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Research paper

Evaluating the sustainable mining contractor selection problems: An imprecise last aggregation preference selection index method

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ABSTRACT

The increasing complexity surrounding decision-making situations has made it inevitable for practitioners to apply ideas from a group of experts or decision makers (DMs) instead of individuals. In a large proportion of recent studies, not enough attention has been paid to considering uncertainty in practical ways. In this paper, a hesitant fuzzy preference selection index (HFPSI) method is proposed based on a new soft computing approach with risk preferences of DMs to deal with imprecise multi-criteria decision-making problems. Meanwhile, qualitative assessing criteria are considered in the process of the proposed method to help the DMs by providing suitable expressions of membership degrees for an element under a set. Moreover, the best alternative is selected based on considering the concepts of preference relation and hesitant fuzzy sets, simultaneously. Therefore, DMs' weights are determined according to the proposed hesitant fuzzy compromise solution technique to prevent judgment errors. Moreover, the proposed method has been extended based on the last aggregation method by aggregating the DMs' opinions during the last stage to avoid data loss. In this respect, a real case study about the mining contractor selection problem is provided to represent the effectiveness and efficiency of the proposed HFPSI method in practice. Then, a comparative analysis is performed to show the feasibility of the presented approach. Finally, sensitivity analysis is carried out to show the effect of considering the DMs' weights and last aggregation approach in a dispersion of the alternatives' ranking values.

1. Introduction

Mining plays an important role in many countries and its acts are considered to be a foundation for development and growth. Meanwhile, selecting an appropriate contractor regarding sustainable competencies is an essential issue when extending the sustainable supply chain partnership (Pimentel, Gonzalez, & Barbosa, 2016). Thereby, a multiple criteria decision-making (MCDM) approach has proven itself as a highly practical tool for presenting the best solution among the potential candidates regarding multiple criteria with different effects. In practice, it is generally out of the question for a solution to completely satisfy all conflicting criteria. Hence, Pareto optimality is implemented within MCDM techniques. In fact, if one alternative obtains an appropriate score with respect to one criterion, it is less likely that in other criteria the same good score is achieved. Candidates which may not be consistent possess their own strengths to various criteria. Therefore, a Pareto optimal solution may be used (Chang, 2010; Mohagheghi, Mousavi, & Vahdani, 2015). In the field of mining problems, MCDM techniques have been widely used for evaluating mining decision making problems.

In this respect, Ataei, Sereshki, Jamshidi, and Jalali (2008) used the order of preference by similarity to the ideal solution (TOPSIS) method to select the best mining design method based on 13 criteria. Azimi, Yazdani-Chamzini, Fouladgar, Zavadskas, and Basiri (2011) made use of an integrated approach by combining the analytic network process (ANP) and TOPSIS methods for ranking the strategies from the internal and external environment which were used to analyze the Iranian mining sector. Pourjavad, Shirouyehzad, and Shahin (2013) used a decision making analysis in which the ANP method is considered for determining the criteria weights and then the TOPSIS technique is applied for selecting the suitable maintenance strategy in mining sector. Shen, Muduli, and Barve (2015) established a sustainable development framework for mining industries in which the analytic hierarchy process (AHP)

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method is utilized to show the priority of each considered criterion. Sivakumar et al. (2015) used a novel approach based on AHP and the Taguchi loss function approach for evaluating and selecting the most suitable vendor in green mining industries.

The complexity of decision making problems makes the practitioners consider the merit of group decision making to analyze all relevant aspects of decision-making problems. In the last decade, practitioners depended on group-MCDM (MCGDM) approaches to get reliable results considering the DMs' assessment instead of an individual DM evaluation. In this way, MCGDM tools try to utilize the techniques to establish a collective solution in some situations where a group of DMs define their preferences on multiple attributes (Foroozesh, Tavakkoli-Moghaddam, Mousavi, & Vahdani, 2017; Mousavi, Vahdani, Tavakkoli-Moghaddam, & Tajik, 2014). In order to find the collective solution, representing, compiling, and fusing experts' judgments is required. Furthermore, providing information in accordance with the fuzzy setting could help the DMs to decrease the complexity of the real world (Ebrahimnejad, Hashemi, Mousavi, & Vahdani, 2015; Mousavi et al., 2014; Vahdani, Mousavi, Tavakkoli-Moghaddam, Ghodratnama, & Mohammadi, 2014).

Recently, several group decision-making methods based on fuzzy preference relations have been presented. Tanino (1984) in his method demonstrated the use of fuzzy preference ordering for group decision-making. Kacprzyk, Fedrizzi, and Nurmi (1992) manipulated an integrated group decision-making approach by combining the fuzzy majority and fuzzy preference approaches. Herrera-Viedma, Herrera, and Chiclana (2002) presented a consensus approach for multi-person decision-making with various preference structures. Xu (2004a, 2004b) applied linguistic aggregation operators with linguistic preference relations to introduce a group decision-making method. Herrera-Viedma, Martinez, Mata, and Chiclana (2005) manipulated a consensus support system approach for MCGDM with multi-granular linguistic preference relations. Xu (2007) applied MCGDM tools with various formats of preference information on criteria. Fedrizzi and Pasi (2008, pp. 19–37) established a group decision-making model by implementing some fuzzy logic approaches in a compromise solution manner. Xu (2008) by also working on multiple types of linguistic preference relations presented a group decision-making method. Mata, Martínez, and Herrera-Viedma (2009) proposed an adaptive consensus support model to cope with group decision-making regarding the multi-granular fuzzy linguistic context. Chiclana, Herrera-Viedma, Alonso, and Herrera (2009) presented a group decision-making approach by focusing on cardinal consistency of reciprocal preference relations. Perez et al. (2010) considered a mobile decision support system for dynamic group decision-making. Chen and Niou (2011) focused on fuzzy preference operations to provide a fuzzy MCGDM method.

In the context of mining problems, Nuong et al. (2011) suggested a number of urgent tasks for providing suitable sustainable development in mining industries in which the fuzzy AHP technique is provided to assess sustainable development and deal with imprecise information. Rahimdel and Karamoozian (2014) proposed an imprecise framework based on the fuzzy positive ideal solution and fuzzy negative ideal solution to choose a suitable primary crusher in the Golegohar iron mine in Iran. Kusi-Sarpong, Bai, Sarkis, and Wang (2015) introduced a green supply chain practices assessment approach combining the fuzzy TOPSIS method and rough set theory. Modak, Pathak, and Ghosh (2017) proposed a framework based on balanced score card (BSC) and the fuzzy AHP approach for the performance assessment of an outsourcing decision in an Indian coal mining organization. In this respect, the BSC approach is used to identify the performance indicators and the triangular fuzzy AHP is utilized to specify the performance of the outsourcing decision.

