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MODIFIED SLIDING WIENER-KHINTCHIN TRANSFORM

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Summary: The article presents the new approach to increase a speed of Sliding Discrete Wiener-Khintchin Transform (SDW-KT) algorithm on the basis of recurrent correlation analysis (CA) algorithm using. In this case it is not necessary to calculate the whole correlation function by each analyzing window position. There is taken note of analyzing window parameters selecting for sliding DW-KT, FFT, and Sliding Periodogram too. Worked out approach predominance over SFFT and Periodogram is demonstrated on examples of short noisy signal recognition.

Keywords: Time-frequency analysis, Wiener-Khintchin transform, Correlation analysis

1. INTRODUCTION

Random signal analysis in the time domain not always allows to obtain sufficient information about a studied signal. Therefore that analysis is complemented by researches in the frequency domain. In this context Discrete Fourier Transform (DFT) is a basic tool. Its algorithm is as follows [2]:

$$X(k)=\frac{1}{N}\sum_{n=0}^{N-1}x_n\exp(-j\frac{2\pi}{N}nk)\,,\quad \text{for } k,n=\overline{0,\ N-1} \tag{1}$$

where:

- $X(k)$ – k-th term of a Fourier's complex spectrum;
- $\{x_n\}_{n=0}^{N-1}$ – signal samples, where $x_n = x(nT_s)$;
- T_s – sampling period;
- N – signal length.

Because of this popular method needs too much operations – about N^2 complex multiplications and N additions – instead of it the fast algorithms (FFT) are used which need about $N\log_2 N$ those operations that allows to use FFT in real time systems [2], [3]. But an immediate usage of "motionless" DFT (1) and FFT to non-stationary signal processing is ineffective because of it does not show signal spectrum changes in time. In opposite to it, the sliding algorithms of DW-KT, Periodogram and FFT have not this downside. However, the correlation analysis which is a base of DW-KT needs many time – consuming operations that limits SDW-KT usage in real time signal processing systems.

Purpose of this work is to compare the sliding algorithms of FFT, Periodogram and DW-KT as well as increase a speed of SDW-KT algorithm on the basis of worked out new recurrent correlation analysis algorithm, select analyzing window parameters and verify the proposed approaches on the basis of short noisy signal recognizing.

2. SLIDING DFT AND PERIODOGRAM ALGORITHMS

Analyzing window of SDFT is being moved along of a signal realization in the next way [4], [5]:

$$X(k,r) = \frac{1}{N_w}\sum_{n=0}^{N_w-1} x_{n+r}\exp(-j\frac{2\pi}{N_w}nk)\text{, for } r=\overline{0,\ N-N_w-1}\text{, } n=\overline{0,\ N_w-1} \tag{2}$$

where:

- r – analyzing window position;
- $X(k,r)$ – k-th Fourier spectrum term at r-th window position;
- N_w – window samples number.

SDFT method resolution (f_{analyz}) [2] depends on N_w and T_s as $f_{analyz} = 1/T_s N_w$, therefore it is not always sufficient for short non stationary signal analysis.
Periodogram is often used to random signal analysis and is built on FFT base:

$$\hat{S}_{xx}^{(p)}(k) = \frac{1}{N}\left|X(k)\right|^2, \tag{3}$$

Usually the Welch procedure is applied to a periodogram in order to improve its accuracy. It boils down to averaging a few periodograms on next parts of a signal realization, that limits its use in a real time analysis. Therefore we can propose using the SFFT algorithm (2) to obtain the sliding periodogram:

$$\hat{S}_{xx}^{(P)}(k,r) = \frac{1}{N_w}\left|\sum_{n=0}^{N_w-1} x_{n+r}\exp(-j\frac{2\pi}{N_w}nk)\right|^2, \tag{4}$$

which allows to analyze a signal in a sample by sample regime in opposite to classical periodogram algorithm when a window steps from subset to subset.

