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Int. J. of Applied Mechanics and Engineering, 2016, vol.21, No.2, pp.393-406
DOI: 10.1515/ijame-2016-0024

THREE-DIMENSIONAL MHD BOUNDARY LAYER FLOW DUE TO AN AXISYMMETRIC SHRINKING SHEET WITH RADIATION, VISCOUS DISSIPATION AND HEAT SOURCE/SINK

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An analysis is made to study a three dimensional MHD boundary layer flow and heat transfer due to a porous axisymmetric shrinking sheet. The governing partial differential equations of momentum and energy are transformed into self similar non-linear ordinary differential equations by using the suitable similarity transformations. These equations are, then solved by using the variational finite element method. The flow phenomena is characterised by the magnetic parameter *M*, suction parameter *S*, porosity parameter K_p, heat source/sink parameter *Q*, Prandtl number Pr, Eckert number Ec and radiation parameter *Rd*. The numerical results of the velocity and temperature profiles are obtained and displayed graphically.

Key words: axisymmetric shrinking sheet, dual solution, radiation, viscous dissipation, FEM.

1. Introduction

The flow over a stretching/shrinking surface is an important problem in many engineering processes with applications in industries such as extrusion, melt-spinning, hot rolling, wire drawing, glass-fiber production, manufacture of plastic and rubber sheets, and cooling of a large metallic plate in a bath, which may be an electrolyte. In industry, polymer sheets and filaments are manufactured by continuous extrusion of the polymer from a die to a windup roller, which is located at a finite distance away. The thin polymer sheet constitutes a continuously moving surface with a nonuniform velocity through an ambient fluid. Sakiadis [1] was the first one to analyze the boundary layer flow on continuous surfaces. After that, Crane [2] studied the boundary layer flow past a stretching plate. Grubka [3] studied the heat transfer characteristics of a continuous stretching surface with variable temperature. Gupta and Gupta [4] investigated heat and mass transfer on a stretching sheet with suction and blowing. The boundary layer flow of an incompressible viscous fluid over a shrinking sheet has received considerable attention of modern day researchers because of a its increasing application to many engineering systems. Wang [5] first pointed out the flow over a shrinking sheet when he was working on the flow of a liquid film over an unsteady stretching sheet. The existence and uniqueness of steady viscous flow due to a shrinking sheet was established by Miklavcic and Wang [6] and they concluded that for some specific value of suction at the sheet, dual solutions exist and also in certain range of value of suction, no boundary layer solution exists. Fang and Zhang [7] recently investigated the heat transfer characteristics of the shrinking sheet problem with a linear velocity. Later on, Noor *et al*. [8] studied the MHD viscous flow due to shrinking sheet using the Adomian decomposition Method (ADM) and they obtained a series solution. Sajid and Hayat [9] applied the homotopy analysis method for the MHD viscous flow due to a shrinking sheet. Midya [10] studied the magnetohydrodynamic viscous flow and heat transfer over a linearly shrinking porous sheet. Muhaimin *et al.* [11] studied the effect of chemical reaction, heat and mass transfer on nonlinear MHD boundary layer past a porous shrinking sheet

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with suction. Following these works, some very important investigations in this direction were made (see [12]-[15]).

Viscous dissipation changes the temperature distributions by playing a role of an energy source, which leads to affected heat transfer rates. The merit of the effect of viscous dissipation depends on whether the plate is being cooled or heated. Viscous dissipation effects are usually ignored in macro scale systems, in laminar flow in particular, except for very viscous liquids at comparatively high velocities. However, even for common liquids at laminar Reynolds numbers, frictional effects in micro scale systems may change the energy equation [16]. Gebhart [17] studied the effects of viscous dissipation for external natural convection flow over a surface. Koo and Kleinstreuer [18] have investigated the effects of viscous dissipation on the temperature field using dimensional analysis and experimentally validated computer simulations.

The heat source/sink effects in thermal convection are significant where there may exist high temperature differences between the surface (e.g., space craft body) and the ambient fluid. For physical situations, the average behaviour of heat generation or absorption can be expressed by some simple mathematical models because its exact modelling is quite difficult. The study of convective heat and mass transfer fluid flow over stretching/shrinking surface in the presence of thermal radiation, heat generation and chemical reaction is gaining a lot of attention. Sparrow and Cess [19] investigated the steady stagnation point flow and heat transfer in the presence of temperature dependent heat absorption. Later, Azim *et al*. [20] discussed the effect of viscous Joule heating on MHD-conjugate heat transfer for a vertical flat plate in the presence of heat generation. Tania *et al.* [21] investigated the effects of radiation, heat generation and viscous dissipation on MHD free convection flow along a stretching sheet. Chamkha [22] studied the effect of heat generation or absorption on hydro magnetic three-dimensional free convection flow over a vertical stretching surface.

