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Application of high resolution methods to underwater data processing

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Abstract - **The statistical approach to the high resolution acoustic signal parameter estimation is considered. The models of the Green's function for a deterministic and a random oceanic waveguide are presented. A brief discussion of popular superresolution methods with application to direction-of-arrival (DOA) and travel time estimation is done. Application of high resolution methods (HRM) to underwater data processing requires that certain modeling assumptions be made. Using maximum likelihood theory, the optimum processing algorithm for amplitudes, angles of arrival and travel times estimation is developed. The proposed techniques take into account the deterministic and the stochastic structure of the acoustic field.**

1. INTRODUCTION

High resolution array signal processing has attracted the great interest of many scientists in underwater acoustics field [1-3]. Fundamental idea of the HRM is to overcome the limitations of the classical methods based on Fourier transform in terms of resolution. Most of the HRM are data-dependent methods, i.e., they start with an estimate of the data covariance. They have a common characteristic which is that the array weight filter is continuously adapted to the noise background. Any array processing methods is based on a model of the propagation between the source and the receiver array. Usually, it is assumed that the received signals have a perfect spatial coherence that is only true for a deterministic, homogeneous medium. Oceanic waveguide is an irregular and a random medium. The ocean surface and floor are statistically rough surfaces. The sound speed field is also become a random due to internal waves, turbulence, inhomogeneities of fine structure and etc. The spatio-temporal variability of hydro-physical parameters of the ocean, sound pulse scattering from the ocean boundary roughness and inhomogeneities lead to such phenomena as multipath, anisotropy of noise, loss of signal coherence, etc. It is well known that HRM are very sensitive to propagation model errors. Thus, to improve efficiency of underwater data processing by HRM it is important to incorporate an adequate propagation model into array processing algorithms.

The aim of this paper is to develop high resolution signal parameter estimator matched to the deterministic and stochastic nature of underwater sound field. The array processing algorithms are synthesized using maximum likelihood theory.

2. OBSERVATION MODEL IN A RANDOM OCEANIC WAVEGUIDE

Consider an array consisting of M hydrophones that receive an acoustic field from the source located in a point with coordinates $\mathbf{r}_S$. In the frequency domain, the acoustic pressure received by the mth hydrophone can be written as follows

$$x_m(\omega) = s_0(\omega)G(\mathbf{r}_S, \mathbf{r}_m, \omega) + n_m(\omega) \qquad (1)$$

where $G(\mathbf{r}_S, \mathbf{r}_m, \omega)$ is the Green's function of the waveguide, $s_0(\omega)$ is the complex spectrum of the emitted signal, $n_m(\omega)$ is the noise pressure at the output of the mth hydrophone. Assuming the source is located in the Fraunhoffer zone of the array and using ray approximation, the Green's function can be represented as a sum of quasiplane wavefields

$$G(\mathbf{r}_S, \mathbf{r}_m, \omega) = \sum_{k=1}^{K} A_k(\omega) \exp\left\{ j\omega t_k + j\frac{\omega}{c}(\mathbf{e}_k, \mathbf{d}_m) \right\} \qquad (2)$$

where t_k is the travel time along the kth ray path, $\mathbf{e}_k$ is the unit vector of the DOA for the kth ray, $\mathbf{d}_m$ is the position vector of the mth array hydrophone, K is the number of rays connecting the source and the receiver, $A_k(\omega)$ is the amplitude of the kth ray taking into account propagation losses and sound absorption, c is the sound speed, ω is a circular frequency. For a linear, equally spaced array, using matrix-vector notation the observation model can be rewritten in the form