3. SLIDING DW-KT WITH USING OF RECURRENT CA ALGORITHM

Discrete variant of the „motionless" Wiener-Khintchin Transform can be presented in the next form [1], [3]:

$$\hat{S}_{xx}(k) = \frac{1}{2P-1} \sum_{m=-(P-1)}^{P-1} \hat{R}_{xx}(m) \exp(-j \frac{2\pi}{2P-1} mk), \tag{5}$$

where:

$\hat{S}_{xx}(k)$ – estimator of k- th term (component) of a spectrum;

$\hat{R}_{xx}(m)$ – correlation function (CF) estimator at m-th time shift;

PT_s – correlation interval, at that $P \le N$, where $m = \overline{-(P-1),\ (P-1)}$.

In turn, estimator of „motionless" CF is defined as follows:

- unbiased estimator

$$\hat{R}_{xx}(m) = \begin{cases} \frac{1}{N-m} \sum_{n=0}^{N-m-1} \overset{\circ}{x}_n \cdot \overset{\circ}{x}_{n+m} & m \ge 0 \\ \hat{R}_{xx}^{*}(-m) & m < 0 \end{cases} \tag{6}$$

- biased estimator

$$\hat{R}_{xx}(m) = \begin{cases} \frac{1}{N} \sum_{n=0}^{N-m-1} \overset{\circ}{x}_n \cdot \overset{\circ}{x}_{n+m} & m \ge 0 \\ \hat{R}_{xx}^{*}(-m) & m < 0 \end{cases} \tag{7}$$

where:

$\overset{\circ}{x}_n = x_n - \bar{x}$ – centered value;

$\bar{x} = \frac{1}{N} \sum_{n=0}^{N-1} x_n$ – mean value;

$\hat{R}_{xx}^{*}$ – complex conjugated CF.

In many practical applications the Blackman-Tukey method, when the product of the CF estimator and the suitable smoothing window is used [6], Common downside of those algorithms is lack of possibility to use them in the time-frequency analysis immediately. Therefore in this context it is expedient to develop existed methods.

3.1. SLIDING DW-KT DIRECT ALGORITHM

We propose that the algorithm was based on connection of sliding CA and FFT. In this case analyzing window moves by sample by sample but does not step by window by window. At that, CF is calculated at each position of window and next a power spectrum of this part of signal realization is calculated with using of FFT. It allows to obtain a time-frequency map of an analyzed process. Sliding direct DW-KT algorithm can be written as

$$\hat{S}_{xx}(k,r) = \frac{1}{2N_w - 1} \sum_{m=-(N_w-1)}^{N_w-1} \hat{R}_{xx}(m,r) \exp(-j\frac{2\pi}{2N_w - 1} mk) \, , \qquad (8)$$

$$\text{for } r = \overline{0,\ N - N_w - 1}\, ,\ m = \overline{-(N_w - 1),\ (N_w - 1)}$$

where:

$\hat{S}_{xx}(k,r)$ – power spectrum estimator of k-th component at r-th position of the window.

At that, CF estimator can be presented as unbiased estimator (9) or biased one (10).

$$\hat{R}_{xx}(m,r) = \begin{cases} \dfrac{1}{N_w - m} \displaystyle\sum_{n=0}^{N_w-m-1} \overset{\circ}{x}_{n+r} \cdot \overset{\circ}{x}_{n+r+m} & m \geq 0 \\ \hat{R}_{xx}^{*}(-m) & m < 0 \end{cases} \qquad (9)$$

$$\hat{R}_{xx}(m,r) = \begin{cases} \dfrac{1}{N_w} \displaystyle\sum_{n=0}^{N_w-m-1} \overset{\circ}{x}_{n+r} \cdot \overset{\circ}{x}_{n+r+m} & m \geq 0 \\ \hat{R}_{xx}^{*}(-m) & m < 0 \end{cases} \qquad (10)$$

where all meanings are analogous to expression (7).