The effect of radiation on an MHD flow and heat transfer problem have become more important industrially. At high operating temperature, the radiation effect can be quite significant. Many processes in engineering areas occur at high temperature and a knowledge of radiation heat transfer becomes very important for the design of the pertinent equipment. Nuclear power plants, gas turbines and the various propulsion devices for aircraft, missiles, satilites and space vehicles are examples of such engineering areas. The study of radiation effects on the various types of flows is quite complicated. In the recent years, many authors have studied radiation effects on the boundary layer of radiating fluids past a plate. Raptis [23] studied the flow of a micropolar fluid past a continuously moving plate by the presence of radiation. Cortell [24] studied the effects of viscous dissipation and radiation on the thermal boundary layer over a non-linear stretching sheet. The thermal radiation and heat generation effects on the MHD convective flow are a new dimension added to the study of stretching surface and has important applications in physics and engineering particularly in space technology and high temperature processes as it plays an important role in controlling the heat transfer process in polymer processing industry. The effect of radiation on heat transfer problems was studied by Hossain and Takhar [25], Takhar *et al*. [26]. Seddeek [27] analyzed the effects of radiation and variable viscosity on an MHD free convection flow past a semi-infinite flat plate with an aligned magnetic field. Hunegnew [28] studied the MHD boundary layer flow and heat transfer over a non-linearly stretching/shrinking sheet. Rajesh [29] investigated chemical reaction and radiation effects on the transient MHD free convection flow of dissipative fluid past an infinite vertical porous plate with ramped wall temperature.

Therefore, in the present paper, the dual solution of a three dimensional MHD boundary layer flow and heat transfer of an electrically conducting fluid due to a porous axisymmetric shrinking sheet with thermal radiation and heat source/sink, have been studied.

2. Mathematical formulation

Consider a three-dimensional MHD viscous incompressible flow of an electrically conducting fluid due to a porous axisymmetric shrinking sheet coinciding with the plane $z=0$ and the flow is confined to $z>0$. The x and y axes are taken along the length and width of the sheet and the z-axis is perpendicular to the sheet, respectively (Fig.1). A constant magnetic field with strength B_0 is applied in the z-direction. The magnetic Reynolds number is taken to be small, so that the induced magnetic field is neglected and a suction is applied normal to sheet to contain the vorticity. All the other fluid properties are assumed constant throughout the motion.

Under the usual boundary layer approximations, the basic governing boundary layer equations with viscous dissipation and heat source/sink are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0\ , \tag{2.1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\frac{\partial^2 u}{\partial z^2}-\frac{\nu}{K}u-\frac{\sigma B_0^2}{\rho}u\ , \tag{2.2}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial v}+\nu\frac{\partial^2 v}{\partial z^2}-\frac{\nu}{K}v-\frac{\sigma B_0^2}{\rho}v\ , \tag{2.3}$$

$$u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\nu\frac{\partial^2 w}{\partial z^2}-\frac{\nu}{K}w\ , \tag{2.4}$$

$$\begin{aligned}
&u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\frac{\alpha}{\rho c_p}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right)+\\
&+\frac{\mu}{\rho c_p}\left[2\left\{\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2\right\}+\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2+\left(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}\right)^2+\left(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right)^2\right]+\\
&+\frac{Q_0(T-T_\infty)}{(\rho c_p}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}
\end{aligned} \tag{2.5}$$

where (u, v, w) are the velocity components along the (x, y, z) directions, respectively, p is the pressure, ϱ is the density of the fluid, μ is the dynamic viscosity, ν is the kinematic viscosity, σ is the electrical conductivity, B_0 is the magnetic induction, α is thermal conductivity, c_p is the specific heat at constant pressure and Q_0 is the volumetric rate of heat generation or absorption.

The boundary conditions applicable to the present flow are

$$\begin{aligned}
&u=-U=-ax, \quad v=-V=-ay, \quad w=-W, \quad T=T_w \quad \text{at} \quad z=0,\\
&u=0, \quad v=0, \quad T=T_\infty \quad \text{at} \quad y\to\infty
\end{aligned} \tag{2.6}$$

where $a>0$ is the shrinking constant, U and V are the shrinking velocities, $W>0$ is the suction velocity, T_w is the sheet temperature and T_∞ is the free stream temperature.

The Rosseland approximation for radiation is

$$\frac{\partial q_r}{\partial y}=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{2.7}$$

where σ^* and k^* are the Stefan-Boltzmann constant and the mean absorption coefficient, respectively. It is assumed that the temperature differences within the flow, such as the term T^4, may be expressed as a linear function of temperature. The Taylor series expanding for T^4 about a free stream temperature T_∞ after neglecting higher-order terms

$$T^4 = 4T_\infty^4 T - 3T_\infty^4 . \tag{2.8}$$

Using Eqs (2.7) and (2.8) , we obtain

$$\frac{\partial q_r}{\partial y} = -\frac{16\sigma^* T_\infty^3}{3k^*}\frac{\partial^2 T}{\partial y^2}. \tag{2.9}$$

Introducing the following similarity transformations

$$u = axf'(\eta), \quad v = ayf'(\eta), \quad w = -2\sqrt{a\nu}\, f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \text{and} \quad \eta = \sqrt{\frac{a}{\nu}}z \tag{2.10}$$

Eq.(2.1) is identically satisfied by the similarity transformations while Eq.(2.4) becomes

$$\frac{p}{\rho} = \nu\frac{\partial w}{\partial z} - \frac{w^2}{2} + \text{Constant} . \tag{2.11}$$

Equations (2.2), (2.3) and (2.5) reduce as

$$f''' + 2ff'' - f'^2 - \left(M + K_p\right)f' = 0 , \tag{2.12}$$

$$\left(1 + \frac{4Rd}{3}\right)\theta'' + \Pr\left[2f\theta' + Q\theta + 12\text{Ec}f'' + \left(\text{Ec}_x + \text{Ec}_y\right)f'^2\right] = 0 . \tag{2.13}$$