$$\mathbf{x}(\omega) = \mathbf{AS}(\omega) + \mathbf{n}(\omega) \qquad (3)$$

where

$$\mathbf{x}(\omega) = [x_1(\omega), x_2(\omega), \ldots, x_M(\omega)]^T,$$

$$\mathbf{S}(\omega) = [S_1(\omega), S_2(\omega), \ldots, S_K(\omega)]^T,$$

$$S_k(\omega) = A_k(\omega) \exp\{j\omega t_k\} s_0(\omega),$$

A is a $M{\times}K$ Vandermonde matrix with column vectors

$$\mathbf{a}(\theta_k)=\left[1,\exp\left\{j\frac{\omega}{c}d\sin\theta_k\right\},...,\exp\left\{j\frac{\omega}{c}d(M-1)\sin\theta_K\right\}\right]^T \quad (4)$$

where θ is the angle of arrival in the vertical plane, d is the hydrophone spacing.. The model of the Green's function (2) is valid for a deterministic, homogeneous ocean. In the scope of a stochastic ocean waveguide, the Green's function averaged on the medium fluctuations has the form (normal law of ocean medium parameters fluctuations is assumed) [4]

$$\bar{G}(\mathbf{r}_S,\mathbf{r}_m,\omega)=\sum_{k=1}^{K}A_k(\omega)\exp\left\{j\omega\bar{t}_k-\frac{1}{2}\omega^2D_k^t+ j\frac{\omega}{c}(\bar{\mathbf{e}}_k,\mathbf{d}_m)-\frac{1}{2}\frac{\omega^2}{c^2}\mathbf{d}_m^T\mathbf{D}_k^e\mathbf{d}_m\right\} \quad (5)$$

where $\bar{t}_k$, $\bar{\mathbf{e}}_k$ are the mean travel time and direction vector for the kth ray, respectively; $D_k^t,\mathbf{D}_k^e$ are the variances of travel time and direction of arrival fluctuations for the kth ray, respectively. Then, for the mean $\mathbf{a}(\theta_k)$ and $S_k(\omega)$ can write as follows

$$\bar{\mathbf{a}}(\theta_k)=\left[1,\exp\left\{j\frac{\omega}{c}d\sin\bar{\theta}_k-\frac{1}{2}\frac{\omega^2}{c^2}d^2D_k^\theta\right\},..., \exp\left\{j\frac{\omega}{c}d(M-1)\sin\bar{\theta}_K-\frac{1}{2}\frac{\omega^2}{c^2}d^2(M-1)^2D_k^\theta\right\}\right]^T \quad (6)$$

$$\bar{S}_k(\omega)=A_k(\omega)\exp\{j\omega t_k\}\exp\left\{-\frac{1}{2}\omega^2D_k^t\right\}s_0(\omega)$$

where $\bar{\theta}$ is the mean angle of arrival, D_k^θ is the variance of angle of arrival fluctuations.

3. REVIEW OF HIGH RESOLUTION METHODS

In this section we briefly consider the known HRM applicable to underwater acoustic signal processing. The estimation of the DOA from noisy data is a major task of sensor array processing. Many methods have been developed such as conventional Bartlett beamformer, Capon's adaptive beamforming [5], maximum entropy method [6], subspace approaches of Pisarenko [7], MUSIC algorithm [8,9], minimum norm algorithm [10], multidimensional search algorithms such as MODE [11], MD-MUSIC and weighted subspace fitting [12,13], maximum likelihood methods [13-15], etc. The aim of designing these methods is to exceed the Rayleigh limit of resolution and provide the accurate DOA estimation.

The resolution of the conventional Bartlett beamformer in which the outputs of each hydrophone is uniformly weighted and summed is proportional to the array size.

$$\Lambda_{BART}(\theta)=E\left\{\left|\mathbf{a}^H(\theta)\mathbf{x}\right|^2\right\}=\mathbf{a}^H(\theta)\mathbf{R}\mathbf{a}(\theta) \quad (7)$$

where $\mathbf{R}=\frac{1}{N}\sum_{n=1}^{N}\mathbf{x}(\omega_n)\mathbf{x}(\omega_n)^H$ is the array cross spectral densities matrix (CSDM) of the received acoustic field, $E\{\ \}$ denotes the mathematical expectation, the superscript 'H' denotes the Hermitian conjugate. So, for small array aperture the resolution is poor. In addition, the Bartlett method has high sidelobes that may mask weak signals. Increasing the array size or working frequency to overcome this problem is unattractive and impractical. Under these circumstances, it is natural to use HRM.

