Current aspects of mathematical modelling of evaluating quality of agricultural products

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Abstract. The paper presents mathematical model of complex evaluation of the quality of production with optimizing indicators of crop growing. Multicriterion model is reduced to the problem of optimizing with the predetermined target function which considers almost all parameters of the prodution quality. The model takes into account limits of modifications of quality characteristics of production by introducing the indistinct description of the quality characteristics of productions. The model was developed on the example of evaluating quality of the barley grain.

Key words: target function, fuzzy restriction, vector-optimizing model.

INTRODUCTION

After signing the political part of Association Agreement with Ukraine, the European Union unilaterally opened the Ukrainian commodities access to the European markets. This gives our country unprecedental conditions supplying of our commodities at the European markets. The best opportunities are possessed by agricultural producers because of cancelling import duties for agricultural products. However, the quality and safety of products they produce and supply should correspond to the requirements of consumers. The described above situation gave birth to the acute demand in developing complex system of quantative evaluation of agricultural production.

ANALYSIS OF RECENT RESEARCH AND PUBLICATIONS

The given problem of systemic evaluation of the quality of agrarian production is in the budding stage now. The work [8] presents the theoretically generalized concept of a virtual measure of quality and of its basis developed structural system of determining the level of the quality of production. The work [9] highlights theoretical foundations of the use of methods of its multidimensional scaling to combine several single parameters of quality of production to the single scale. Developing such a scale is a rather complicated objective. We believe it would be more expedient to apply calculating methods of evaluating quality of production with setting a functional dependence of a complex indicator of quality from a number of single quality indicators or input parameters of manufacturing (growing) agricultural products.

Prof. A. Dolzhanskyi [5] suggests a mathematical model of quality evaluation where the single quality indicators are presented in rows. The rows are formed on the basis of experimental data focused on the problem of influence of technical, technological and organization parameters on a single quality characteristic of production. Complex indicator of quality is presented by the functional (the average value of functions of the single quality indicators). The suggested mathematical model is studied on the example of calculating complex quality indicators of a steel wire after dragging two single indicators of quality. The work determined maximum value of the composite indicator of quality, but this value is reached by the functional at unrealistic values of input parameters. Thus, the suggested mathematical model of complex evaluation of quality ignores the restriction of real values of input parameters.

To create a general system of obtaining flax fibers with predictable properties, the work [4] developed a mathematical model of predicted properties of the production and quality control in the technological process. Using the developed mathematical model you can optimize the properties of the resulting product. However, the paper presents only the results of optimization flax fibers and ignores the mathematical mechanism of this model.

Safety and quality of agricultural products are studied in various aspects. The work [13] analyzes regional and national food staff safety at the state level in Ukraine and Russia. The work [14] offers economically-mathematical modeling of factors influencing the formulation of competitiveness of milk production and procession. One of the factors influencing the competitiveness of milk is its quality. Thus the problem of quality of agricultural production is important in terms of the improving competitiveness of enterprises.

Research in the area of quality evaluation of production is carried out in the direction of complex evaluation of quality and evaluation of quality characteristics of production. In particular, the problem of quality of water is discussed in the work [15].

In our opinion, the problem of complex evaluation of quality of production is advisable to reduce to optimizing with application of the theory of fuzzy sets. This idea has been suggested in the work [3]. The author has restricted his analysis to studying optimal indices of quality of production under conditions of certainty. However, the production may be of a high quality in case it does not necessarily reach a certain specified value and stays within the predetermined limits. To account the admissible modifications of some separate characteristics of quality at mathematical modeling of the complex indicator of the production quality it is advisable to apply the theory of fuzzy sets. It is also advisable to work out some recommendations on providing progressive norms of the quality of production on the basis of its mathematical model.

OBJECTIVES

The given research is aimed at the development and studies of mathematical model of complex avaluation of the quality of agricultural production.