Due to the existence of situations with unknown or incomplete values in preference relations, some methods have been developed to estimate the unknown or incomplete values in incomplete fuzzy preference relations. Xu (2004a, 2004b) focused on additive consistent incomplete fuzzy preference relations and multiplicative consistent incomplete fuzzy preference relations in order to the priority vector of several incomplete fuzzy preference relations to develop two goal programming models. Alonso et al. (2008) introduced a consistency-based method to predict the missing pair wise preference values. Alonso, Herrera-Viedma, Chiclana, and Herrera (2010) integrated individual and social strategies to deal with situations of ignorance in multi-person decision-making. Wang and Chen (2010) applied incomplete fuzzy linguistic preference relations to enable DMs to cope with imprecise or vague environments. Genc et al. (2010) introduced a fuzzy method in order to detect whether a preference relation based on interval fuzzy is consistent or not and also to handle the priority vector of a consistent preference relation under an interval fuzzy environment. Büyüközkan and Çifçi (2012) used group decision-making based on fuzzy logic to deploy a quality function with incomplete preference relations. In addition, to demonstrate the efficiency of their method a collaborative software development example was also provided.

Over the last decade, several studies have been carried out to propose compromise solution techniques for MCGDM problems under a fuzzy environment (Mousavi, Gitinavard, Vahdani, & Foroozesh, 2016; Vahdani, 2016). For instance, Chen (2000) focused on the TOPSIS method for group decision-making under fuzzy uncertainty. Chen, Lin, and Huang (2006) presented a fuzzy TOPSIS method for supplier evaluation and selection in supply chain management. Candidate ratings and weight assessment for the main criteria were analyzed using linguistic terms and triangular fuzzy numbers. Vahdani, Mousavi, and Tavakkoli-Moghaddam (2011) elaborated a modified fuzzy group TOPSIS technique for solving the issue of robot selection and rapid prototyping process selection problems. Ebrahimnejad, Mousavi, Tavakkoli-Moghaddam, Hashemi, and Vahdani (2012) made us of an imprecise approach based on fuzzy TOPSIS technique in order to rank higher risks in mega projects.

The hesitant fuzzy sets (HFSs), among the most suitable theories, has received a great deal of attention recently and was first introduced by Torra and Narukawa (2009) and Torra (2010) who properly dealt with hesitant conditions to solve MCDM problems based on imprecise information. Hence, Wei (2012) put forward some imprecise hesitant fuzzy MCDM approaches where various priority levels were used to describe the criteria with regards to some extended prioritized aggregation operators. Zhang, Wang, Tian, and Li (2014) provided some aggregation relations for the HFSs and interval-valued HFSs regarding their various properties. Moreover, the proposed relations were implemented to solve the MCGDM problems.

Consequently, the HFSs are widely accepted as powerful tools for illustrating uncertain information in a hesitant situation. Therefore, several scholars have utilizied HFSs in decision-making problems. Zhang and Wei (2013) presented a VIKOR technique for TOPSIS methods in the HFSs setting to solve MCDM problem and then compared the results of proposed methods. Xu and Zhang (2013) developed a TOPSIS method for solving MCDM problems in which the information about the evaluation of a candidate described under a hesitant fuzzy set environment and the information related to the attribute weight was incomplete. Liao and Xu (2013) elaborated a hesitant fuzzy VIKOR method regarding the hesitant normalized Manhattan distance measure in the procedure of the ranking method. Consequently, the recent literature gap is

presented in Table 1 and was created by reviewing relevant studies which are focused on preference selection index method or contractor selection problems. This table indicates that the last aggregation approach, the determination of the DMs' weights and the assigning of some membership degrees are not considered in recent relevant literature as three important properties for preventing data loss and decreasing errors.

The survey of relevant studies suggests that there is a gap that could be filled by the proposing of a novel imprecise last aggregation preference selection index method by considering the DMs' weights for solving the mining contractor selection problems to overcome the issues of uncertainty and risk. In this paper, the classical preference selection index method from Maniya and Bhatt (2010) is developed based on novel indexes, DMs' weight determinations and hesitant fuzzy setting information. In this method it is not required to specify the relative significance of criteria; hence, the overall preference value of each criterion is assessed by utilizing the statistics concept.

However, a novel hesitant fuzzy preference selection index (HFPSI) method, according to the risk preferences of DMs, is developed for solving MCGDM problems. HFS theory is taken into account to illustrate the uncertain information in real-life situations under hesitant conditions. The presented approach places the DMs' difficulties in the procedure of membership function definition for an object into a set through multi-criteria analysis. Furthermore, the DMs' weights are computed regarding proposed hesitant fuzzy compromise solution technique to avoid judgment error. Also, the HFPSI method is presented based on the last aggregation approach for aggregating the DMs' opinions during the last stage to prevent data loss. Moreover, some operations which were required in order to propose the approach were developed. Finally, a real case study about the mining industry is provided to highlight the details of the proposed HFPSI method. In this respect, the obtained results are reported and also compared with the recent hesitant fuzzy TOPSIS method proposed by Zhang and Wei (2013).

The rest of the paper is arranged as follows. The preliminaries for the HFS, its concept and relations are outlined in section 2. The proposed HFPSI method is described in section 3. The proposed method is implemented for a case study and the corresponding results are reported in section 4. Meanwhile, in section 5, comparative analysis is considered to show the validity and efficiency of the proposed method. Furthermore, sensitivity analysis is represented in section 6, to show the sensitivity/robustness of the proposed approach regarding DM's weight and the last aggregation approach. Finally, section 7 includes the concluding remarks and future work suggestions.

2. Preliminaries

In this section, a brief review of preliminaries for HFS is given. Meanwhile, the concepts and relations of HFS will also be expressed. Then, the extensions of summation and multiplication operators are proved by using several HFSs.

Definition 1. (Xia & Xu, 2011). If Z is the discourse universe, then X as a HFS on Z is demonstrated by $S_x(z)$ that when applied to Z returns to subset of [0, 1].

$$X = \{ < z, S_x(z) > | z \in Z\} \tag{1}$$

where $S_x(z)$ is denoting a set of membership degrees for an element [0, 1], representing the membership degrees of elements $z \in Z$ to X.

Definition 2. (Xia & Xu, 2011). Some operations/relations are defined regarding the relationship between the intuitionistic fuzzy sets (IFSs) and the hesitant fuzzy elements (HFEs).

$$S_1 \oplus S_2 = \cup_{\mu_1 \in S_1, \mu_2 \in S_2} \{\mu_1 + \mu_2 - \mu_1 . \mu_2\} \tag{2}$$

$$S_1 \otimes S_2 = \cup_{\mu_1 \in S_1, \mu_2 \in S_2} \{\mu_1 . \mu_2\} \tag{3}$$

$$S^{\lambda} = \cup_{\gamma \in S} \left\{ \mu^{\lambda} \right\} \tag{4}$$

$$\lambda S = \cup_{\mu \in S} \left\{ 1 - (1 - \mu)^{\lambda} \right\} \tag{5}$$

Definition 3. Consider $E = \{S_1, S_2, ..., S_n\}$ as a collection of HFEs, then the multiplication and summation operators could be developed based on definition 2, as follows:

$$S_1 \otimes S_2 \otimes .... \otimes S_n = \cup_{\mu_1 \in S_1, \mu_2 \in S_2, ..., \mu_n \in S_n} \left\{ \prod_{i=1}^{n} \mu_i \right\} \tag{6}$$

$$\overset{n}{\underset{i=1}{\oplus}} S_i = \cup_{\mu_1 \in S_1, \mu_2 \in S_2, ..., \mu_n \in S_n} \left\{ \left(1 - \prod_{i=1}^{n} (1 - \mu_i) \right) \right\} \tag{7}$$

Theorem 1. Let S_1, S_2, S_3 be three HFEs; then, we have:

$$S_1^c \oplus S_2^c \oplus S_3^c = (S_1 \otimes S_2 \otimes S_3)^c \tag{8}$$

Table 1
Preference selection index methods and their properties.