3.2. WORKED – OUT FAST RECURRENT CORRELATION ANALYSIS ALGORITHM FOR SDW-KT

We can write down the Sliding CA algorithm for r-th position of the window in next form:

$$\hat{R}_{xx}(m,r) = \frac{1}{N_w - m} \sum_{n=0}^{N_w-m-1} \overset{\circ}{x}_{n+r} \cdot \overset{\circ}{x}_{n+r+m} \qquad (11)$$

for all positions of window $r = \overline{0,\ N - N_w - 1}$ and time shifts $m = \overline{-(N_w - 1),\ (N_w - 1)}$

Here:

$\hat{R}_{xx}(m,r)$ – CF estimator at r-th position of window

and for (r-1) – th position of the window

$$\hat{R}_{xx}(m,r-1) = \frac{1}{N_w - m} \sum_{n=0}^{N_w-m-1} \overset{\circ}{x}_{n+r-1} \cdot \overset{\circ}{x}_{n+r-1+m} \qquad (12)$$

For obtaining the recurrent algorithm, it is necessary to apply backward differences to CF by adjacent positions of a window:

$$\nabla_r \hat{R}_{xx}(m,r) = \hat{R}_{xx}(m,r) - \hat{R}_{xx}(m,r-1) \tag{13}$$

Then differential CF is:

$$\nabla_r \hat{R}_{xx}(m,r) = \frac{1}{N_w - m}\left[\sum_{n=0}^{N_w-m-1} \overset{\circ}{x}_{n+r} \cdot \overset{\circ}{x}_{n+r+m}\right] - \frac{1}{N_w - m}\left[\sum_{n=0}^{N_w-m-1} \overset{\circ}{x}_{n+r-1} \cdot \overset{\circ}{x}_{n+r-1+m}\right] =$$

$$= \frac{1}{N_w - m}\left[\overset{\circ}{x}_{1+r} \cdot \overset{\circ}{x}_{1+r+m} + \overset{\circ}{x}_{N_w-1+r-m} \cdot \overset{\circ}{x}_{N_w-1+r} - \overset{\circ}{x}_{r-1} \cdot \overset{\circ}{x}_{r-1+m}\right],$$

where for $r<1$ it takes place $\overset{\circ}{x}_{r-1} \cdot \overset{\circ}{x}_{r-1+m} = 0$.
Hence the CA recurrent algorithm (for $r \geq 1$ only):

$$\hat{R}_{xx}(m,r) = \hat{R}_{xx}(m,r-1) + \nabla_r \hat{R}_{xx}(m,r) =$$

$$= \hat{R}_{xx}(m,r-1) + \frac{1}{N_w - m}\left[\overset{\circ}{x}_{1+r} \cdot \overset{\circ}{x}_{1+r+m} + \overset{\circ}{x}_{N_w-1+r-m} \cdot \overset{\circ}{x}_{N_w-1+r} - \overset{\circ}{x}_{r-1} \cdot \overset{\circ}{x}_{r-1+m}\right] \tag{14}$$

where:

$\overset{\circ}{x}_{r-1} \cdot \overset{\circ}{x}_{r-1+m} = 0$ for r < 1 by the same initial conditions.

Hence SDW-KT on the basis of the recurrent CA algorithm (14) is:

$$\hat{S}_{xx}(k,r) = \frac{1}{2N_w - 1}\sum_{m=-(N_w-1)}^{N_w-1}\{\hat{R}_{xx}(m,r-1) +$$

$$+ \frac{1}{N_w - m}\left[\overset{\circ}{x}_{r+1} \cdot \overset{\circ}{x}_{r+m+1} + \overset{\circ}{x}_{N_w+r-m-1} \cdot \overset{\circ}{x}_{N_w+r-1} - \overset{\circ}{x}_{r-1} \cdot \overset{\circ}{x}_{r+m-1}\right]\}\exp(-j\frac{2\pi}{2N_w - 1}mk) \tag{15}$$

We can see that it is not necessary to calculate the whole correlation function by each analyzing window position. There is enough to add only averaged sums of 3 products of suitable samples to the CF at previous position of the window.