The first high resolution method was proposed by Capon [5]. This adaptive beamforming method attempts to reject interfering sources at the array output while maintaining unit gain and zero phase shift for each look direction. The angular spectrum of the Capon's method has the form

$$\Lambda_{CAP}(\theta)=\left[\mathbf{a}^H(\theta)\mathbf{R}^{-1}\mathbf{a}(\theta)\right]^{-1} \quad (8)$$

where $\mathbf{R}^{-1}$ is the inverse CSDM of the received signals. The angles of arrival are found by the locations of the spectrum peaks. The peak level also provides an estimate of the power. The Capon's estimator has considerably better resolution and lower level of sidelobes than the Bartlett beamformer.

Another group of the HRM consists of parametric methods such as autoregressive-maximum entropy [6] or linear prediction [16]. These methods are frequently used for time series analysis. The main idea is to assume a parametric model generating the observation. By using a parametric model, these methods predict the spatially sampled acoustic waveform beyond the antenna aperture and, then, "artificially" extend it. The linear predictive array output is defined by the following expression

$$\Lambda_{LP}(\theta)=\frac{\mathbf{u}_n^T\mathbf{R}^{-1}\mathbf{u}_n}{\left|\mathbf{u}_n^T\mathbf{R}^{-1}(\omega)\mathbf{a}(\theta)\right|^2} \quad (9)$$

where $\mathbf{u}_n$ is the column vector which has the number one in the position corresponding to the reference hydrophone and zeros elsewhere. The linear prediction methods are applied for equally spaced arrays.

Advanced techniques for array processing based on the eigendecomposition of the CSDM of the received signals have been discussed extensively in the literature the last two decades [8-14,16-20]. Schmidt and Bienvenue were pioneers in the application of the subspace method previously used by Pisarenko for estimation of sinusoids embedded in noise. Their method known as MUSIC based on a more complete modeling of the observation acoustic field than above mentioned ones. It requires the background noise to be spatially uncorrelated or of known correlation so that it can be whitened before using the MUSIC algorithm. The number of sources that are present in the acoustic field (signal's rays) is also assumed to be known and less than the number of the array hydrophones. High resolution feature of the MUSIC algorithm is based on the asymptotical properties of the eigendecomposition of $\mathbf{R}$,i.e., on the orthogonality between the noise subspace and the signal subspace spanned by the position vectors $\mathbf{a}(\theta)$. The MUSIC spectrum is defined as

$$\Lambda_{MUSIC}(\theta)=\left[\mathbf{a}^{H}(\theta)\sum_{i=p+1}^{M}\mathbf{V}_i\mathbf{V}_i^{H}\mathbf{a}(\theta)\right]^{-1} \quad (10)$$

where $\mathbf{V}_i$ are the eigenvectors of $\mathbf{R}$ associated with the smallest eigenvalues. The MUSIC algorithm provides a very good resolution and unbiased angle estimates. However, it is required more computations than other above described methods. The estimate of the number of the signal's rays can find using the information theoretical criteria AIC, MDL [21,22] or applying a multiple alternative hypothesis testing problem [13]. The described algorithm of the DOA estimation can be applied to the problem of travel time (time delay) estimation of multipath signal [23]. In this case, correlation matrix is formed by averaging on K realizations of the measured data vector $\mathbf{x}_k$ $\hat{\mathbf{R}}=\frac{1}{K}\sum_{k=1}^{K}\mathbf{x}_k\mathbf{x}_k^{*}$, where the vector $\mathbf{x}_k=\left[x_k(\Delta T),x_k(2\Delta T),\ldots,x_k(M\Delta T)\right]^{T}$, M is the length of the kth realization at the output of some hydrophone, ΔT is the sample rate. In this case, the steering vector describes time signature of the emitted signal