Authors	Methodology properties					
	Last aggregation approach	Uncertainty modeling	Assigning some membership degrees	Utilizing the linguistic terms	Computing the DMs' weights	Group decision-making
Jaskowski et al. (2010)	×	✓	×	×	×	✓
Maniya and Bhatt (2010)	×	×	×	✓	×	×
Hadipour et al. (2012)	×	✓	×	✓	×	✓
Zhang and Wei (2013)	×	✓	×	✓	×	×
Vahdani et al. (2014)	×	✓	×	✓	×	✓
Attri and Grover (2015)	×	×	×	✓	×	×
This paper	✓	✓	✓	✓	✓	✓

$$S_1^c \otimes S_2^c \otimes S_3^c = (S_1 \oplus S_2 \oplus S_3)^c \tag{9}$$

Proof. For summation operator regarding the three HFEs S_1,S_2 and S_3, we have:

$$\begin{aligned} S_1 \oplus S_2 \oplus S_3 &= \cup_{\mu_1 \in S_1, \mu_2 \in S_2, \mu_3 \in S_3} \{\mu_1 + \mu_2 + \mu_3 - \mu_1\mu_2 \\ &\quad - \mu_1\mu_3 - \mu_2\mu_3 + \mu_1\mu_2\mu_3\} \\ S_1^c \oplus S_2^c \oplus S_3^c &= \cup_{\mu_1 \in S_1, \mu_2 \in S_2, \mu_3 \in S_3} \{1 - \mu_1.\mu_2.\mu_3\} = (S_1 \oplus S_2 \oplus S_3)^c + \end{aligned} \tag{10}$$

In addition, for a multiplication operator, we have:

$$\begin{aligned} &S_1 \otimes S_2 \otimes S_3 = \cup_{\mu_1 \in S_1, \mu_2 \in S_2, \mu_3 \in S_3} \{\mu_1.\mu_2.\mu_3\} \\ &S_1^c \otimes S_2^c \otimes S_3^c = \cup_{\tilde{\gamma}_1 \in \tilde{h}_1, \tilde{\gamma}_2 \in \tilde{h}_2, \tilde{\gamma}_3 \in \tilde{h}_3} \{(1-\mu_1)(1-\mu_3)(1-\mu_3)\} \\ &= 1 - \mu_1 - \mu_2 - \mu_3 + \mu_1\mu_2 + \mu_1\mu_3 + \mu_2\mu_3 - \mu_1\mu_2\mu_3 \\ &= (S_1 \oplus S_2 \oplus S_3)^c + \end{aligned} \tag{11}$$

However, the generalized equation (10) which is equivalent to equation (7) is represented as follows:

$$S_1 \oplus S_2 \oplus \ldots \oplus S_n = \cup_{\mu_1 \in S_1, \mu_2 \in S_2, \ldots, \mu_n \in S_n} \left\{ \sum_{i=1}^{n} \mu_i - \sum_{\substack{i=1 \\ \forall j=i+1,\ldots,n}}^{n-1} \mu_i.\mu_j + \sum_{\substack{i=1 \\ \forall j=i+1,\ldots,n-1 \\ \forall k=j+1,\ldots,n}}^{n-2} \mu_i.\mu_j.\mu_k - \ldots + \prod_{i=1}^{n} \mu_i \right\} \quad \forall i \neq j \neq k \tag{12}$$

Therefore, the proof of theorem 1 is complete.

Definition 4. (Xu & Xia, 2011). Let S_M and S_N be two HFEs, the generalized distance measure is defined, as follows:

$$d_{gh}(S_M, S_N) = \left(\frac{1}{l_{x_i}} \sum_{j=1}^{l_{x_i}} \left| S_M^{\sigma(j)}(x_i) - S_N^{\sigma(j)}(x_i) \right|^{\lambda} \right)^{\frac{1}{\lambda}} \tag{13}$$

where $S_M^{\sigma(j)}$ and $S_N^{\sigma(j)}$ are the jth largest value in S_M and S_N. Also, if $\lambda = 1$, the hesitant fuzzy Hamming distance measure is obtained, and if $\lambda = 2$, the hesitant fuzzy Euclidean distance is achieved.

Definition 5. (Xia & Xu, 2011). Hesitant fuzzy weighted averaging (HFWA) and hesitant fuzzy weighted geometric (HFWG) are defined as follows; let $S_j (j = 1, 2, \ldots, n)$ be some hesitant fuzzy elements, then:

$$\begin{aligned} HFWA(S_1, S_2, \ldots, S_n) &= \overset{n}{\underset{j=1}{\oplus}} (w_j S_j) \\ &= \cup_{\mu_1 \in S_1, \mu_2 \in S_2, \ldots, \mu_n \in S_n} \left\{ 1 - \prod_{j=1}^{n} \left(1 - \mu_j\right)^{w_j} \right\} \end{aligned} \tag{14}$$

$$\begin{aligned} HFWG(S_1, S_2, \ldots, S_n) &= \overset{n}{\underset{j=1}{\otimes}} (S_j)^{w_j} \\ &= \cup_{\mu_1 \in S_1, \mu_2 \in S_2, \ldots, \mu_n \in S_n} \left\{ \prod_{j=1}^{n} \left(\mu_j\right)^{w_j} \right\} \end{aligned} \tag{15}$$

where the weight vector of $S_j (j = 1, 2, \ldots, n)$ is denoted by $w = (w_1, w_2, \ldots, w_n)^T$. In this study, the last aggregation method is considered by aggregating the preferences' DMs' judgments during the last stage of the proposed approach to prevent data loss. Moreover, the weight of each DM should be applied in the procedure of judgment aggregation in order to consider the expertise of each DM. Thus, developing new aggregation operators to consider the DMs' weight is needed. Let, $S_k (k = 1, 2, \ldots, k)$ be a set of k HFEs, which is presented by kth DM for a candidate under the conflicted criteria. Therefore, the HFWA and HFWG aggregation operators regarding the DMs' weight are obtained based on the basic relations for HFSs as follows:

$$\begin{aligned} HFWA(S_1, S_2, \ldots, S_k) &= \overset{K}{\underset{k=1}{\oplus}} (\wp_k S_k) \\ &= \cup_{\mu_1 \in S_1, \mu_2 \in S_2, \ldots, \mu_k \in S_k} \left\{ 1 - \prod_{k=1}^{K} (1 - \mu_k)^{\wp_k} \right\} \end{aligned} \tag{16}$$

$$\begin{aligned} HFWG(S_1, S_2, \ldots, S_k) &= \overset{K}{\underset{k=1}{\otimes}} (S_k)^{\wp_k} \\ &= \cup_{\mu_1 \in S_1, \mu_2 \in S_2, \ldots, \mu_k \in S_k} \left\{ \prod_{k=1}^{K} (\mu_k)^{\wp_k} \right\} \end{aligned} \tag{17}$$

where the weight of each DM is denoted by $\wp_k$.