MAIN PRESENTATION

Quality of production depends on many characteristics. In mathematical modeling of complex evaluation of quality we shall take into account the most essential features. The irrelevant features will not be introduced into the mathematical model. But their influence will be taken into consideration by means of approximation of non-linear problems by linear problems with fuzzy coefficients. Thus, we shall accepts the decision based on the mathematical model building where purposes and restrictions are formulated undistinctly. Analysis of difficult qualimetric systems built with the use of fuzzy set theory can correctly describe the status of the product and compare it with the basic product. A fuzzy set $\tilde{A}$ is defined by the base scale X and the belonging function $\mu_{\tilde{A}}(x)$. Functions of belonging accept values in the interval [0, 1]. A fuzzy set $\tilde{A}$ - is a totality of pairs of the form [12]:

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)),\ x \in X\}. \qquad (1)$$

If the basic scale is discrete and finite, that is $X = \{x_i\}_{i=1}^{n}$. Then the fuzzy set can be written as:

$$\tilde{A} = \sum_{i=1}^{n} \frac{x_i}{\mu_{\tilde{A}}(x_i)}, \qquad (2)$$

where: x_i - i-meaning of the base scale.

Functions of belonging $\mu_{\tilde{A}}(x)$ determines the degree of the expert confidence that the given base value of the scale corresponds to the fuzzy set.

If the functions of belonging is continuous, it can be presented in the bell, triangular or trapezoidal form.

In addition, for producers is important both to evaluate the production and to receive recommendations of getting sufficient output and the highest quality of the products. Thus, we put two objectives of reaching maximum results in quality and quantity of production. Sometimes they are contradictionary. One should, therefore, find a compromising decision. In such case it is advisable to apply the vector-optimizing model [11]:

$$Z(x) = \begin{pmatrix} z_1(x) \\ \vdots \\ z_n(x) \end{pmatrix} = \begin{pmatrix} C_1 \cdot x \\ \vdots \\ C_n \cdot x \end{pmatrix} \to \max, \qquad (3)$$

under conditions:

$$g_i(x) \equiv a_i \cdot x \tilde{\le} b_i, b_i + d_i \quad i = \overline{1, m_1}$$
$$g_i(x) \equiv a_i \cdot x \le b_i, \quad i = \overline{m_1 + 1, m}. \qquad (4)$$

Vectors:

$x = (x_1;...;\ x_n)$.

$C_k = (c_{1k};\ ....;c_{nk})$; $k = \overline{1, K}$.

$a_i = (a_{i1};\ ....;\ a_{in})$, $i = \overline{1, m}$.

and numbers

b_i $i = \overline{1, m}$, $d_i > 0$ $i = \overline{1, m_1}$.

are real:

Let us formulate a multicriterion linear optimizing model of evaluation of quality of agricultural production with the simultaneous optimizing parameters of growing.

$$Z(x_1, x_2, ..., x_n) = \begin{pmatrix} z_1(x_1, x_2, ..., x_n) \\ \vdots \\ z_r(x_1, x_2, ..., x_n) \end{pmatrix} =$$

$$= \begin{pmatrix} 1 - \sqrt{\sum_{j=1}^{p} w_j^2 (\sum_{i=0}^{n} a_i x_i - \mu_j(x_1, x_2, ..., x_n))^2} \\ \vdots \\ \sum_{i=0}^{n} a_i x_i \end{pmatrix} \to \max, \qquad (5)$$

where $\mu_j(x_1, x_2, ..., x_n)$ - functions of belonging, which may have a triangular, a trapezoidal, a bell-shaped and other forms. The function of belonging determines the rate of assurance of an expert in the position, that the given value of the basic scale corresponds to the fuzzy sets,

w_j - coefficient of the weightiness of the j-index of quality.

In such situation the conditions (4) and x_0=1 should be valid.

When functions of belonging have a trapezoidal form, then in (5) $\mu_j(x_1, x_2, ..., x_n)$ is determined by the kernel $[\underline{q_1}, \overline{q_1}]$ and carrier $[\underline{q_0}, \overline{q_0}]$.

To solve this problem we use the position of Bellman-Zadeh. According to the mentioned idea the solution means the crossing objectives and restrictions.

