PI observer stability and application
in an induction motor control system

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Abstract. The paper discusses the problem of stability of a proportional-integral Luenberger observer, designated for the state variables reconstruction of a linear, time-invariant dynamical system. It is proven, that there exists such a class of observed systems, for which the observer is always unstable, independently of its gains. Stability can be provided in every possible case after application of proposed modifications to the structure of the observer. It is proven, that stability of the modified observer depends only on its gains. It is shown, that an induction motor is the exemplary observed system, for which application of the unmodified observer is impossible due to its lack of stability, while the modified observer provides proper operation of the control system. Finally, some experimental results are presented, obtained in the multiscalar control system of the induction motor, equipped with the modified proportional-integral observer.

Key words: Luenberger observer, induction motor, multiscalar control system.

1. Introduction

A proportional (P) observer is the most basic of all Luenberger observers. It is applied, among the others, to reconstruction of induction motor state variables. There are also known other types of observers, equipped with more complex feedbacks. These are: an observer with integrators [1, 2], a modified integral observer [3, 4], a proportional integral (PI) observer [3], function observers [5], and the others. Until now, the PI observer was not applied in control systems of induction motors.

The PI observer is characterised by stronger attenuation of reconstruction errors and provides better reconstruction quality than the P observer. Every of non-proportional observers mentioned above, in some cases can remain unstable independently of its gains selection. Therefore, the practical application of non-proportional observers requires some modifications to be introduced to their structures.

In this paper, conditions that determine instability of the PI observer are specified. Proper modification of its mathematical model is also proposed, providing stability in every possible case.

2. The mathematical model of the PI observer

The mathematical model of the observed system is the basis for the observer design. It is a linear time-invariant dynamical system with p inputs included in the vector $\boldsymbol{u}$, n state variables included in vector $\boldsymbol{x}$ and q outputs included in vector $\boldsymbol{y}$, described with a matrix differential state equation and a matrix algebraic output equation, using dimensionless quantities:

$$\begin{cases} t_b\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} \\ \boldsymbol{y} = \boldsymbol{C}\boldsymbol{x} \end{cases}, \qquad (1)$$

where: t_b - base time; $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ – real constant matrices.

State variables of the system described with (1) can be reconstructed with use of a PI observer, described with the following system of equations:

$$\begin{cases} t_b\dot{\widehat{\boldsymbol{x}}} = \boldsymbol{A}\widehat{\boldsymbol{x}} + \boldsymbol{B}\boldsymbol{u} + \boldsymbol{K}_P\left(\boldsymbol{C}\widehat{\boldsymbol{x}} - \boldsymbol{y}\right) + \boldsymbol{w} \\ t_b\dot{\boldsymbol{w}} = \boldsymbol{K}_I\left(\boldsymbol{C}\widehat{\boldsymbol{x}} - \boldsymbol{y}\right) \end{cases}, \qquad (2)$$

where $\boldsymbol{w}$ – additional state vector of size n; $\widehat{\boldsymbol{x}}$ - reconstructed state vector of the observed system; $\boldsymbol{K}_P$ and $\boldsymbol{K}_I$ – real constant feedback matrices of the proportional and the integral unit correspondingly. The block diagram of the observed system and the PI observer are shown in Fig. 1. Observer gains, that is, the elements values of matrices $\boldsymbol{K}_P$ and $\boldsymbol{K}_I$, should be chosen so that the observer is stable and exhibits desired dynamical properties [3, 4].
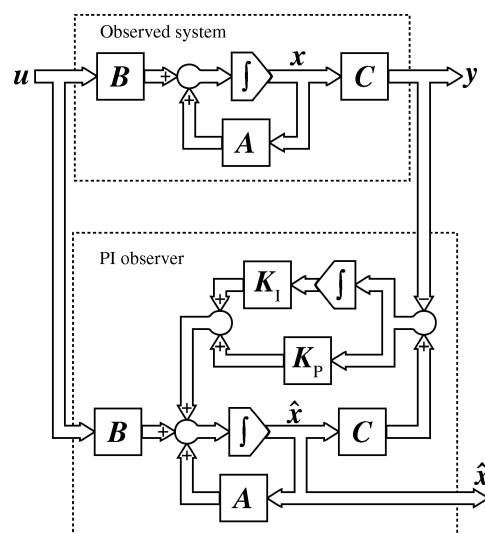