Definition 6. (Zhu, Xu, & Xia, 2012). Considering the hesitant fuzzy decision matrix as $H = (S_{ij})_{m \times n}$, the normalized hesitant fuzzy decision matrix $(B = (b_{ij})_{m \times n})$ is obtained as follows:

$$b_{ij} = \cup_{t_{ij} \in b_{ij}} = \begin{cases} \{\mu_{ij}\} & PC_j \\ \{1 - \mu_{ij}\} & NC_j \end{cases} \forall i = 1, \ldots, m; j = 1, \ldots, n \tag{18}$$

where the PC_j and NC_j are defined as the positive and negative criteria.

3. Proposed imprecise preference selection index method

In this section, the procedure of the proposed hesitant fuzzy preference selection index method (HFPSI) is explained. In this approach, the weight of each DM and the last aggregation approach is tailored to decrease errors and data loss. Thereby, the conceptual framework of the proposed approach is depicted in Fig. 1.

3.1. The steps of the approach

Step I. Establish a group of DMs. These experts evaluate m alternatives A_i (i = 1,2, …,m) among n criteria C_j (j = 1,2, …,n) which are defined as benefit or cost types for assessment of the problem.

Step II. Construct a hesitant fuzzy decision matrix regarding the DM's opinions.

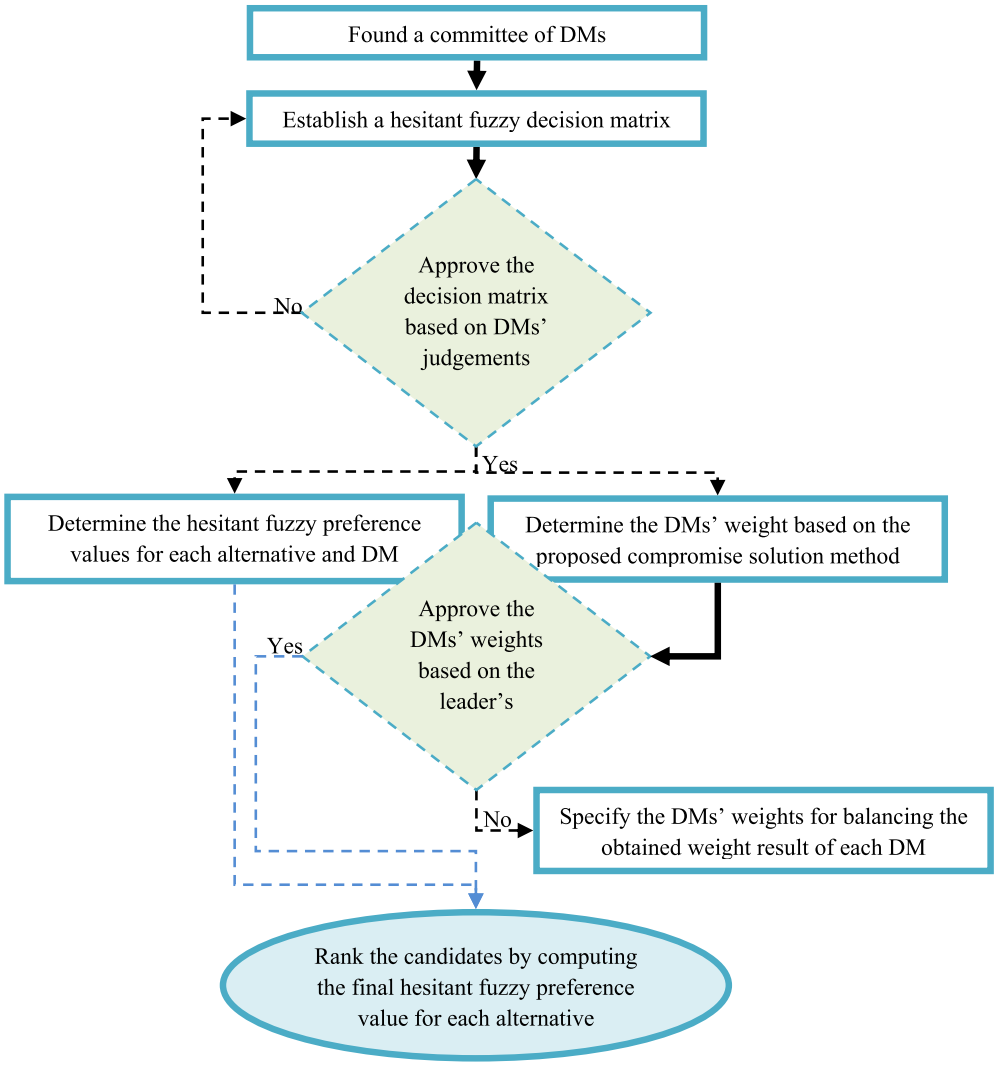

Fig. 1. The schematic structure of the proposed approach.

Step III. Establish the normalized hesitant fuzzy decision matrix (ν^k) for each DM based on definition 6.

$$\nu^k = \begin{array}{c} \\ A_1 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{ccc} C_1 & \cdots & C_n \end{array} \\ \begin{bmatrix} \left\{R_{11}^1, R_{11}^2, \ldots, \mu_{11}^k\right\} & \cdots & \left\{R_{1n}^1, R_{1n}^2, \ldots, R_{1n}^k\right\} \\ \vdots & \ddots & \vdots \\ \left\{R_{m1}^1, R_{m1}^2, \ldots, R_{m1}^k\right\} & \cdots & \left\{R_{mn}^1, R_{mn}^2, \ldots, R_{mn}^k\right\} \end{bmatrix}_{m\times n} \end{array} \quad \forall k \tag{19}$$

Step IV. Determine the hesitant fuzzy preference value ($HFPV_j^k$) by applying the following equation:

$$HFPV_j^k = 1 - \prod_{i=1}^{m}\left(\frac{2\overline{R}_j^k\left(R_{ij}^k - 1\right) - \left(R_{ij}^k\right)^2 + 1}{1 + \left(\overline{R}_j^k\right)^2 - 2\overline{R}_j^k}\right) \quad \forall j, k \tag{20}$$

where $\overline{R}_j^k$ is the mean of the normalized value of criteria j that is computed as $\overline{R}_j^k = \frac{1}{m}\sum_{i=1}^{m} R_{ij}^k \quad \forall j, k$.

Step V. Specify the hesitant fuzzy overall preference value (ψ_j^k) for each DM by considering the deviation (φ_j^k) in hesitant fuzzy preference value as follows:

$$\varphi_j^k = 1 - HFPV_j^k \quad \forall j, k \tag{21}$$

$$\psi_j^k = \frac{\varphi_j^k}{\sum_{j=1}^{n}\varphi_j^k} \quad \forall j, k \tag{22}$$

The total hesitant fuzzy overall preference value should be equal to one, $\sum_{j=1}^{n}\psi_j^k = 1 \quad \forall k$.