The corresponding boundary conditions are

$$\begin{aligned} &f = S, \quad f' = -1, \quad \theta = 1 \quad \text{at} \quad \eta = 0, \\ &f' \to 0, \quad \theta \to 0, \quad \text{as} \quad \eta \to \infty \end{aligned} \tag{2.14}$$

where $S = \frac{W}{2\sqrt{a\acute{\imath}}}$ is the suction parameter, $M = \frac{\sigma B_0^2}{a\rho}$ is the magnetic parameter, $K_p = \frac{\nu}{aK}$ is the porosity parameter, $\Pr = \frac{\mu c_p}{\alpha}$ is the Prandtl number, $Q = \frac{Q_0}{a\rho c_p}$ is the heat source/sink parameter, $\text{Ec} = \frac{\mu a}{\rho c_p\left(T_w - T_\infty\right)}$ is the Eckert number, $\text{Ec}_x = \frac{U^2}{c_p\left(T_w - T_\infty\right)}$ and $\text{Ec}_y = \frac{V^2}{c_p\left(T_w - T_\infty\right)}$ are the local Eckert numbers based an the variables x and y, respectively.

The physical quantity of interest is the local skin friction co-efficient C_f on the surface along the x and y directions, which are denoted by C_{f_x} and C_{f_y}, respectively and the local Nusslet number Nu, i.e., surface heat transfer are given by

$$C_{f_x}=\frac{\tau_{wx}}{\rho U^2/2}=\frac{\mu\left(\frac{\partial u}{\partial z}\right)_{z=0}}{\rho U^2/2}=\frac{2}{\sqrt{\mathrm{Re}_x}}f''(0),\qquad C_{f_y}=\frac{\tau_{wy}}{\rho V^2/2}=\frac{\mu\left(\frac{\partial v}{\partial z}\right)_{z=0}}{\rho v^2/2}=\frac{2}{\sqrt{\mathrm{Re}_y}}f''(0),$$

$$\mathrm{Nu}=\frac{x\left(\frac{\partial T}{\partial z}\right)_{z=0}}{\left(T_w-T_\infty\right)}=-\sqrt{\mathrm{Re}_x}\,\theta'(0)$$

where τ_{wx} and τ_{wy} are the wall shear stresses along the *x* and *y* directions, respectively and $\mathrm{Re}_x=\frac{Ux}{\nu}$ and $\mathrm{Re}_y=\frac{Vy}{\nu}$ are the local Reynolds numbers.

3. Method of solution

FEM is a numerical and computer-based technique of solving a variety of practical engineering problems that arise in different fields. It is recognized by developers and users as one of the most powerful numerical analysis tools ever devised to analyze complex problems of engineering. It has been applied to a number of physical problems, where the governing differential equations are available. The method essentially consists of assuming the piecewise continuous function for the solution and obtaining the parameters of the functions in a manner that reduces the error in the solution.

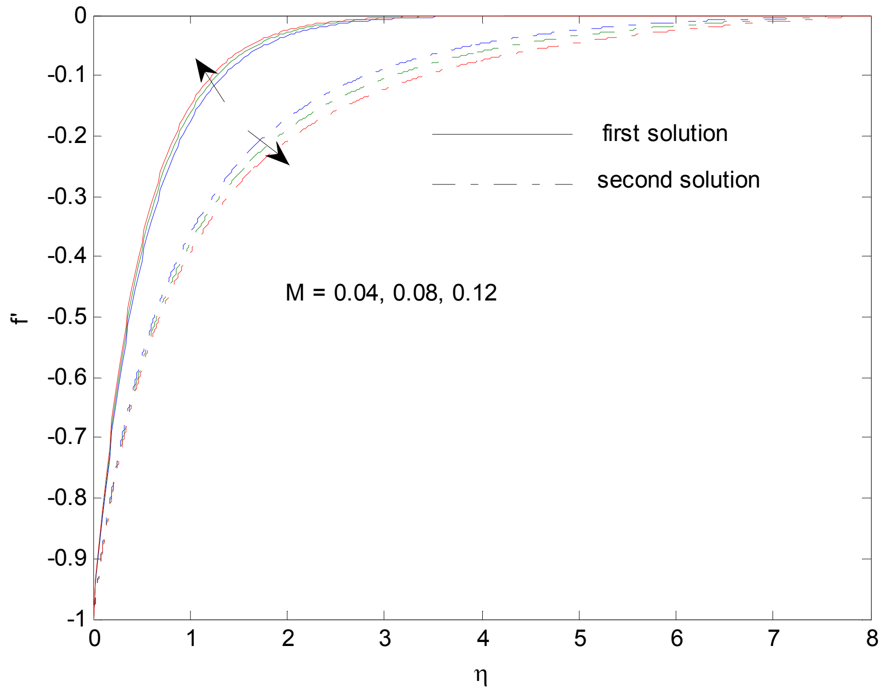
The entire flow domain is divided into 1000 linear elements of equal size. Each element has two nodes, and therefore, the whole domain contains 1001 nodes. At each node two functions are to be evaluated; hence after assembly of the element equations, we obtain a system of 2002 equations which are non-linear. Therefore, an iterative scheme must be utilized in the solution. After imposing the boundary conditions, a system of equations has been obtained which is solved by the Gauss elimination method while maintaining an accuracy of 10^{-5}.

