MODELLING OF THE FINITE AMPLITUDE WAVES INTERACTION PROBLEM INSIDE TRUNCATED CONE

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The paper presents the results of theoretical investigations of the finite amplitude waves interaction in water. The mathematical model and some results of numerical investigations are presented. The acoustic pressure amplitude changes for different frequency waves (especially difference frequency wave) along the sound beam were investigated. The mathematical model was built on the basis of the KZK equation. It was assumed that a circular piston was the source of two different frequency finite amplitude waves. The problem was considered as an axial symmetric one. The solution of the problem was looked for inside a truncated cone. To solve the problem the finite-difference method was applied.

INTRODUCTION

The finite amplitude waves interaction problem has been considered in both experimental and theoretical investigations for many years (for example [1,3,4,5,6]). Mathematical model of this problem is often built on the basis on the KZK equation. This equation describes the acoustic pressure changes along the sound beam. It allows including nonlinearity, dissipation of medium and sound beam diffraction. Since the analytical solution of this equation is not known, it is necessary to solve it numerically. It is possible to solve this equation using some different numerical methods. The finite-difference method is one of them. The waves propagate in half-infinitive space but the solution of the problem is looked for inside bounded space. Due to sound beam diffraction this space must be suitably big. Assuming axial symmetry of the source it is comfortably to solve the KZK equation on the cylinder with fixed value of its radius [2]. It is possible to solve this equation inside truncated cone, too.

The aim of this paper is the numerical analysis of the finite amplitude waves interaction problem on the truncated cone. The paper presents mathematical model and some examples of the results of theoretical investigations.

1. MATHEMATICAL MODEL

We assume that a circular piston with a fixed radius a is a source of two harmonic waves. The source of the waves is placed in plane yOz and the waves are propagated in the x direction. It means that x axis corresponds to sound beam axis (Fig. 1)

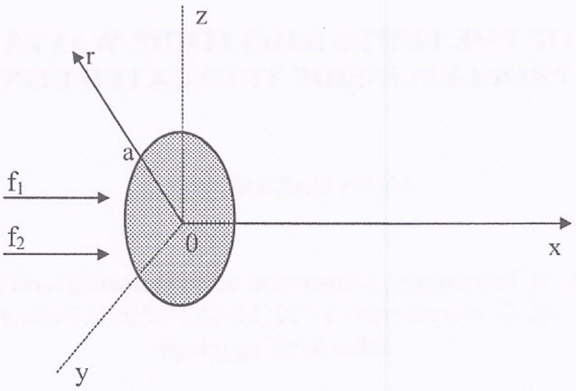

Fig. 1 The geometry of the problem

Moreover we assume axial symmetry of the source. Then the waves distribution on the piston is defined by:

$$p'(x=0,r,\tau) = -p_o \sin\omega_1\tau - p_o \sin\omega_2\tau \tag{1}$$

for r≤a and p'(x=0,r,t)=0 for r>a, where $r=\sqrt{y^2+z^2}$ and $\omega_1 = 2\pi f_1$, $\omega_2 = 2\pi f_2$ are angular frequencies of primary waves respectively.

The mathematical model of the finite amplitude waves interaction problem is built on the basis of the KZK equation:

$$\frac{\partial}{\partial\tau}\left(\frac{\partial p'}{\partial x} - \frac{\varepsilon}{\rho_o c_o^3} p'\frac{\partial p'}{\partial\tau} - \frac{b}{2\rho_o c_o^3}\frac{\partial^2 p'}{\partial\tau^2}\right) = \frac{c_o}{2}\left(\frac{\partial^2 p'}{\partial r^2} + \frac{1}{r}\frac{\partial p'}{\partial r}\right) \tag{2}$$

where $p'=p-p_o$ denotes an acoustic pressure, variable $\tau=t-x/c_o$ is the time in the coordinate system fixed in the zero phase of the propagating wave, ρ_o - medium density at rest, c_o - speed of sound, b - dissipation coefficient of the medium, ε - nonlinearity parameter.

The solution of Eq. (2) is look for inside the truncated cone for $\tau \in [0,T]$.

The finite-difference method is used to solve the problem numerically. Function p'(x,r,τ) is discretized in both space and time. To solve Eq. (2) numerically the rectangular net is constructed. The pressure changes along the sound beam are the result of computer calculations. The knowledge of pressure changes allows to calculate the spectrum changes. The fast Fourier transform (FFT) is used to calculate spectrum.

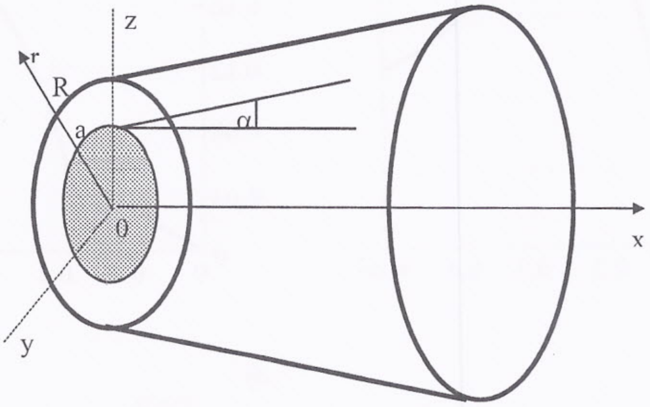

Fig. 2 Three - dimensional space

2. NUMERICAL INVESTIGATIONS

The calculations were carried out using own computer program that was worked out on the basis of obtained algorithm.