Step VI. Determine the hesitant fuzzy positive preference solution (ξ_i^{k*}) and the hesitant fuzzy negative preference solution (ξ_i^{k-}) as follows:

$$\xi_i^{k*} = \max_j\left\{R_{ij}^k\psi_j^k\right\} \quad \forall i, k \tag{23}$$

$$\xi_i^{k-} = \min_j\left\{R_{ij}^k\psi_j^k\right\} \quad \forall i, k \tag{24}$$

Step VII. Determine the hesitant fuzzy preference selection index

(HFI_i^k) for each DM by following:

$$HFI_i^k = \lambda\left(\xi_{\min}^{k*}\sum_{i=1}^{m}\xi_i^{k*} - \xi_i^{k-}\xi_i^{k*}\sum_{i=1}^{m}\left(\frac{\xi_{\min}^{k*}}{\xi_i^{k*}}\right)\left(\xi_{\min}^{k*}\sum_{i=1}^{m}\xi_i^{k*} - \xi_i^{k*} \times \sum_{i=1}^{m}\left(\frac{\xi_{\min}^{k*}}{\xi_i^{k*}}\right)\right)\right) \quad \forall i,k \tag{25}$$

$$\lambda = \left(\xi_i^{k*}\sum_{i=1}^{m}\left(\frac{\xi_{\min}^{k*}}{\xi_i^{k*}}\right)\right)^{-1} \tag{26}$$

$$\xi_{\min}^{k*} = \min_i\left\{\xi_i^{k*}\right\} \quad \forall k \tag{27}$$

Step VIII. Calculate the DMs' weights based on the proposed hesitant fuzzy compromise solution technique as following procedure:

Step VIII.1. Specify the imprecise positive ideal solution (Δ^+) and the imprecise negative ideal solution (Δ^-) based on the following:

$$\Delta^+ = \left[\Delta_{ij}^+\right]_{m\times n} = \begin{cases} \max_k\left\{\left[R_{(ij)k}\right]_{m\times n}\right\} & \forall PC_j \\ \min_k\left\{\left[R_{(ij)k}\right]_{m\times n}\right\} & \forall NC_j \end{cases} \tag{28}$$

$$\Delta^- = \left[\Delta_{ij}^-\right]_{m\times n} = \begin{cases} \min_k\left\{\left[R_{(ij)k}\right]_{m\times n}\right\} & \forall PC_j \\ \max_k\left\{\left[R_{(ij)k}\right]_{m\times n}\right\} & \forall NC_j \end{cases} \tag{29}$$

Step VIII.2. Compute the aggregated hesitant fuzzy positive ($\Im_k^+$) and negative ($\Im_k^-$) solutions' indices as follows:

$$\Im_k^+ = \prod_{i=1}^{m}\left(l_{x_q}\sqrt[2m]{\frac{\sum_{j=1}^{n}\sum_{\lambda=1}^{l_{x_q}}\left(R_{(ij)k}^{\sigma(\lambda)}(x_q) - \Delta_{(ij)}^{+\sigma(\lambda)}(x_q)\right)^2}{l_{x_q}}}\right) \quad \forall k \tag{30}$$

$$\Im_k^- = \prod_{i=1}^{m}\left(l_{x_q}\sqrt[2m]{\frac{\sum_{j=1}^{n}\sum_{\lambda=1}^{l_{x_q}}\left(R_{(ij)k}^{\sigma(\lambda)}(x_q) - \Delta_{(ij)}^{-\sigma(\lambda)}(x_q)\right)^2}{l_{x_q}}}\right) \quad \forall k \tag{31}$$

Step VIII.3. Calculate the experts' weight ($\wp_k$) based on the following:

$$\wp_k = \frac{1-\left(\prod_{k=1}^{K}\left(1-\Im_k^-\right)\right)^{\min_k\{\Im_k^-\}} - \Im_k^+\left(\prod_{k=1}^{K}\left(1-\frac{\min_k\{\Im_k^-\}}{\Im_k^-}\right)\right)^{\Im_k^-} + \Im_k^+\left(\prod_{k=1}^{K}\left(1-\Im_k^-\right)\right)^{\min_k\{\Im_k^-\}}}{1-\left(\prod_{k=1}^{K}\left(1-\frac{\min_k\{\Im_k^-\}}{\Im_k^-}\right)\right)^{\Im_k^-}} \quad \forall k \tag{32}$$

Step VIII.4. Determine the final weight of each expert ($\wp_k^F$) regarding the leader's judgments as follows:

$$\wp_k^F = \frac{\partial_k \wp_k}{\sum_{k=1}^{K}(\partial_k \wp_k)} \quad \forall k \tag{33}$$

where ∂_k is the weight of each expert which is defined based on the leader's judgments for balancing the experts' final weights.

Step IX. Specify the final hesitant fuzzy preference selection values (HFI_i^F) based on definition 5.

$$HFI_i^F\left(HFI_i^1, HFI_i^2, \ldots, HFI_i^k\right) = \left(\bigoplus_{k=1}^{K}\left(\wp_k^F HFI_i^k\right)\right) = 1 - \prod_{k=1}^{K}\left(1-HFI_i^k\right)^{\wp_k^F} \quad \forall i \tag{34}$$

Step X. Rank the candidate alternatives by selecting the maximum value of the hesitant fuzzy preference selection index.

3.2. Procedure of the proposed approach

To summarize, the procedure of the proposed imprecise last aggregation preference selection index method is categorized based on the following phases:

Phase 1. Evaluate the selection problem by establishing a group of DMs and constructing the hesitant fuzzy group decision matrix (Steps I and II).

Phase 2. Specify the hesitant fuzzy preference value for each alternative based on equations (19)–(27), (Steps III to VII).

Phase 3. Compute the DMs' weights by proposing the hesitant fuzzy compromise solution technique based on equations (28)–(33), (Step VIII).

Phase 4. Determine the final hesitant fuzzy preference selection

values regarding the hesitant fuzzy weighted averaging operator and DMs' weights based on equation (34), (Step IX).

4. Case study: mining contractor selection problem

4.1. Description of the problem

To further demonstrate the proposed HFPSI method, a real case study for selecting the best mining contractor is presented. Environmental issues, aboriginal land claims, and communities surrounding mining are regularly highlighted as important issues. These are only some of the problems which could serve to distract mining companies from their work. The needs of mine management is far greater than in the past. Thereby, statutory processes, safety, planning approvals, environmental competencies, servicing customer and marketing requirements, corporate bureaucracy and industrial reforms all reduce the time that mine management have to focus on sustainability, cost and production. Other activities such as exploration, transport, long term acquisitions and planning are also ongoing. These issues result in the efforts of mining companies being largely unsuccessful because their focus still remains on many areas that get lost in standardization and processes.

Moreover, candidate mining contractors have carried out specialist work, such as fault driveages, drift and shaft work, specialized strata control and excavations. Thus, the use of a mining contractor allows mining companies to succeed in many areas, while the contractor focuses on efficient mining. Therefore, the selection of a mining contractor is an integral part of surface and underground mining industry. The case in question is a mining company in Iran which has performed a wide range of mining, stripping and excavating projects in Iran. The company outsourced some important activities to the most suitable international/domestic mining contractors regarding sustainable competencies. Therefore, selecting and ranking the mining contractor candidate was a significant issue for this company. Some projects, such as the Chadormalu iron ore mine, the Miduk copper mine, and the Choghart iron ore mine which have been successfully carried out by mining contractor candidates, are represented in Fig. 2.