4. Results and discussion

To analyze the physical insight into the problem the numerical computations were carried out for governing parameters, namely: magnetic parameter *M*, suction parameter *S*, porosity parameter K_p, heat source/sink parameter *Q*, Prandtl number Pr, Eckert number Ec, local Eckert numbers Ec_x, Ec_y and radiation parameter *Rd*. In ordered to illustrate the salient features of the model the numerical results are presented graphically in Figs 1-8.

Figure 1a shows that the effect of the magnetic parameter *M* in the dimensionless velocity profile *f*'. It is observed that the velocity profile at a point increases with the increase in *M* for the first solution and the velocity profile decreases with the increase in *M* for the second solution. Figure 1b is drawn to analyze the influence of magnetic parameter *M* on temperature. It can be seen from the figure that temperature at a point decreases with the increase of *M* for the first solution and the temperature profile increase with the increase of *M* for the second solution. This is the consequence of the fact that temperature field is influenced by the advection of the fluid velocity above the sheet. Interestingly, the effect of *M* on the second solution is very significant in comparison to the first solution.

a.



b.

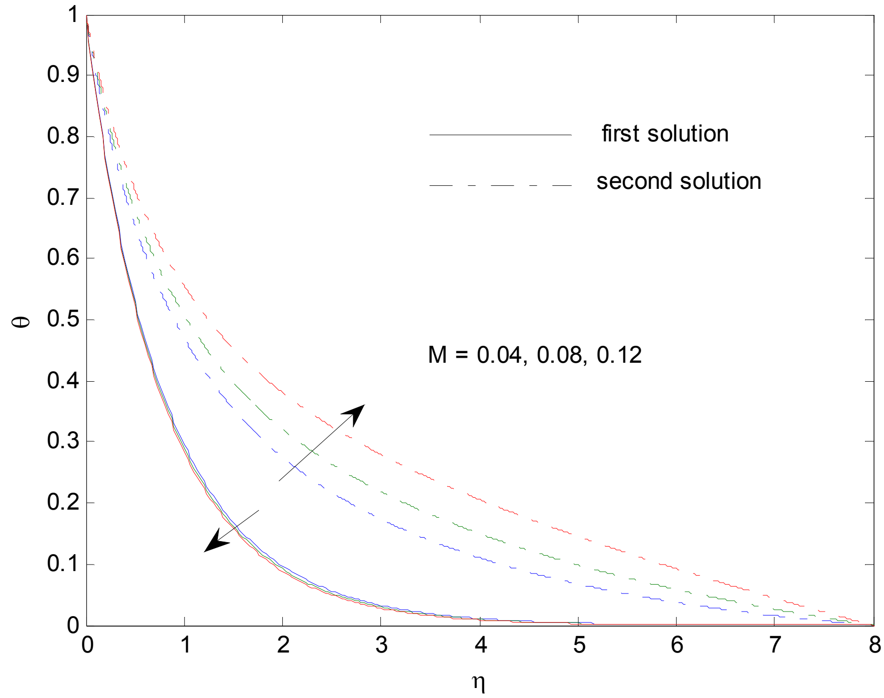
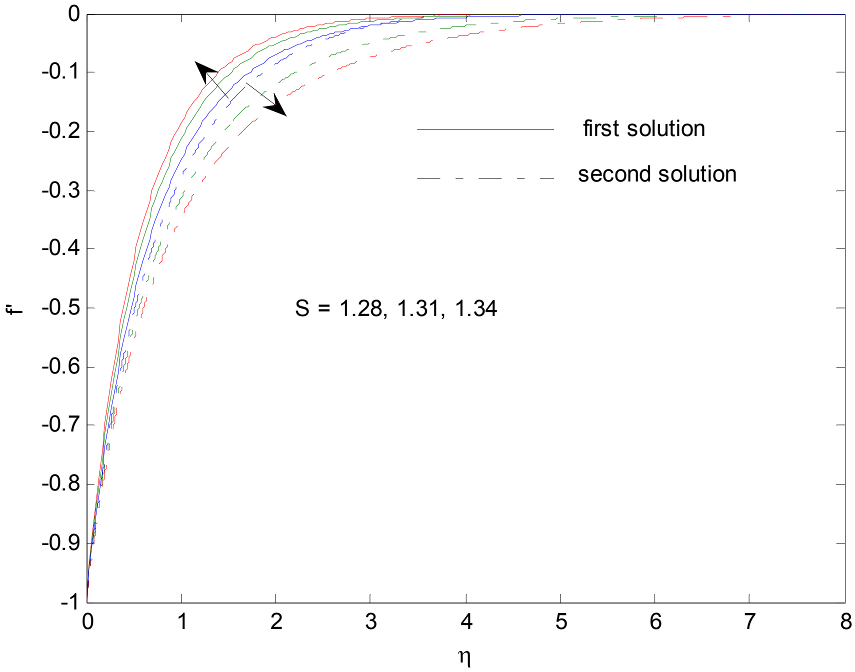


Fig.1. Dual velocity and temperature profiles for various values of *M*.

Figures 2a and 2b are drawn respectively for velocity and temperature profiles for several values of the suction parameter *S*. The velocity profile *f'* is to increase with the effect of the suction parameter *S* for the first solution and opposite is true for the second solution. Furthermore, it is evident that in every case of dual solution of velocity profiles the momentum boundary layer thickness is larger than that of the first solution.

It follows from Fig.2b that the temperature profiles decrease with the increase of S for the first solution and temperature profiles increases with the increase of S for the second solution. It is important to note that the effect of suction is more significant in the second solution.

a.



b.