Here we shall compare various objectives:

$$\max_{x \in X_U}(Z_1(x);....; Z_k(x); \mu_1(x);....; \mu_{m_1}(x)) . \quad (6)$$

Let us construct the optimization model:

$$\begin{aligned} &\lambda \to \max \\ &\lambda \le \mu_z(x) \\ &\lambda \le \mu_i(x) \qquad \forall i = \overline{1, m_1} \\ &0 \le \lambda \le 1 \quad 0 \le \lambda \le 1 \quad x \in \bigcup \end{aligned} , \quad (7)$$

$\mu_z(x) \mu_i(x)$ - functions of belonging of targets and limitation:

$$\lambda = \min(\mu_z(x), \mu_1(x),..., \mu_{m_1}(x)) .$$

Each target value $Z_k = Z_k(x)$ will be compared with the measure $\tilde{\mu}_{z_k}(z_k)$:

The optimal solution $X_k^{**} = \max_{x \in X} Z_k(x)$, (8)

where:

$$\overline{X} = \begin{Bmatrix} x \in X^n \mid & g_i(x) = a_i \cdot x \le b_i + d_i \forall i = \overline{1, m_1} \\ & g_i(x) = a_i \cdot x \le b_{i\,i} \forall i = \overline{m_1 + 1, m} \end{Bmatrix}.$$

Maximum value:

$$\overline{Z_k} = Z_k(x_k^{**})$$

All functions of belonging are linear and the fuzzy problem of optimization will be reduced to the determined form:

$$\begin{aligned} &\lambda \to \max \\ &d_0 \lambda - c \cdot x \le -(w_0 - d_0) \\ &\lambda d_i + a_i \cdot x \le b_i + d_i \forall i = \overline{1, m_1} \\ &a_i \cdot x \le b_i \forall i = \overline{m_1 + 1, m} \\ &x \ge 0; \lambda \ge 0. \end{aligned} \quad (9)$$

$\underline{w} = \max\ z(x)$,

$$a_i \cdot x \le b_i \forall i = \overline{1, m} .$$

x≥0,

$\overline{w} = \max\ z(x)$

$$a_i \cdot x \le b_i + d_i \forall i = \overline{1, m_i} .$$

$$a_i \cdot x \le b_i \forall i = \overline{m_1 + 1, m} .$$

x≥0

Studies of the model are carried out on the example of evaluating quality of the brewery barley with maximization of the grains mass depending upon the applied mineral fertilizers in crop growing. One should give some recommendations on the amount of fertilizers applied during the brewery barley growing to get 8% of protein in a grain but less than 9-12%. The starch content should be more than the 60-70% [2]. The barley grain should get the maximum value of the mass of each grains.

Let us formulate the mathematical model:

$$Z(x_1, x_2, x_3) =$$

$$\left(1 - \sqrt{ \begin{matrix} w_1^2 \cdot (2{,}47 x_1 + 2{,}47 x_2 - 2{,}07 x_3 + 7{,}7 - [9;11])^2 + \\ w_2^2 \cdot (-0{,}8 x_1 - 0{,}8 x_2 - 0{,}54 x_3 + 67{,}83 - [60;75])^2 \end{matrix} } , \atop 2{,}93 x_1 + 2{,}93 x_2 - 2{,}33 x_3 + 37{,}23 \right) \quad (10)$$

$\to \max$

At the conditions:

$$\begin{aligned} &g_1(x) = -0{,}247 \cdot x_1 - 0{,}247 \cdot x_2 + 0{,}207 \cdot x_3 \le -0{,}03 \\ &g_2(x) = 0{,}247 \cdot x_1 + 0{,}247 \cdot x_2 - 0{,}207 \cdot x_3 \le 0{,}13;0{,}13 + 0{,}3 \\ &g_3(x) = 0{,}08 \cdot x_1 + 0{,}08 \cdot x_2 + 0{,}054 \cdot x_3 \le 0{,}78 \end{aligned} \quad (11)$$

$$\begin{aligned} &g_4(x) = x_1 \ge 0{,}1 \\ &g_5(x) = x_1 \le 0{,}3 \\ &g_6(x) = x_2 \ge 0{,}1 \\ &g_7(x) = x_2 \le 0{,}3 \\ &g_8(x) = x_3 \ge 0{,}15 \\ &g_9(x) = x_3 \le 0{,}4 \end{aligned} \quad (12)$$

x≥0.