The formation of the different frequency waves, another that primary one, is the result of waves propagation in the same direction. Figure 3 shows normalized on-axis pressure amplitude for f_1, $2f_1$, f_2, $2f_2$, difference and sum frequency wave as a function of distance from the source. The calculations were carried out assuming that circular piston was the source of two waves. The piston radius was equal a=24 mm, pressure p_o=150 kPa, primary waves frequencies f_1=600 MHz and f_2=800 MHz respectively. The numerical investigations were carried out assuming that waves are propagated in water where speed of sound c_o=1500 m/s, medium density ρ_o=1000 kg/m^3, nonlinearity parameter ε=3.5 and dissipation coefficient of the medium b=0.004. In this example the truncated cone radius was equal R=2a and angle $\alpha=8^o$.

As a result of calculations we obtain the pressure changes along whole sound beam, not only on the beam axis. Normalized pressure amplitude for the difference frequency wave as a function of distance from the beam axis at distance from the source x=0.08 m is presented at Fig. 4. The calculations were done using the same values of physical and numerical parameters as earlier.

The correct choice of numerical parameters (step sizes, size of the space) is very important during numerical calculations. Figure 5 shows normalized on-axis pressure amplitude for f_1 and difference frequency waves as a function of distance from the source. Calculations were done for different values of the truncated cone parameters. Line number 1 shows the result of numerical calculations obtained for R=a, $\alpha=1.4^o$, line number 2 presents analogous results for R=2a and $\alpha=1.4^o$, line number 3 was obtained after calculations for R=2a and $\alpha=8^o$.

a)