4.2. Implementation and results

Three DMs from several departments in the mining company worked together to evaluate the sustainable mining contractor selection problem. They had 14 years of experience in mining industry in Iran. Also, the risk preference of each DM was unique and considered as pessimistic, moderate, and optimistic for DM_1, DM_2 and DM_3, respectively. In the mining contractor selection problem, three candidates were first selected and 15 conflicted criteria were then chosen for the evaluation (Step I). These criteria are described in Table 2. In addition, the linguistic variables for pessimistic, moderate, and optimistic are defined based on the hesitant fuzzy element in Table 3. In this respect, the hesitant fuzzy decision matrix is established by DMs' evaluations which are shown with linguistic terms (Table 4, and the normalized hesitant fuzzy decision matrix is then obtained regarding hesitant fuzzy elements (Steps II and III).

Table 2
The assessment criteria of the sustainable mining contractor selection problem.

Notation	Criteria
C_1	Tender price
C_2	Past client/contractor relationship
C_3	Financial statement
C_4	Social aspects
C_5	Cost overruns
C_6	Delay
C_7	Quality
C_8	Environmental competence
C_9	Failure to have contract completed
C_{10}	Experience
C_{11}	Human resources
C_{12}	Physical resources
C_{13}	Financial references
C_{14}	Current workload
C_{15}	Safety performance

Table 5 shows the computational results of hesitant fuzzy preference value ($HFPV_j^k$) regarding the mean of the normalized value ($\overline{R}_j^k$) for each criterion (Step IV). Also, the hesitant fuzzy overall preference value (ψ_j^k) is computed using the deviation in hesitant fuzzy preference value (Step V). The result is presented in Table 6. Then, the hesitant fuzzy positive preference solution (ξ_i^{k*}) and the hesitant fuzzy negative preference solution (ξ_i^{k-}) are determined based on Step VI. Therefore, as indicated in Table 7, the hesitant fuzzy preference selection indices are determined by regarding each DM and each potential alternative (Step VII). Moreover, we utilized the proposed hesitant fuzzy compromise solution technique to compute the DMs' weight ($\wp_k^F$) (Step VIII). In this respect, the leader considers the same weight for each DM (∂_k). The computational results of determining the DMs' weight are reported in Table 8.

Finally, the hesitant fuzzy weighted averaging for the aggregation of the hesitant fuzzy preference selection index is applied to specify the relative significance of each mining contractor candidate (Step IX). The results are provided in Table 9. Finally, as indicated in Table 9, a preferred solution is considered to show the validation and verification of the proposed approaches. The HFPIS method selects the first alternative as a suitable mining contractor among the other potential alternatives for the mining company.

5. Comparative analysis

In this section, the comparative analysis is prepared for the

Chadormalu iron ore project Miduk copper project Choghart iron ore project

Fig. 2. The projects which have been successfully carried out by candidate mining contractors.

Table 3
Linguistic variables for rating the possible alternatives.

Linguistic variable	Interval-valued hesitant fuzzy sets	DM's risk preferences		
		Pessimist	Moderate	Optimist
Very very poor (VVP)	[0.10, 0.10]	0.10	0.10	0.10
Very poor (VP)	[0.10, 0.25]	0.10	0.175	0.25
Poor (P)	[0.25, 0.40]	0.25	0.325	0.40
Moderately poor (MP)	[0.40, 0.50]	0.40	0.45	0.50
Fair (F)	[0.50, 0.60]	0.50	0.55	0.60
Moderately good (MG)	[0.60, 0.70]	0.60	0.65	0.70
Good (G)	[0.70, 0.80]	0.70	0.75	0.80
Very good (VG)	[0.80, 0.90]	0.80	0.85	0.90
Very very good (VVG)	[0.90, 0.90]	0.90	0.90	0.90
Extremely good (EG)	[1.00, 1.00]	1	1	1

Table 4
The hesitant fuzzy decision matrix based on linguistic terms.

Criteria	Mining contractors	Decision makers		
		DM_1	DM_2	DM_3
C_1	A_1	G	VG	VG
	A_2	F	G	G
	A_3	F	F	G
C_2	A_1	F	MG	MG
	A_2	MG	F	MG
	A_3	MP	F	F
C_3	A_1	F	MG	MG
	A_2	F	MG	F
	A_3	MG	MP	MG
C_4	A_1	G	VG	VG
	A_2	MG	G	G
	A_3	G	MG	MG
C_5	A_1	G	G	MG
	A_2	F	MG	F
	A_3	F	MG	F
C_6	A_1	P	MP	MP
	A_2	MP	P	P
	A_3	MP	VP	VP
C_7	A_1	MG	MG	MG
	A_2	MP	F	F
	A_3	F	F	MP
C_8	A_1	G	G	MG
	A_2	F	MG	F
	A_3	MG	F	F
C_9	A_1	MP	F	MP
	A_2	P	MP	MP
	A_3	P	F	F
C_{10}	A_1	MP	P	MP
	A_2	MP	MP	P
	A_3	P	P	P
C_{11}	A_1	MG	F	MG
	A_2	P	MP	MP
	A_3	P	P	P
C_{12}	A_1	MG	MG	MG
	A_2	F	MP	F
	A_3	MP	F	F
C_{13}	A_1	G	G	MG
	A_2	G	F	F
	A_3	MG	F	F
C_{14}	A_1	G	MG	G
	A_2	MG	MG	F
	A_3	F	F	F
C_{15}	A_1	MG	F	MG
	A_2	F	F	MG
	A_3	MP	MP	MG

proposed HFPS method versus the Wei (2012) and Xu and Zhang (2013) methods to represent the efficiency of the proposed approach. In this respect, the case study is solved by using the recent hesitant fuzzy TOPSIS method proposed by Zhang and Wei (2013) that leads to the same results as the proposed approach. In addition, to show the reliability of these results, the case study is

Table 5
The hesitant fuzzy preference values regarding the mean of the normalized value of criteria j.

Criteria	$\overline{R}_j^k$			$HFPV_j^k$		
	$k=1$	$k=2$	$k=3$	$k=1$	$k=2$	$k=3$
C_1	0.600	0.616	0.666	0.298	0.121	0.354
C_2	0.500	0.583	0.600	0.253	0.109	0.121
C_3	0.633	0.616	0.666	0.416	0.121	0.354
C_4	0.600	0.616	0.666	0.298	0.295	0.215
C_5	0.416	0.475	0.466	0.366	0.366	0.192
C_6	0.300	0.408	0.433	0.136	0.149	0.075
C_7	0.500	0.583	0.666	0.253	0.109	0.215
C_8	0.666	0.716	0.733	0.581	0.683	0.472
C_9	0.466	0.441	0.466	0.071	0.229	0.131
C_{10}	0.300	0.408	0.350	0.300	0.093	0.163
C_{11}	0.600	0.616	0.666	0.142	0.295	0.215
C_{12}	0.500	0.550	0.633	0.126	0.260	0.128
C_{13}	0.500	0.616	0.666	0.273	0.121	0.144
C_{14}	0.700	0.683	0.833	0.409	0.154	0.354
C_{15}	0.500	0.550	0.666	0.239	0.260	0.144

Table 6
The hesitant fuzzy overall preference values.