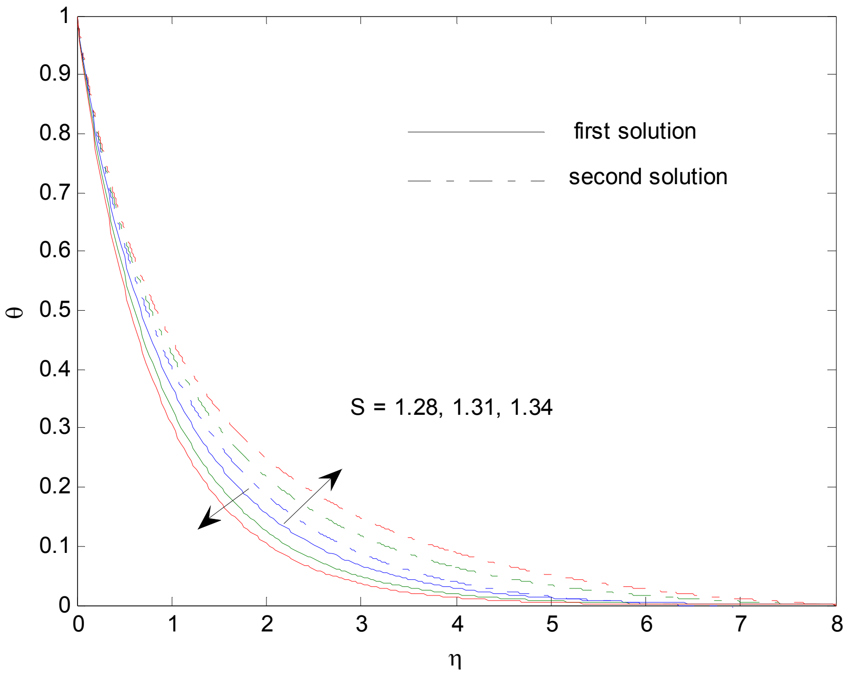
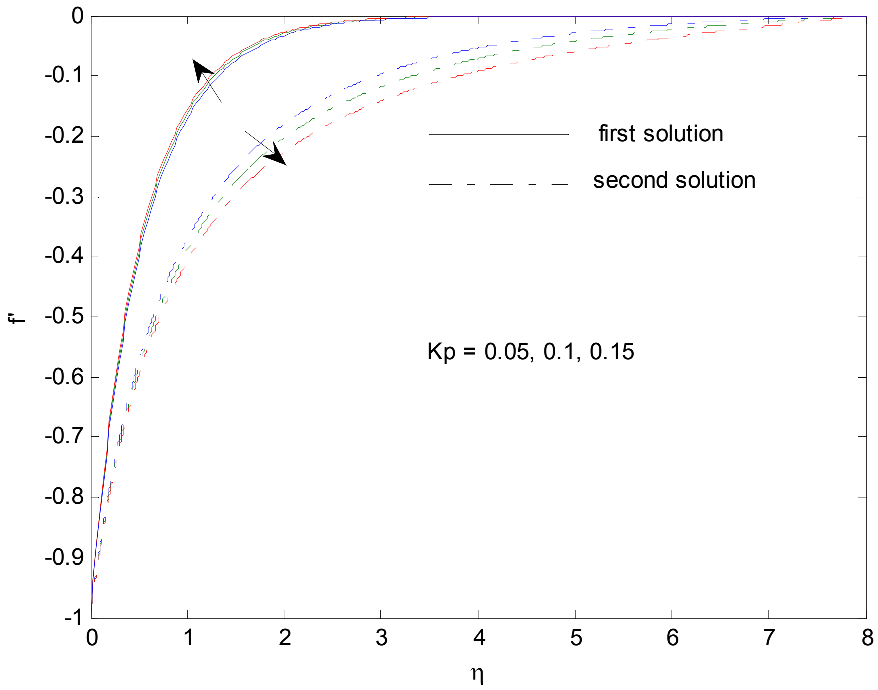


Fig.2. Dual velocity and temperature profiles for various values of S.

Figures 3a and 3b have been plotted to demonstrate the effect of the porosity parameter K_p on dual velocity and temperature profiles respectively. For the first solution it is noticed that the velocity increases as the porosity parameter increases for the case of the first solution and the opposite result is noticed for the second solution. It is due to the fact that the increasing porosity causes the momentum boundary layer thickness and consequently enhances the flow near the solid surface. The temperature profiles for selected values of K_p are presented in Fig.3b. It can be seen that with the increase of the porosity parameter K_p the temperature profiles decrease for first solution. It is also evident that with the increase of K_p the temperature profile increases significantly in the case of second solution.

a.



b.

