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THE SIMULATIONS OF SEQUENTIAL OF ESTIMATORS FOR OBJECTS WITH A SERIAL STRUCTURE

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The paper concerns the results of simulation of a certain approach, in which estimation of object state associated with the decentralization of calculations. It is realized by dividing the optimization problem into sub-problems of smaller dimensionality. The main idea of the method is to assign individual quality indicators to each sub-object and to carry out the synthesis of estimators in a sequential manner, starting with the last sub-object. Implementation of the estimation process requires knowledge about the measurements of the individual sub-objects. The parameters of the filter sequential gains are calculated based on Riccati equation solutions for sub-objects and certain bilinear equations for cross-linkage connections. In the simulation tests the influence of types of connection between the sub-objects, the intensity disturbances of measurements and system on the values of coefficients of gains, as well as the estimation errors is presented.

Keywords: complex systems, dynamics equations, filter sequential, Riccati equation, bilinear equations, simulation

1. Introduction

The following paper presents simulation studies in a decentralized approach that assigns enhancements necessary for estimating the state of an object with a serial structure. Then the gains of filters are calculated for the tasks of lower dimensionality by dividing the optimization problem into sub-problems [8, 9]. The major idea of the method consists in assigning individual quality indicators to the objects and carrying out the synthesis of estimators in a sequential manner. A structure corresponding to the local (classical) Kalman filter and cross-coupling is obtained [1, 3]. It comes down to solving Riccati differential equations for sub-objects and bilinear equations for cross-linkage connections.

2. Filtering problem in the multi-user Nash game framework

It is here assumed that the knowledge about the condition of sub-objects is provided in the form of inaccurate measurements. There are multiple approaches to the estimation of the state of complex objects [1, 7]. One of possible solutions is to exclude the object structure; it leads to multidimensional calculations which in turn pose reasoned difficulties. There are solutions known from the literature, regarding the problem of control for the objects with complex structure [2, 4, 5, 6, 10]. A different approach to filtering is to extend the Özgüner - Perkins approach of the one presented in the work [8, 9]. However, the approaches that are presented there seem to allow a shortened calculation process.

Filtering problem is formulated in a deterministic manner to adapt the mentioned approach in the possibly least complicated way. An example of the structure of complex objects which will be included in the studies is presented in Fig. 1, where A_1, A_2, A_3 – represent the matrix of sub-objects state, and A_{21} and A_{32}-connections matrices, x_i, W_i, Y_i – state vector, disturbances and measurements respectively.

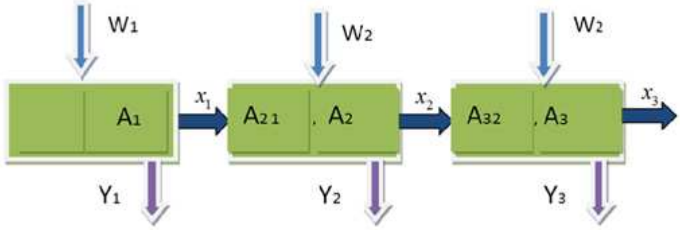

Figure 1. A block diagram of the complex object with a serial structure

A model of the object considered here looks as follows:

$$\frac{d}{dt}x(t) = Ax(t) + \sum_{i=1}^{N}\tilde{D}_i w_i(t), \qquad x(t_o) = x_o$$

$$y_i(t) = \tilde{C}_i x(t) + \upsilon_{pi}(t), \qquad i = 1, \ldots, N \qquad (1)$$

where: x-state vector, y_i-measurement in the sub-object υ_{pi}(t) means a measurement error, w_i- system disturbance, while A-state matrix is determined as follows:

$$A = \begin{bmatrix} A_1 & 0 & & 0 \\ A_{21} & A_2 \ddots & & \vdots \\ 0 & A_{i,i-1} & A_i & \\ \vdots & & \ddots & . \\ 0 & \cdots & A_{N,N-1} & A_N \end{bmatrix}, \tag{1b}$$

The diagonal elements correspond to the matrix of sub-objects state and the elements below correspond to the diagonal connections between the sub-objects.