4. EXAMPLES OF PROPOSED ALGORITHMS APPLICATION

Worked out algorithms were verified on examples of short noisy signals recognition. It was realized with using a worked out program and MATLAB where were assigned aperture $N = 1024$ samples and sampling rate $f_s = 8000\ Hz$. Compound analyzed signal consisted of two sine parts and broadband LFM signal (chirp). Frequency of the sine signal was 2000 Hz and amplitude was A = 1. Character of the chirp was as follows:

$$x_n = x(nT_s) = A\cos\left[2\pi\left(\frac{\Delta f}{2N_{ch}}n + f_1\right)nT_s + \varphi_0\right] \tag{16}$$

by next parametrs: $\{x_n\}$, $n = \overline{257,\ 384}$, $N_{ch} = 128$ – the chirp samples number, $\Delta f = f_2 - f_1$ – the frequency increase, $f_1 = 2000\ Hz$ – initial frequency, $f_2 = 4000\ Hz$ – final frequency, φ_0 – initial phase. The chirp duration (τ_{ch}) was 16 ms, $A = \sqrt{2}$, and $BT = \Delta f \tau_{ch} = 32$. A frequency- time characteristic of the signal is shown in Fig. 1.

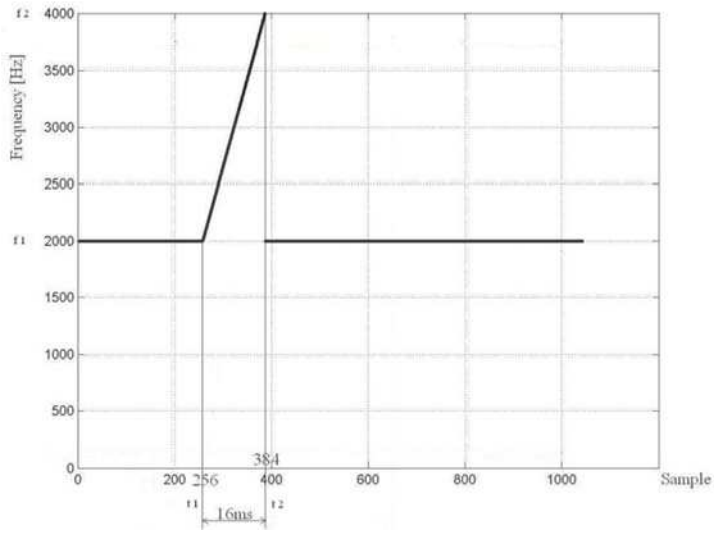

Fig. 1. Frequency-time characteristic of studying signal

This whole signal realization is shown in Fig. 2a and its motionless FFT result – in Fig. 2b. By that there can be done wrong decision that all signal is monochrome whilst it concludes the chirp which amplitude is bigger than one of the sine signal.