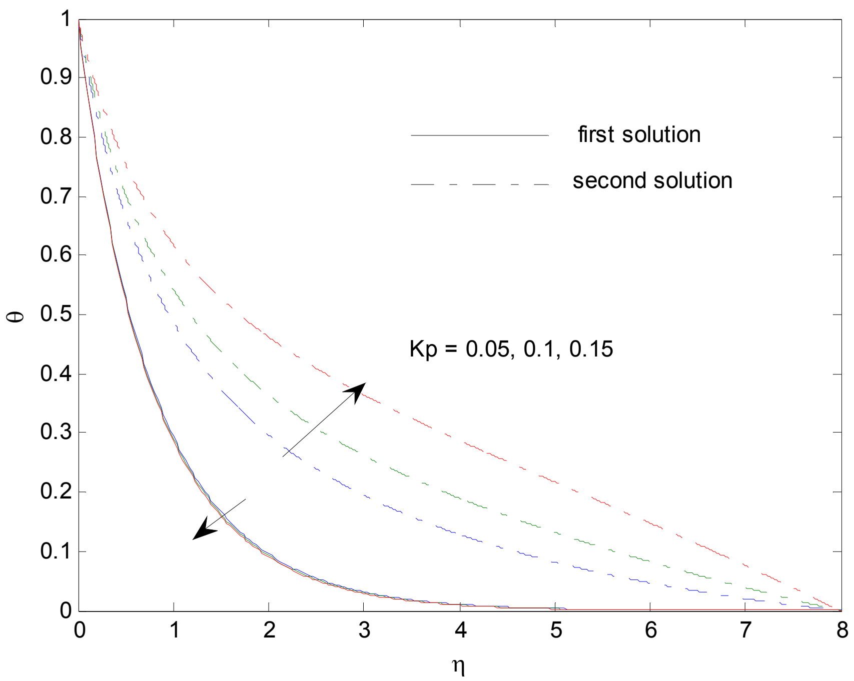


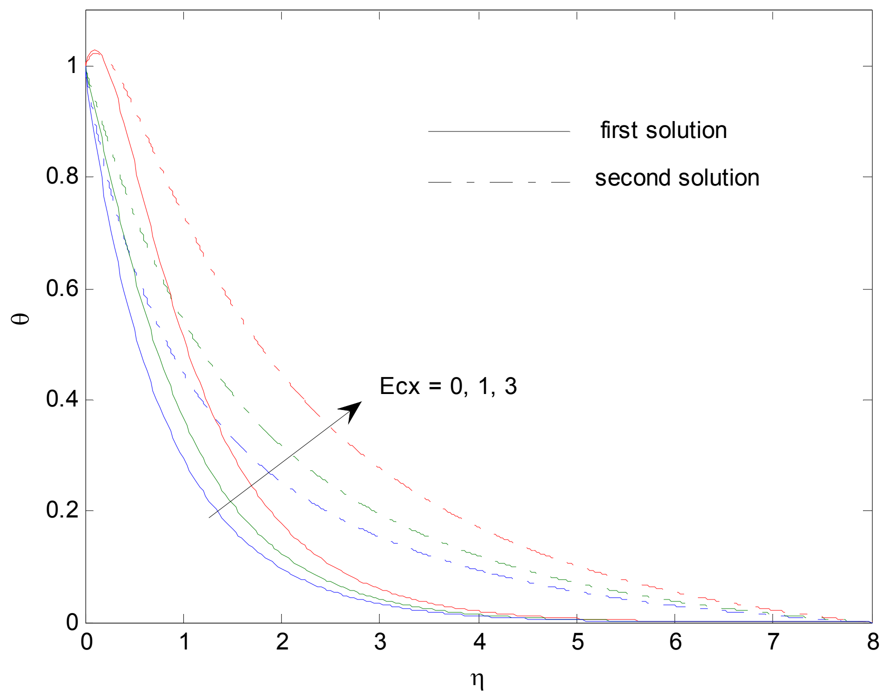
Fig.3. Dual velocity and temperature profiles for various values of *Kp*.

The effect of the local Eckert numbers Ecx and Ecy on dual temperature profiles is shown in Figs 4a and 4b respectively. It is evident from the figure that the temperature profiles increase with the increasing values of both Ecx and Ecy. However, the temperature field is effected by the local Eckert numbers more in the second solution when compared with the first solution. The dual temperature profiles for various values of the viscous dissipation parameter Ec is shown in Fig.5. The parameter Ec called the fluid motion controlling parameter. From the figure it is noticed that with the increase of Ec the temperature profiles increase for both solutions, this is due to the fact that viscous dissipation creates frictional heating which is stored in the fluid as heat energy and this heat energy increases the thickness of thermal boundary layer as result. The Eckert number Ec represents the relationship between kinetic energy in the flow and the enthalpy. It embodies the kinetic energy into internal energy by work done against the viscous fluid stresses.

From Fig.6 it can be observed that the temperature profiles increase with the increase of the radiation parameter *Rd* for both solutions: the effect of the radiation parameter *Rd* causes an increase in the radiative heat flux.

It is observed from Fig.7 that there exists a dual solution for different values of the Prandtl number Pr and it is evident that the temperature profiles decrease with the increase of the Prandtl number Pr. The dual temperature profiles for various values of the heat source/sink parameter Q are shown in Fig.8. It is observed that the temperature at a point in the boundary layer increases for an increasing heat source($Q>0$) and the temperature profile decrease for an increase of heat sink ($Q<0$) and the thickness of the thermal boundary layer reduces the increase of the heat sink parameter but the thermal boundary layer increases with the heat source parameter. This result is very much significant for the flow where the heat transfer is given prime importance.

a.



b.