Flexible ratios are composed on the basis of the structural matrix:

$$A = \begin{pmatrix} -0{,}247 & -0{,}247 & 0{,}207 \\ 0{,}247 & 0{,}247 & -0{,}207 \\ 0{,}08 & 0{,}08 & 0{,}054 \end{pmatrix}, \quad (13)$$

received from the experimental data of the influence of fertilizers on the brewery quality of the spring barley grain and conditions corresponding to the indices of brewery barley grain.

w_1 and w_2 - coefficients of the weightness of the percent content of the protein and the starch are determined by the method of experts.

We shall receive the value:

$\underline{w} = z(x_1, x_2, x_3) = z(1;1;1{,}5) = 3{,}5$, and

$\overline{w} = z(x_1, x_2, x_3) = z(1;1;1{,}75) = 3{,}75$.

Let us put down the functions of belonging:

$$\mu_z(x) = \begin{cases} 0 & w = z(x) < \underline{w} \\ \dfrac{z(x) - \underline{w}}{\overline{w} - w} & \underline{w} \le w = z(x) < \overline{w} \\ 1 & w = z(x) \ge \overline{w} \end{cases}$$

$$\mu_z(x_1, x_2, x_3) =$$

$$= \begin{cases} 0 & x_1 + x_2 + x_3 < 3{,}5 \\ \dfrac{x_1 + x_2 + x_3}{0{,}25} & 3{,}5 \le x_1 + x_2 + x_3 < 3{,}75 \\ 1 & x_1 + x_2 + x_3 \ge 3{,}75 \end{cases},$$

$$\mu_z(x_1, x_2, x_3) =$$

$$= \begin{cases} 1 & aa \le 0{,}13 \\ 1 - \dfrac{aa - 0{,}13}{0{,}3} & 0{,}13 < aa \le 0{,}43 \\ 0 & aa > 0{,}43 \end{cases}$$

Where:

$$aa = 0,247\, x_1 + 0,247\, x_2 - 0,207\, x_3$$

Having solved the problem (11) - (12), we shall find the maximum value:

$$\overline{Z_k} = Z_k(x_k^{**})$$

$$\overline{Z}_1 = Z_1(1;1;1,5) = 1 \quad \text{for } x_1^{**}=(1;1;1,5),$$

$$Z_1(x_1^*) = Z_1(3;2;4) = 0,99 \quad \text{for } x_1^*(3;2;4).$$

$Z_2(x_1, x_2, x_3) = 2,93x_1 + 2,93x_2 - 2,33x_3 + 37,23 \rightarrow \max$ under conditions (11)-(12) we shall get optimal solution $(x_1, x_2, x_3) = (3;2;4)$ with the maximum value of the target function:

$Z_2^*=Z_2^*(x_1^*, x_2^*, x_3^*) = Z_2(3;2;4)= 42,84.$

$Z_2^*=Z_2^*(x_1^*, x_2^*, x_3^*) = Z_2(1;1;1,5)= 39.$

Thus, we shall get the determined quality of production having applied the fertilizers N_{60} P_{45} K_{120}. We may also apply the fertilizers N_{30} P_{30} K_{45}. But the mass of grain in such a can will be less.

CONCLUSIONS

1. The problem of evaluating quality of agricultural production with the simultaneous optimization of parameters of growing can be solved by constructing mathematical multicriterion model of the evaluation of quality.
2. We presented the dual-purpose optimizing model of evaluating quality of production with the simultaneous optimizing the indices of growing. The model takes into account the limits of modifications of quality characteristics of production by means of introduction of fuzzy descriptions of characteristics of the production quality.
3. To find the solution of the suggested model we used the multi-purpose approach of Bellman-Zadeh.
4. The approbation of the suggested model of evaluating quality of agricultural production with the simultaneous optimization of the grains mass and the ammounts of mineral fertilizers is carried out on the basis of the analysis of the influence of mineral fertilizers on the quality and mass of brewery barley grain.

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