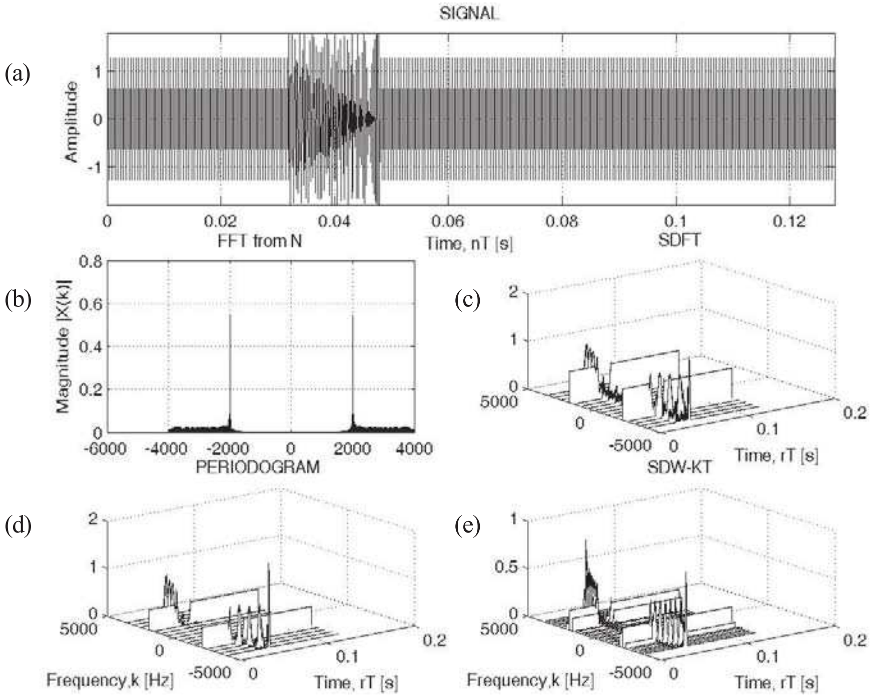

Fig. 2. (a) input noiseless signal, $N = 1024$. (b) "motionless" FFT result of the whole signal realization. A spatial view: (c) SDFT result (d) SPERIODOGRAM result, (e) SDW-KT result. For (c), (d) and (e) the window was $N_w = 16$

Analysis of SDFT, Sliding PERIODOGRAM and SDW-KT algorithms was fulfilled for next window length: 8, 16, 32, 64, and 128 samples. These windows slid along a signal realization in a sample by sample regime. Longer windows allow to obtain a better frequency resolution but a worse time one. Note that SDW-KT has twice as big frequency resolution in comparison to SDFT and PERIODOGRAM for the same window length without losing of the time resolution. This fact takes place because of CA, which is part of DW-KT, according to (7), (9), and (10) leads to obtain a double number of CF samples and, accordingly, double number of the power spectrum components (Figs. 2b, 2c, and 2d). Fig. 3 shows the same results but using a view on a plane.

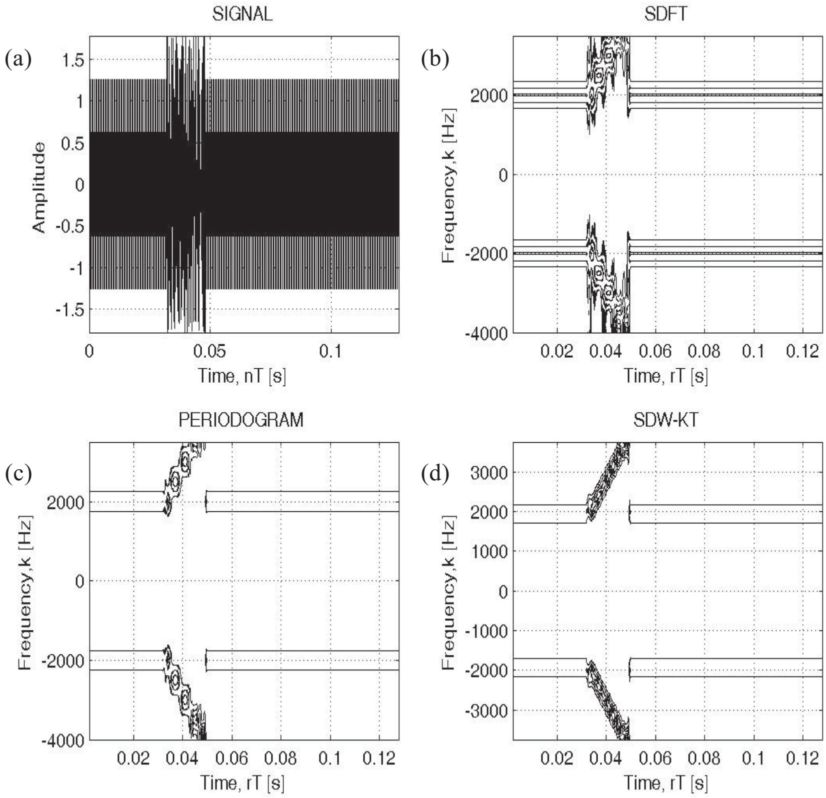

Fig. 3. (a) input noiseless signal, length is $N = 1024$. The view on a plane: (b) SDFT result, (c) Sliding PERIODOGRAM result, (d) SDW-KT result. For (b), (c), (d) the window was $N_w = 16$