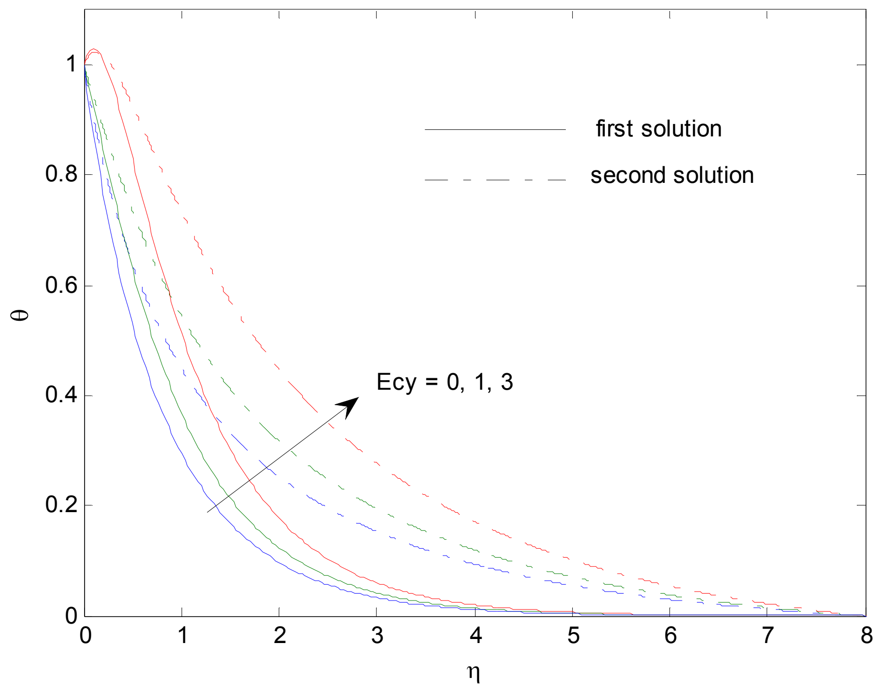


Fig.4. Dual temperature profiles for various values of Ec_x and Ec_y.

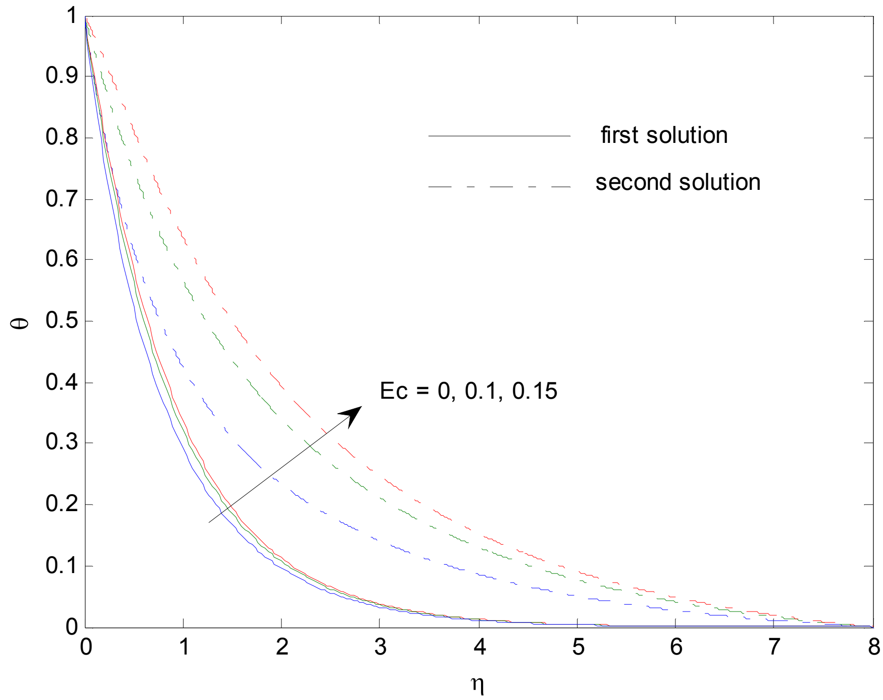


Fig.5. Dual temperature profile for various values of Ec.

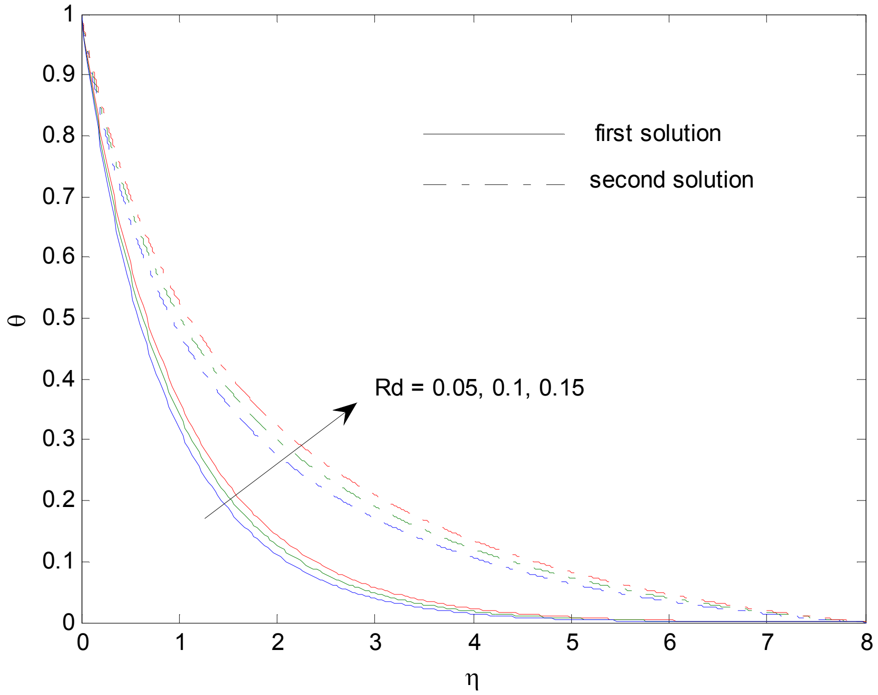


Fig.6. Dual temperature profile for various values of *Rd*.