Fig. 1. The block diagram of a linear system and a PI observer

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3. Error equation

The error equation of the observer is the basis for the analysis of its stability and dynamical properties. The reconstruction error vector is defined as:

$$\varepsilon = \widehat{x} - x. \tag{3}$$

Due to the mathematical model of the observer, consisting of two differential equations and containing two state vectors $\widehat{x}$ and w, a new generalized state vector $\widehat{\xi}$ must be introduced, in order to derive one error equation. The state vector of the observed system has to have the same size as the state vector of the observer. Therefore, the state vector x of the observed system is to be correspondingly extended with null vector of size n. The resulting extended state vector of the observed system ξ, and the state vector of the observer are given as:

$$\widehat{\xi} = \left[\begin{array}{c} \widehat{x} \\ w \end{array} \right], \qquad \xi = \left[\begin{array}{c} x \\ 0_{n \times 1} \end{array} \right]. \tag{4}$$

On introducing new state vectors, equations of the observed system assume following new form:

$$\left\{ \begin{array}{l} t_b \dot{\xi} = A_\xi \xi + B_\xi u \\ y = C_\xi \xi \end{array} \right. , \tag{5, 6}$$

where the matrices are given as follows:

$$A_\xi = \left[\begin{array}{cc} A & 1_n \\ 0_{n \times n} & 0_{n \times n} \end{array} \right], \quad B_\xi = \left[\begin{array}{c} B \\ 0_{n \times p} \end{array} \right],$$

$$C_\xi = \left[\begin{array}{cc} C & 0_{q \times n} \end{array} \right]. \tag{7}$$

1_n denotes identity matrix of n-th order. The identity matrix included in the matrix A_ξ does not result from the mathematical model of the observed system and was introduced in order to keep consistency with the model of the observer. Equations (6), (7) and (4) show that the identity matrix 1_n corresponds with null vector included in the vector ξ, therefore, it has no impact on properties of the observed system.

On introducing new state vectors, the mathematical model of the observer is given as follows:

$$t_b \dot{\widehat{\xi}} = A_\xi \widehat{\xi} + B_\xi u + K_\xi \left(C_\xi \widehat{\xi} - y \right), \tag{8}$$

where K_ξ – a new gain matrix defined as:

$$K_\xi = \left[\begin{array}{c} K_P \\ K_I \end{array} \right]. \tag{9}$$

On having mathematical models of the observed system and the observer modified, the error equation of the observer can be derived from (5), (6) and (8):

$$t_b \dot{\varepsilon}_\xi = E \varepsilon_\xi \quad \text{where} \quad E = A_\xi + K_\xi C_\xi. \tag{10}$$

4. Stability of the PI observer

Basing on the error equation (10), it can be proved that there exist such observed systems described with (1), for which the PI observer described with (2) is always unstable, independently of its gains selection.

In order to ensure stability, all the eigenvalues of the state matrix E of the observer have to be placed in the left half of the complex plane. It is derived from (10), that the rank of E depends on ranks of matrices A_ξ, C_ξ and K_ξ:

$$\text{rank}(E) \leq \text{rank}(A_\xi) + \min(\text{rank}(K_\xi), \ \text{rank}(C_\xi)). \tag{11}$$