As Figures 2 and 3 show, above mentioned methods not only allow to see a signal changes character but determine their appearance moment as well. The best result was obtained for SDW-KT which has twice as big a frequency resolution without worsen a time one.

Next, there was analyzed the SDFT, SPERIODOGRAM and SDW-KT usefulness for recognizing of short noisy signals. At that the computer Gauss' noise „*randn*" was added according to next SNRs: +10 dB, +3 dB, and 0 dB. Here:

$$SNR = 10\lg\frac{\Psi_x^{\ 2}}{\Psi_\xi^{\ 2}}\,, \tag{17}$$

and a signal and a noise powers are:

$$\Psi_x^{\ 2} = \frac{1}{N}\sum_{n=0}^{N-1} x_n^{\ 2}\,, \quad \Psi_\xi^{\ 2} = \frac{1}{N}\sum_{n=0}^{N-1} \xi_n^{\ 2}$$

The noisy signal realization by $SNR = 0\ dB$ is given in Fig. 3a and Fig. 4a. Other figures show the examples of the signal processing on the basis of SDFT, SPERIODOGRAM, and SDW-KT (the spatial views and views on a plane accordingly).

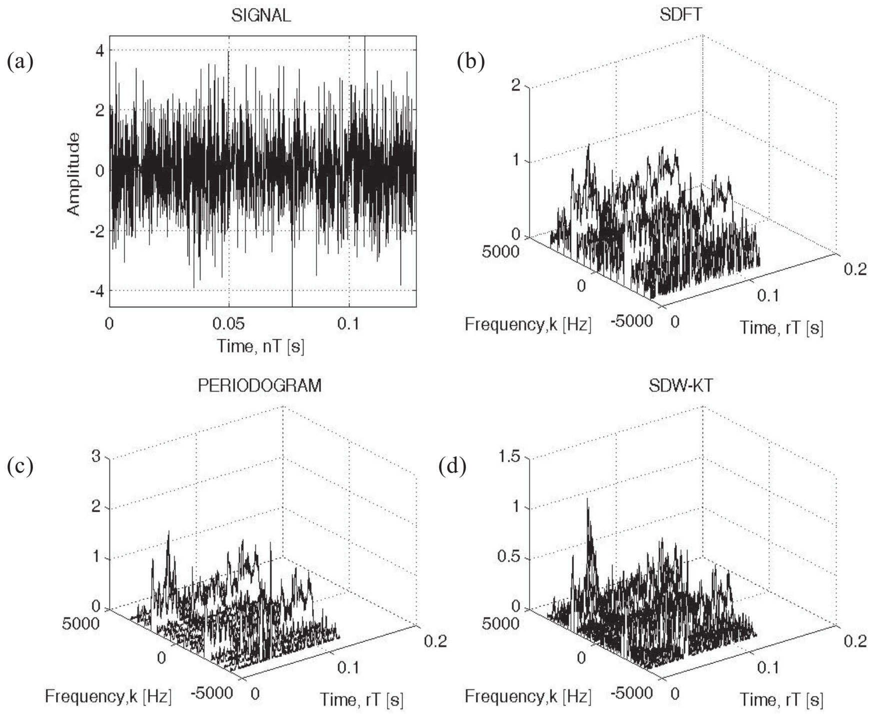

Fig. 4. (a) input noisy signal. The spatial view: (b) SDFT result, (c) SPERIODOGRAM result, (d) SDW-KT result. Here signal length is $N = 1024$, the window is $N_w = 16$, $SNR = 0\ dB$ by using the „randn” noise