Criteria	ψ_j^k		
	$k=1$	$k=2$	$k=3$
C_1	0.064798	0.075572	0.055077
C_2	0.068930	0.076602	0.075036
C_3	0.053871	0.075572	0.055077
C_4	0.064798	0.060560	0.066940
C_5	0.058445	0.054480	0.068939
C_6	0.079678	0.073157	0.078966
C_7	0.068930	0.076602	0.066940
C_8	0.038676	0.027188	0.045022
C_9	0.085683	0.066273	0.074120
C_{10}	0.064586	0.077970	0.071387
C_{11}	0.079185	0.060560	0.066940
C_{12}	0.080649	0.063586	0.074427
C_{13}	0.067040	0.075572	0.073026
C_{14}	0.054538	0.072723	0.055077
C_{15}	0.070191	0.063586	0.073026

Table 7
The values of the hesitant fuzzy preference selection index.

i	k		
	HFI_i^1	HFI_i^2	HFI_i^3
A_1	0.462351	0.463706	0.49089
A_2	0.394742	0.422885	0.44686
A_3	0.351112	0.417993	0.43830

Table 8
The computational results of the DMs' weight.

Decision makers	$\Im_k^+$	$\Im_k^-$	$\wp_k^F$
DM_1	0.04263	0.06844	0.34327
DM_2	0.03419	0.05025	0.33152
DM_3	0.04150	0.05820	0.32521

also solved by the Xu and Zhang's (2013) method. The comparative analysis is reported in Table 9.

Although, the comparative analysis in Table 9 showed that the ranking results obtained from the proposed HFPS method, Zhang and Wei (2013), and Xu and Zhang (2013) are the same, but this study's approach might be suitable when dealing with imprecise situations, because of some of the merits and advantages that enhance the evaluation approach. Indeed, the dispersion of the candidates' ranking values in the proposed approach is higher than that of Zhang and Wei's (2013) and Xu and Zhang's (2013) studies in which it may be more difficult for DMs to select the most suitable candidate when ranking values are very close together. In fact, the proposed approach takes advantage of the last aggregation method to prevent data loss as much as possible and compute the DMs' weight based on the proposed hesitant fuzzy compromise solution technique to decrease judgment errors. The obtained candidate ranking values based on the proposed approach and that of two other existing studies are depicted in Fig. 3 to better present the dispersion ranking values.

Moreover, some other comparative factors, such as agility in the decision procedure, modeling of uncertainty, the impact of DMs' weights, and the impact of the last aggregation approach are considered when comparing the obtained results from the three methods. Thereby, the agility of the decision process factor evaluates the amount of judgments which are needed from DMs. All three methods are sufficient in this comparative factor, because k judgments are required for each m alternative which the agility in the decision procedure factor for all three methods is $k \times m$. Also, the modeling of uncertainty for all three methods is sufficient, because these studies considered the hesitant fuzzy theory in order to cope with incomplete information. Thus, the computation of the DMs' weights and last aggregation approach factors are provided in this study, whereas these aspects were not considered in the methods proposed by Zhang and Wei (2013) and Xu and Zhang (2013). Therefore, by indicating the impact of these factors in ranking of final candidates shows that the proposed HFPSI method could deal with real situations appropriately.

In this case, as represented in Fig. 4, the variations of final hesitant fuzzy preference selection values according to each candidate are depicted regarding the various DMs' weights. Meanwhile, this analysis is decomposed based on the first and last aggregation approach which are shown in Fig. 4(a) and Fig. 4(b), respectively. As surveyed in the sensitivity analysis section, the last aggregation approach versus the first aggregation approach leads the obtained ranking results to more dispersal. On the other hand, the last aggregation approach could rank the candidates by making a great difference in final hesitant fuzzy preference selection values. Fig. 4 indicates that changing the weight of DMs could affect the final hesitant fuzzy preference selection values. Thus, the obtained ranking results could become more reliable by computing and considering the DMs' weights as well as the last aggregation approach in the procedure of the proposed approach. Consequently, the efficiency of the proposed HFPSI method when compared to Zhang and Wei's (2013) and Xu and Zhang's (2013) methods is validated.

6. Sensitivity analysis

In this section, the mining contractor selection problem is solved trice. We first solve the problem with the proposed approach. Then, the weight of each DM and the last aggregation approach in the procedure of the proposed method is ignored and the problem is

Table 9
Ranking the potential alternatives and comparative analysis.

HFI_i^F		Ranked by proposed HFPSI method	The Relative closeness coefficient based on Zhang and Wei (2013)' method	Ranked by Zhang and Wei (2013)' TOPSIS method	The Relative closeness coefficient based on Xu and Zhang (2013)' method	Ranked by Xu and Zhang (2013)' method
HFI_1^F	0.472246	1	0.364684	1	0.59374	1
HFI_2^F	0.421414	2	0.350739	2	0.57903	2
HFI_3^F	0.402788	3	0.348967	3	0.56612	3
Standard deviation	0.035952		0.008608		0.01382	

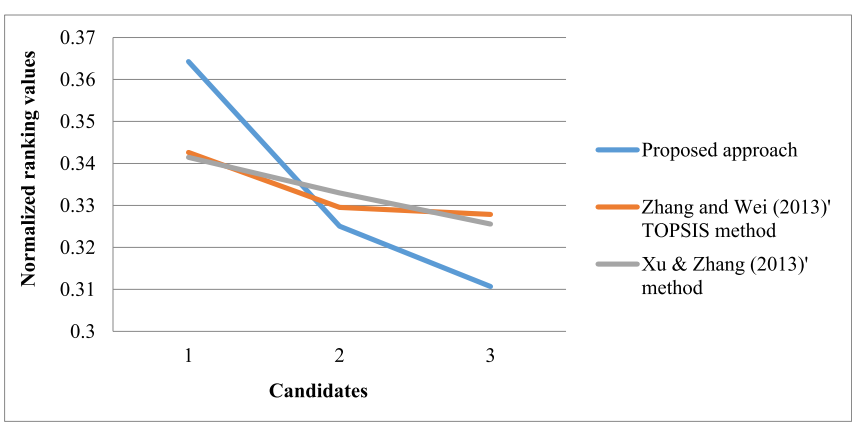

Fig. 3. The dispersion of candidates ranking values based on the proposed approach vs. two other studies.