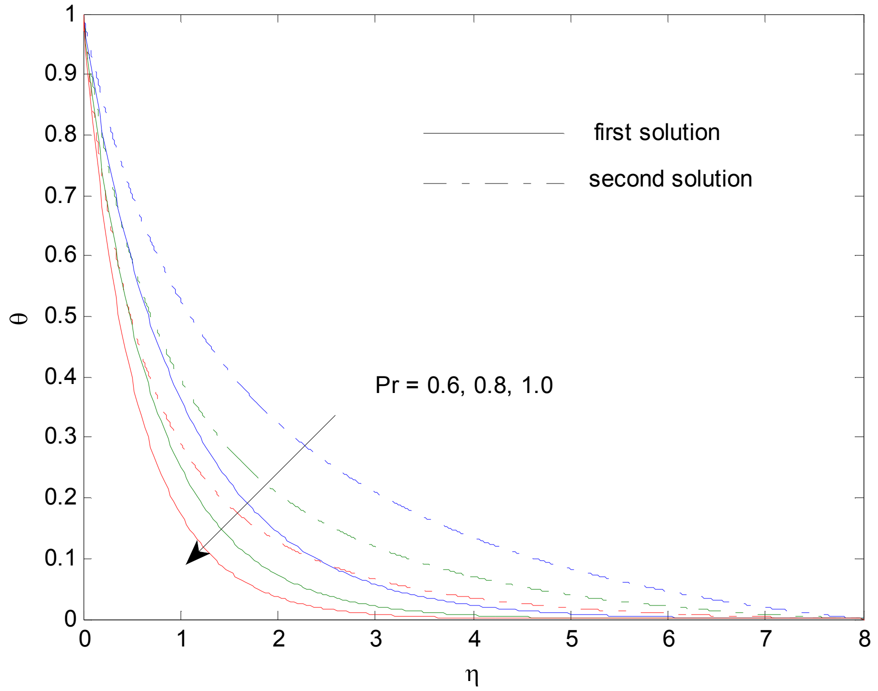


Fig.7. Dual temperature profile for various values of Pr.

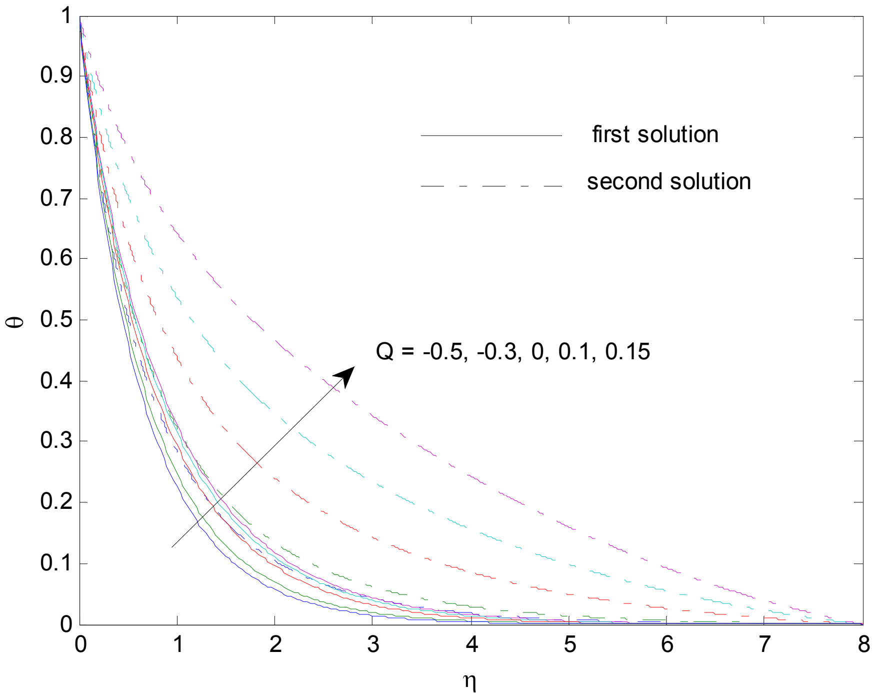


Fig.8. Dual temperature profile for various values of Q.

Nomenclature

a	– shrinking constant
B_0	– magnetic induction
C_{f_x}	– local skin friction coefficient
Ec	– Eckert number
$\mathrm{Ec}_x, \mathrm{Ec}_y$	– local Eckert numbers
f	– dimensionless stream function
K_p	– porosity parameter
M	– magnetic field parameter
Nu	– local Nusselt number
Pr	– Prandtl number
Q	– heat source/sink parameter
Rd	– radiation parameter
$\mathrm{Re}_x, \mathrm{Re}_y$	– local Reynolds numbers
S	– suction parameter
T	– fluid temperature
u, v, w	– velocity components
x, y, z	– Cartesian coordinates
α	– thermal diffusivity
η	– similarity independent variable
θ	– dimensionless temperature
μ	– dynamic viscosity
ν	– kinematic viscosity of the fluid
ψ	– stream function

Subscripts

w – conditions at the wall
∞ – ambient condition

Superscripts

' – prime denotes the derivative with respect to η

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Received: August 17, 2015

Revised: March 10, 2016