Ranks of matrices A_ξ, C_ξ and K_ξ, on the other hand, depend on their structures. It is derived from (7), that the matrix A_ξ has n linearly independent rows, therefore:

$$\text{rank}(A_\xi) = n. \tag{12}$$

It is derived from (8), that the number of rows of C_ξ equals q and is the lesser of its dimensions; similarly, we derive from (9) and (10), that the number of columns of K_ξ equals q and is the lesser of its dimensions, therefore:

$$\text{rank}(C_\xi) \leq q, \quad \text{rank}(K_\xi) \leq q. \tag{13}$$

It is derived from equations (11) to (13), that the rank of the state matrix E satisfies the following inequality:

$$\text{rank}(E) \leq n + q. \tag{14}$$

It is derived from (10), that the number of rows of the square matrix E equals $2n$. Therefore, it can be derived from inequality (14), that depending on the values of the elements of matrices K_P and K_I, the observer has at least $(n-q)$ eigenvalues equal to zero. Thus, if the number of outputs of the observed system q is less than the number of its state variables n, then the observer always has poles placed in the origin of coordinates on complex plane and does not satisfy the necessary condition of asymptotical stability (theorem 7.1.3 in [6]).

In order to provide stability of the PI observer in case of every possible observed system, independently of its number of outputs and state variables, the modification of the structure of the observer was proposed. Modification consists in replacing of integration in the feedback of the observer with a 1-st order inertia. A similar solution was proposed in [7]. The inertia introduces into the mathematical model of the observer an additional negative feedback:

$$\left\{ \begin{array}{l} t_b \dot{\widehat{x}} = A\widehat{x} + Bu + K_P \left(C\widehat{x} - y \right) + w \\ t_b \dot{w} = K_I \left(C\widehat{x} - y \right) - \omega_c 1_n w \end{array} \right. , \tag{15}$$

where ω_c – a vector of size n and positive non-zero elements, containing inverses of inertia time constants in dimensionless quantities. Introduction of the additional feedback has an impact on the form of the matrix A_ξ:

$$A_\xi = \left[\begin{array}{cc} A & 1_n \\ 0_{n \times n} & -\omega_c 1_n \end{array} \right]. \tag{16}$$

This change in the form of A_ξ has no impact on properties of the observed system, as it was in case of previously introduced identity matrix 1_n. The new form of A_ξ has an impact on its rank, previously determined by (12); after taking into consideration (16), it is given as:

$$\text{rank}(A_\xi) = 2n, \tag{17}$$

and the rank of the state matrix E:

$$\text{rank}(E) \leq 2n, \tag{18}$$

assuming that A is non-singular. The assumption of non-singularity of matrix A is equivalent to an assumption of the linear independence of the state variables of the observed system, and it is always satisfied while its mathematical model is appropriately formed.

The introduced modification enables in every possible case such a selection of observer gains that the state matrix E is non-singular and the observer is stable.

5. The mathematical model of the induction motor and experimental results

In case of considered mathematical model of the induction motor, vector sizes have following values: $n = 4$, $q = 2$. Therefore the PI observer, described with (2), is always unstable and the application of proposed modifications is necessary for its proper operation.

The basis for the observer design is the mathematical model of an induction motor assuming sinusoidal flux distribution in the air gap, the linear and lossless magnetic core and one rotor circuit [8], described in stationary Cartesian $\alpha-\beta$ coordinate system using dimensionless quantities. The electrical angular speed ω is treated as a parameter and it is reconstructed in the observer by the adaptation mechanism described in [9, 10]. The following forms of vectors were assumed for the motor:

$$\boldsymbol{x} = \left[\begin{array}{cccc} \psi_{s\alpha} & \psi_{s\beta} & \psi_{r\alpha} & \psi_{r\beta} \end{array} \right]^T,$$

$$\boldsymbol{u} = \left[\begin{array}{cc} u_{s\alpha} & u_{s\beta} \end{array} \right]^T, \qquad (19)$$

$$\boldsymbol{y} = \left[\begin{array}{cc} i_{s\alpha} & i_{s\beta} \end{array} \right]^T,$$

where $\psi_{s\alpha}$, $\psi_{s\beta}$ – magnetic fluxes coupled with the stator winding; $\psi_{r\alpha}$, $\psi_{r\beta}$ – magnetic fluxes coupled with the rotor winding; $u_{s\alpha}$, $u_{s\beta}$ – stator winding supply voltages; $i_{s\alpha}$, $i_{s\beta}$ – stator winding currents. Then the matrices, presented in a simplified block form, are given as:

$$\boldsymbol{A} = \left[\begin{array}{cc} \gamma R_s L_r \boldsymbol{1}_2 & -\gamma R_s L_m \boldsymbol{1}_2 \\ -\gamma R_r L_m \boldsymbol{1}_2 & \gamma R_r L_s \boldsymbol{1}_2 + \omega \boldsymbol{J} \end{array} \right],$$

$$\boldsymbol{B} = \left[\begin{array}{c} \boldsymbol{1}_2 \\ \boldsymbol{0}_{2\times2} \end{array} \right], \qquad \boldsymbol{C} = \gamma \left[\begin{array}{cc} -L_r \boldsymbol{1}_2 & L_m \boldsymbol{1}_2 \end{array} \right], \qquad (20)$$

where R_s, R_r, L_s, L_r and L_m – the parameters of the equivalent circuit of the motor [8]. The matrix $\boldsymbol{J}$ and the parameter γ are defined as:

$$\boldsymbol{J} = \left[\begin{array}{cc} 0 & -1 \\ 1 & 0 \end{array} \right], \qquad \gamma = \frac{1}{L_m^2 - L_s L_r}. \qquad (21)$$

The PI observer with proposed modifications, designed for an induction motor, was applied in a multiscalar control system [11, 12] and investigated during laboratory tests. Figure 2 shows transient waveforms recorded during investigations.
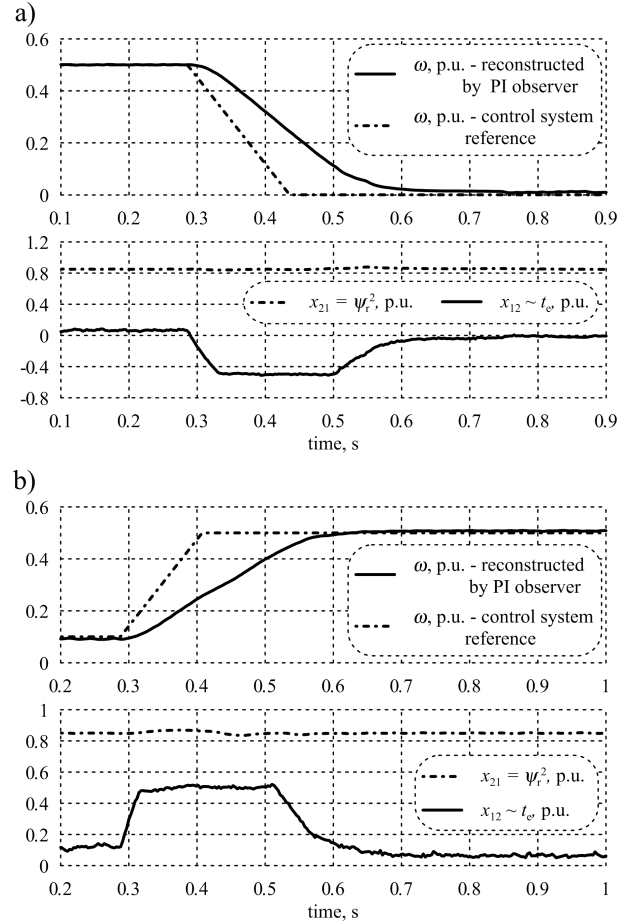

Fig. 2. Exemplary transient waveforms recorded during laboratory investigations in the multiscalar control system equipped with PI observer; a) braking at 0.5 of nominal speed; b) acceleration from 0.1 to 0.5 of nominal speed; t_e – motor torque

6. Conclusions

The PI observer provides better reconstruction quality than the P observer, but its structure is much more complicated. Without additional modifications the PI observer can be applied only to the systems with equal numbers of state variables and outputs. In case of systems with lower number of outputs, due to stability of the observer, the modification of the integral unit is necessary.

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