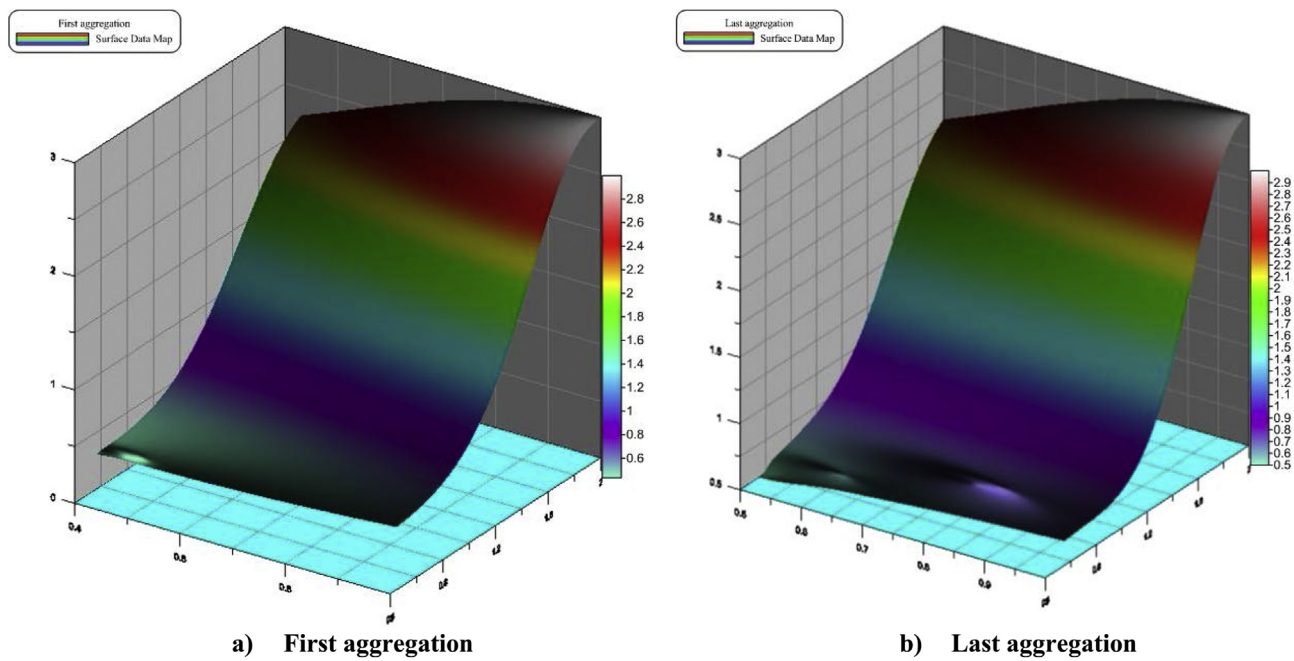

Fig. 4. Representing the impact of the DMs' weight regarding the first and last aggregation.

Table 10
The results of sensitivity analysis.

HFI_i^F	Ranking values based on proposed approach	Ranking values by eliminating the DMs' weights	Ranking values by eliminating the last aggregation approach
HFI_1^F	0.472246	0.472482	0.458365
HFI_2^F	0.421414	0.421887	0.439624
HFI_3^F	0.402788	0.403605	0.430751
Standard deviation	0.035952	0.035679	0.014098

solved again to highlight the sensitivity and robustness of the candidates' rankings to these processes. Therefore, changing the DMs' weights shows that the ranking results remained unchanged. This means that the ranking values in versus, considering the proposed hesitant fuzzy compromise solution technique, are robust. As indicated in Table 10, the dispersion of the ranking values becomes smaller by eliminating the proposed hesitant fuzzy compromise solution technique for determining the DMs' weight in the procedure of the proposed approach. On the other hand, when the last aggregation method is eliminated the dispersion of ranking values gets worse. Therefore, both the DMs' weights and the last aggregation method can help with ranking the candidates in a

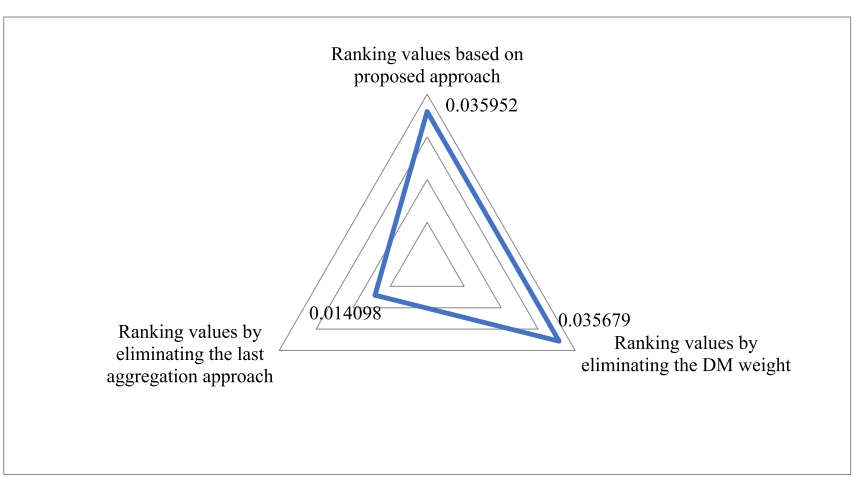

Fig. 5. The radar representation for indicating the elimination of each considered approach vs. the proposed method.

more reliable manner with a considerable dispersion in ranking values. In addition, the lower dispersion of the ranking values in the studies of Zhang and Wei (2013) and Xu and Zhang (2013) might be derived from ignoring the DMs' weight and the lack of the last aggregation method. Finally, the dispersion effect of each eliminating approach is depicted in Fig. 5.

7. Conclusions and future courses of action

The multi-criteria analysis method has focused on assessing and ranking a set of alternatives in a hesitant fuzzy situation while multiple conflicting criteria are considered. HFS was utilized to address the imprecision of the criteria and the corresponding importance of the chosen criteria. In this paper, a new soft computing method of multi-criteria decision-making, HFPSI, by a group of decision makers (DMs) or experts with risk preferences was proposed. Thereby, the proposed approach has applied the concept of HFSs for the evaluation and selection of problems under uncertainty. The introduced method improved the fuzzy based preference relation approach by considering uncertainty in a more practical and sophisticated way. It also helped the DMs to gather data in linguistic terms, which enables the method to be applied in real-life situations. Moreover, it was based on hesitant fuzzy and preference relation outcomes and was able to provide the practitioner with a justified ranked ordering of alternatives based on the opinions of a group of experts. Moreover, the hesitant fuzzy compromise solution method is presented in order to provide the weight of each DM to decrease judgment error. In addition, the last aggregation approach is considered in the procedure of the proposed method to prevent data loss.

Finally, a case study from the mining industry was outlined and solved using the proposed HFPSI method. The results show that the first mining contractor was selected as the most suitable candidate. Furthermore, two recent studies of hesitant fuzzy decision-making approaches were also implemented in the case study. However, the same ranking results were obtained from three approaches that the better dispersion of ranking values in the proposed approach makes it more reliable and accurate for real-world decision-making problems. The advantages and merits of the proposed approach (determining DMs' weights and considering the last aggregation method) were then examined to show the efficiency and usefulness of the presented approach by comparing it to two other existing studies. The results of sensitivity analysis indicated that the proposed hesitant fuzzy compromise solution technique and the last aggregation method could increase the dispersion of candidates ranking values by decreasing errors and preventing data loss.

For further research, providing a comprehensive decision-making framework based on the proposed method would be a good direction, which can be implemented to solve real-life complex decision-making problems. Moreover, the proposed approach can be enhanced by developing an expert evaluation system to assess the potential candidates under the conflicted criteria. In addition, considering the hierarchical structure and creating a procedure to compute the interdependencies between and within the criteria are some potentially interesting approaches in presenting hesitant fuzzy decision-making methods. Future work in the field of the sustainable mining contractor selection problem can compare its findings to this study to test the sustainability assurance.

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Appendix A. Supplementary data

Supplementary data related to this article can be found at https://doi.org/10.1016/j.jsm.2017.12.006.

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