0.025**
0.19	**0.100**	**0.025**
0.22	**0.100**	**0.037**

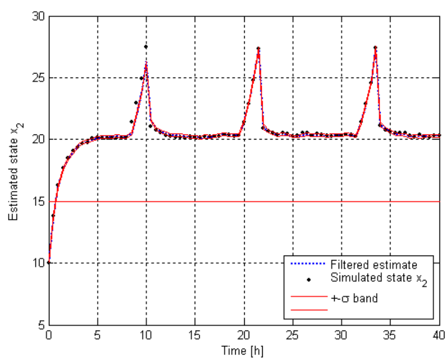

Figure 6. Particle filter-estimated mean of the fluid temperature (dotted line), with $\pm 1\hat{\sigma}_k$ uncertainty

bands (solid line) and simulated fluid temperature (dots).

4. Conclusion

In this paper, a Monte Carlo-based filter has been devised for estimating both the continuous states and the discrete modes of a controlled system, whose transitions between the discrete modes are autonomously triggered by the continuous states. Comparison with a crude algorithm which bases its estimates directly on the observed measurements, shows the higher performance of the particle filter on a wider range of measurements noises, thus counterbalancing the larger computational effort required.

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