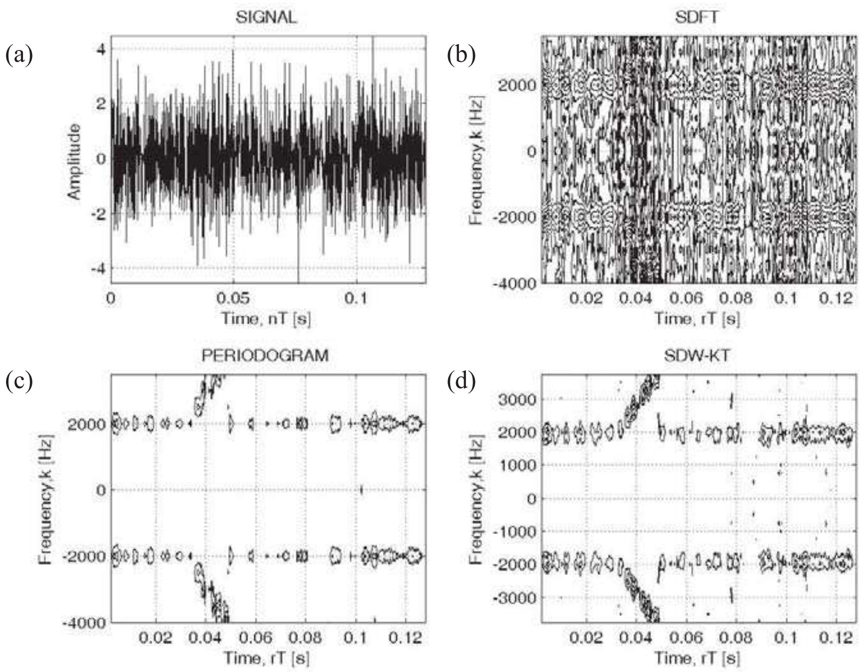

Fig. 5. (a) input noisy signal. The view on a plane: (b) SDFT result, (c) SPERIODOGRAM result, (d) SDW-KT result. Here $N = 1024$, $N_w = 16$, $SNR = 0\ dB$ by using the „randn" noise

As we can see, the best result is obtained for SDW-KT when the appearance moment of the noisy chirp signal is the most expressive (Fig. 5d).

5. CONCLUSION

A comparison between the sliding algorithms of FFT, Periodogram and Discrete Wiener-Khintchin Transform (SDW-KT) is made in this work.

Apart from it, the article presents the new approach to increase a speed of SDW-KT algorithm on the basis of fast recurrent correlation analysis algorithm using. In this case it is not necessary to calculate the whole correlation function by each analyzing window position but there is enough to add only averaged sums of 3 products of suitable samples to the CF at previous position of the window.

There is taken note of analyzing window parameters selecting for the sliding algorithms. Worked out approach predominance over SFFT and Sliding Periodogram is demonstrated on examples of short noisy signals recognition.

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ZMODYFIKOWANE ŚLIZGAJĄCE PRZETWARZANIE WIENERA-CHINCZYNA

Streszczenie

W artykule przedstawiono opracowany nowy szybki rekurencyjny algorytm, ślizgającej dyskretnej analizy korelacyjnej (SDW-KT- Sliding Discrete Wiener-Khitnchin Transform), pozwalający na istotne zmniejszenie ilości operacji przy kolejnych przemieszczeniach okna analizy, co prowadzi do zwiększenia szybkości przetwarzania SDW-KT. Zwrócono także uwagę na dobranie parametrów okien dla ślizgających przetwarzań DW-KT, FFT (Fast Fourier Transform) i PERIODOGRAMU. Przewagi opracowanych podejść nad SFFT i ślizgającym periodogramem przedstawiono na przykładach wykrywania krótkotrwałych zaszumionych sygnałów.

Słowa kluczowe: analiza czasowo-częstotliwościowa, przetwarzanie Wienera-Chinczyna, analiza korelacyjna