$$\mathbf{a}(\tau_i)=\left[s(\Delta T-\tau_i),s(2\Delta T-\tau_i),\ldots,s(M\Delta T-\tau_i)\right]^{T} \quad (11)$$

where τ_i is the travel time (time delay) of the ith signal's ray. For the Capon method, the processor output has the form

$$\Lambda_{CAP}(\tau)=\left[\mathbf{a}^{T}(\tau)\mathbf{R}^{-1}\mathbf{a}(\tau)\right]^{-1} \quad (12)$$

For the MUSIC algorithm, we have the following formula

$$\Lambda_{MUSIC}(\tau)=\left[\mathbf{a}^{H}(\tau)\sum_{i=p+1}^{M}\mathbf{V}_i\mathbf{V}_i^{H}\mathbf{a}(\tau)\right]^{-1} \quad (13)$$

It is well known that HRM are very sensitive to propagation model errors. The assumptions of a known array response and noise covariance are never satisfied in hydroacoustics. As ocean is a random medium and its parameters are fluctuated, the assumed model for the array steering vector $\mathbf{a}(\theta)$ (4) may be differ from the actual one. As a rule, in the HRM one neglect the medium influence to the model of the steering vector and scanning is made by using the "deterministic" steering vector (4). The mismatch between reality and assumed model might cause serious estimation problems. To improve efficiency of underwater data processing by HRM it is important to incorporate an adequate propagation model that taking into account the propagation in a fluctuating ocean into processing algorithm. Applying the steering vector (6) to HRM, we will have a more robust response at the processor output.

4. STATISTICAL APPROACH TO HIGH RESOLUTION ARRAY SIGNAL PROCESSING

According to the statistical theory of making decisions the optimum processing algorithm is a log-likelihood ratio [24]. We consider the following statistics of the received fields: the deterministic signal with the unknown amplitude embedded in the Gaussian noise with zero mean and unknown CSDM $\mathbf{R}_N(\omega)=E\{\mathbf{n}(\omega)\mathbf{n}(\omega)^{H}\}$.

Under hypotheses H_0 (signal absent) and H_1 (signal present), the joint probability density functions of the random vectors $\mathbf{X}(\omega_n)$ for $n=1,\ldots,N$ are given, respectively, by

$$\begin{aligned}P(\mathbf{x}/H_1)&=\prod_{n=1}^{N}\left|\pi\mathbf{R}_N(\omega_n)\right|^{-1}\cdot\\&\exp\{-[\mathbf{x}(\omega_n)-\mathbf{G}(\omega_n)s_0(\omega_n)]^{H}\mathbf{R}_N^{-1}(\omega_n)[\mathbf{x}(\omega_n)-\mathbf{G}(\omega_n)s_0(\omega_n)]\}\\P(\mathbf{x}/H_0)&=\prod_{n=1}^{N}\left|\pi\mathbf{R}_N(\omega_n)\right|^{-1}\exp\{-\mathbf{x}(\omega_n)^{H}\mathbf{R}_N^{-1}(\omega_n)\mathbf{x}(\omega_n)\}\end{aligned} \quad (14)$$

where $|\cdot|$ denotes the matrix determinant. The log-likelihood ratio test for this detection problem is given by

$$\Lambda(\mathbf{x})=\ln\frac{P(\mathbf{x}/H_1)}{P(\mathbf{x}/H_0)}\begin{matrix}\geq\eta,\ \text{then } H_1\\<\eta,\ \text{then } H_0\end{matrix} \quad (15)$$

where η is the threshold of the test. Successively applying the maximum likelihood method for estimating the unknown parameters $A_k(\omega)$ and $\mathbf{R}_N(\omega)$ we obtain the following form of the decisive statistic (DS)

$$\Lambda_D(\mathbf{x},\theta_k,t_k)=\frac{\left|\mathbf{a}(\theta_k)^{H}\hat{\mathbf{R}}^{-1}\sum_{n=1}^{N}\mathbf{x}(\omega_n)\exp\{-j\omega_n t_k\}s_0^{*}(\omega_n)\right|^{2}}{\mathbf{a}(\theta_k)^{H}\hat{\mathbf{R}}^{-1}\mathbf{a}(\theta_k)} \quad (16)$$