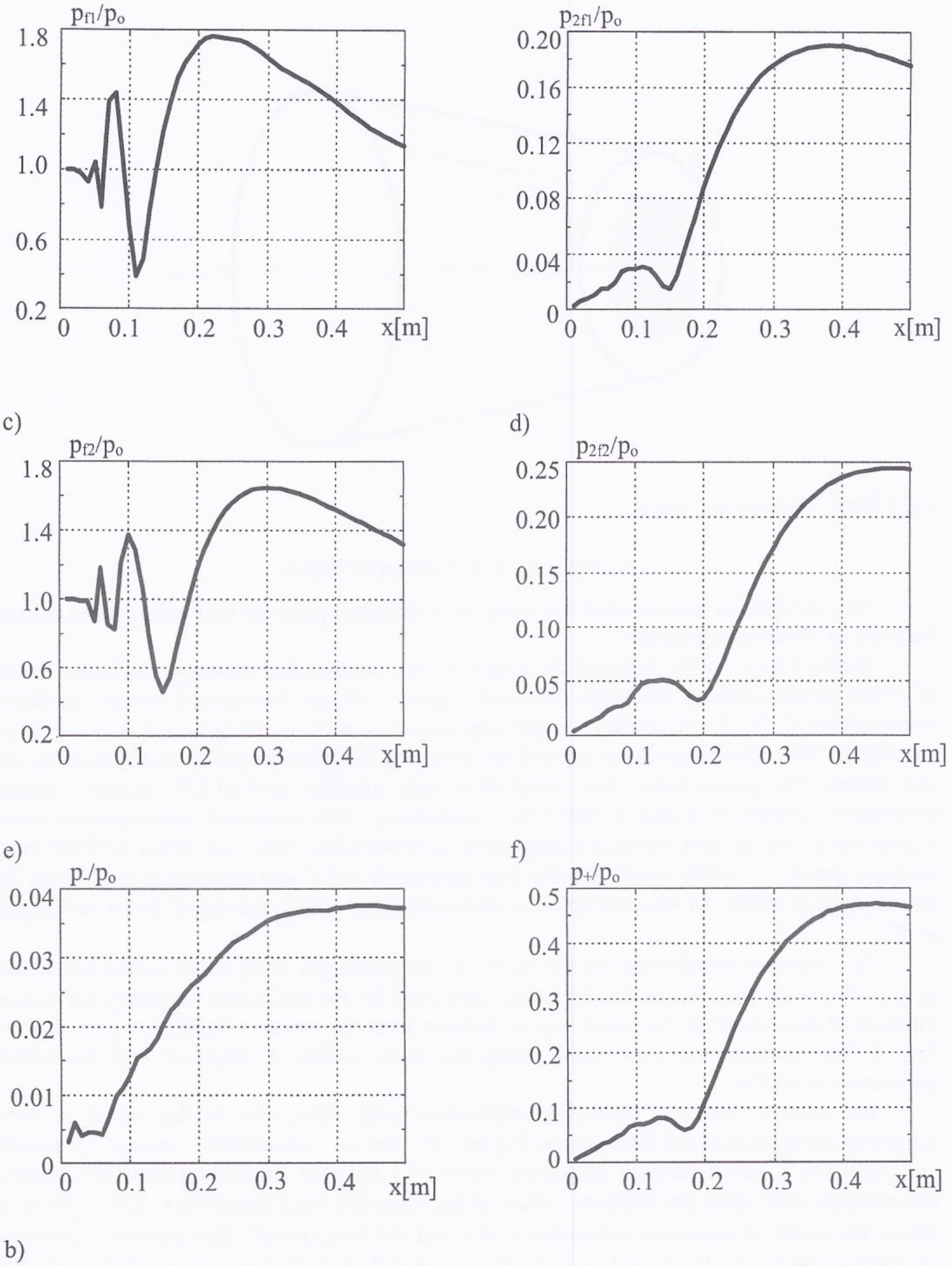

Fig. 3 Normalized on-axis pressure amplitude for different frequency waves as a function of distance from the source: a - f_1 , b - $2f_1$, c - f_2 , d - $2f_2$, e - difference, f - sum frequency wave

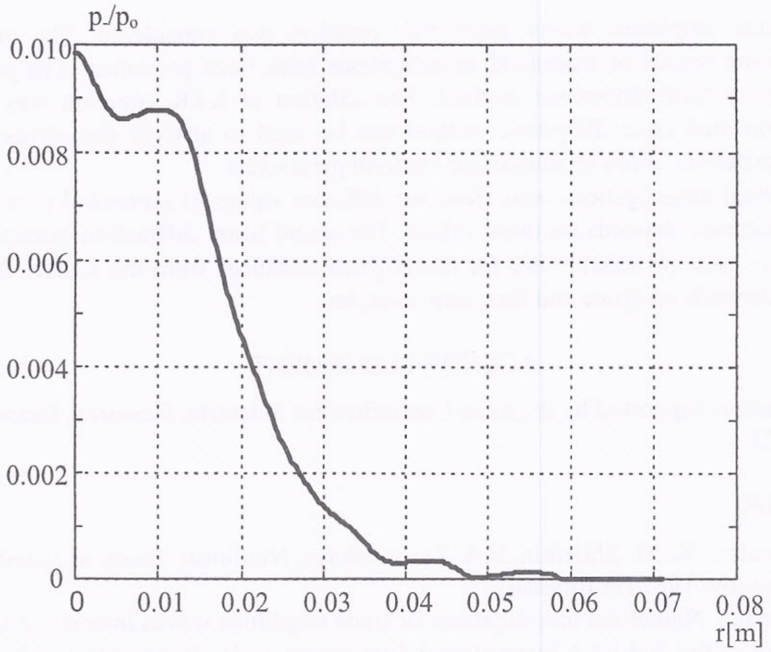

Fig. 4 Normalized pressure amplitude for the difference frequency wave as a function of distance from the beam axis at distance from the source x=0.08 m

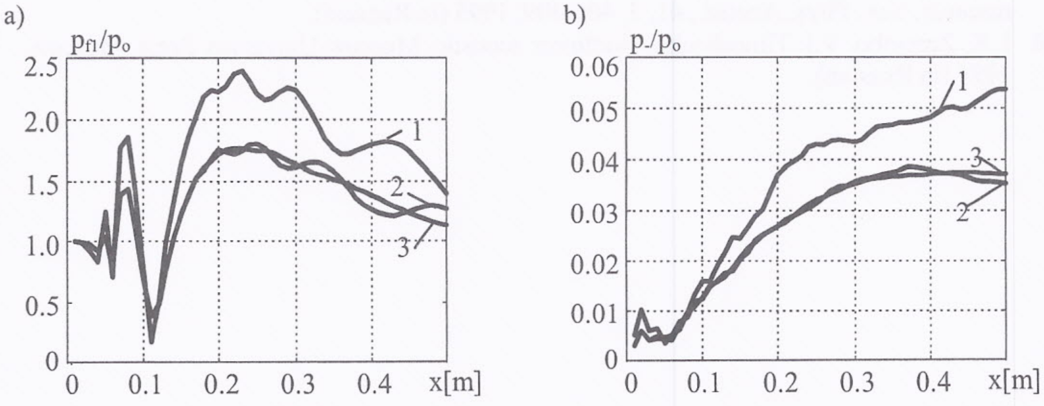

Fig. 5 Normalized on-axis pressure amplitude for f_1 (a) and difference (b) frequency waves as a function of distance from the source: 1-R=a, $\alpha=1.4^{\circ}$, 2-R=2a, $\alpha=1.4^{\circ}$, 3-R=2a, $\alpha=8^{\circ}$

3. CONCLUSIONS

The finite amplitude waves interaction problem was considered. The mathematical model and some results of numerical investigations have been presented. The problem was calculated using finite-difference method. The solution of KZK equation was looked for inside the truncated cone. Proposed method can be used to analyse the waves interaction problem for different values of source and medium parameters.

Theoretical investigations were done for different values of numerical parameters. The calculation accuracy depends on these values. The sound beam diffraction causes that radius R and angle α must be suitably big for investigated distances from the source. Accuracy of calculations depends on space and time step sizes, too.

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