At derivation of the (16) the following assumptions have been made: a weak dependence of the array steering vector and the noise CSDM on frequency within signal bandwidth, and signal-to-noise ratio at a single frequency is a low. So, an estimate of the noise CSDM can be replaced by the CSDM of the received signals $\hat{\mathbf{R}}=\frac{1}{N}\sum_{n=1}^{N}\mathbf{x}(\omega_n)\mathbf{x}(\omega_n)^{H}$. Then, the DS (16) is built in angle-time coordinates. The threshold η determined according to a given false alarm probability is formed. In each point of the grid $\theta - t$ the DS is compared with the threshold. If the threshold is exceeded then the problem of detection of the signal's ray is solved positively and coordinates of this maximum are accepted as estimates of the angle of arrival and the travel time. The proposed algorithm is derived from log-likelihood ratio and is the optimum for parameter estimation of the quasideterministic signal in the Gaussian noise background.

DS (16) is an optimum for a deterministic medium. In a random, fluctuating ocean, the likelihood functions (14) are needed to average with respect to ocean medium parameters. According to the Bayesian approach these pdfs are determined as follows

$$\overline{P}(\mathbf{x} / H_1) = \int_{\Theta} P(\mathbf{x} / H_1, \Theta) P(\Theta) d\Theta, \qquad \overline{P}(\mathbf{x} / H_0) = \int_{\Theta} P(\mathbf{x} / H_0, \Theta) P(\Theta) d\Theta \tag{17}$$

where Θ is the vector of unknown medium parameters, $P(\mathbf{x}/H_{1(0)},\Theta)$ is the joint likelihood function for the corresponding hypothesis, $P(\Theta)$ is pdf of medium parameters. Assuming normal law of medium parameters fluctuations this averaging can be made analytically. As only the Green's function depends on medium parameters, we can build the processing algorithm matched to a random medium substituting to (16) the mean Green's function $\overline{G}(\mathbf{r}_S, \mathbf{r}_m, \omega)$ or $\overline{\mathbf{a}}(\theta_k)$ and $\overline{S}_k(\omega)$

$$\Lambda_R(\mathbf{x}, \theta_k, t_k) = \frac{\left| \overline{\mathbf{a}}(\theta_k)^H \hat{\mathbf{R}}^{-1} \sum_{n=1}^{N} \mathbf{x}(\omega_n) \exp\{-j\omega_n t_k\} \exp\left\{-\frac{1}{2}\omega^2 D_k\right\} s_0^*(\omega_n) \right|^2}{\overline{\mathbf{a}}(\theta_k)^H \hat{\mathbf{R}}^{-1} \overline{\mathbf{a}}(\theta_k)} \tag{18}$$

The presented processing algorithm (18) is optimum one matched to a deterministic and a stochastic structure of the oceanic waveguide. Unlike the algorithm (16) taking into account only deterministic structure of sound field, in the algorithm (18), the spatio-temporal filtering window is broadened with respect to the variance of travel time and angle of arrival of signal. Variances of angle of arrival and travel time fluctuations can be computed by using *a priori* information about correlation functions of inhomogeneities of ocean environment.

4. CONCLUSION

The major problem of high resolution methods is their sensitivity to deviations from the assumed propagation model. The presented optimum processing algorithm provides a more robust performance in a random oceanic waveguide.

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