For each particular sub-objects the following functional has been defined:

$$J_i = (x_o - \overline{x}_o)^T P_{oi}^{-1}(x_o - \overline{x}_o) + \int_T \left\{ w_i^T W_i^{-1} w_i + (y_i - \tilde{C}_i x)^T V_{pi}^{-1}(y_i - \tilde{C}_i x) \right\} dt \tag{2}$$

in which the matrices P_o, W, V are symmetric and positive definite. In a stochastic formulation of the filtering problem they would correspond to the covariances. Considerations concern a situation when T = [t_o, t_k], t_k is finite.

Assuming that the multi-user Nash game is taken into consideration in which w_i means control, i-the player, and $J_i(w_1 \ldots w_i)$ its quality functional. An optimal solution $\{w_1 \ldots w_N\}$ is determined by the condition of Nash equilibrium.

$$J_i(w_1^*, \ldots, w_i^*) \leq J(w_1^*, \ldots, w_{i-1}^*, w_i) \qquad \text{for } i = 1, \ldots, N \tag{3}$$

The problem with filtering is formulated as a task which aims at finding the estimate *x(t)* present state, for which the condition (3) is met with respect to variable w_i.

A solution for the problem with filtering (1, 2, 3) is the estimate $\hat{x}(t)$ satisfying the equation of the form

$$\frac{d}{dt}\hat{x}(t) = A\hat{x}(t) + \sum_{i=1}^{N} K_i \left[y_i(t) - \tilde{C}_i x(t) \right], \quad \hat{x}(t_o) = \hat{x}_o, \tag{5}$$

where coefficients of enhancement K_i yield the formulas

$$K_i = P_i \tilde{C}_i V_{pi}^{-1}, \qquad i = 1, \ldots, N, \tag{6}$$

and matrices P_i are the solution of the Riccati equations

$$\frac{d}{dt} P_i = AP_i + P_i A^T + P_i \tilde{C}_i^T V_{pi}^{-1} \tilde{C}_i P_i - \sum_{j=1}^{N} (P_i \tilde{C}_j^T V_{pj}^{-1} P_j + P_j \tilde{C}_j^T V_{pj}^{-1} \tilde{C}_j P_j) + \\ + \tilde{D}_i W_i \tilde{D}_i^T, \; P_i(t_o) = P_{oi} \tag{7}$$

The equation (5) will be called here the filter equation (alternatively the estimate equation). For the object with a serial structure (Fig.1) *i-th* the sub-object is defined in the equations.

$$\frac{d}{dt}x_i(t) = A_i x_i(t) + A_i x_{i-1}(t) + D_i w_i(t)$$

$$y_i(t) = C_i x_i(t) + \upsilon_{pi}(t), \tag{8a}$$

Matrices in the quality indicator (2) and in (1) look as follows:

$$W_i = diag[0,...,0,W_i,0,...,0], \qquad P_{oi} = diag[0,...,0,P_{oi},0,...,0]$$

$$\tilde{D}_i = col[0,...,0,D_i,0,...,0], \qquad \tilde{C}_i = col[0,...,0,C_i,0,...,0] \tag{8b}$$

Determining the matrix P_i as $[P^i_{kl}]$, $k, l=1,\ldots N$ and substituting the matrices $\tilde{A}$ and $W, P_o, \tilde{D}, \tilde{C}$ from (8b) to (5 - 7) we obtain the state estimators for the objects with a serial structure in the form of equations:

$$\frac{d}{dt}\hat{x}_1 = A_1\hat{x} + K_1[y_1 - C_1\hat{x}_1], \qquad \hat{x}_1(t_0) = \overline{x}_{01}$$

$$\frac{d}{dt}\hat{x}_2 = A_2\hat{x}_2 + A_{21}\hat{x}_1 + K_{21}[y_1 - C_1\hat{x}_1] + K_2[y_2 - C_2x_2], \qquad \hat{x}_2(t_0) = \overline{x}_{02} \tag{9}$$

$$\frac{d}{dt}\hat{x}_N = A_N\hat{x}_N + A_{N,N-1}\ \hat{x}_{N-1} + \sum_{j=1}^{N}\{K_{Nj}[y_j - C_jx_j]\}, \qquad x_N(t_0) = \overline{x}_{0N}$$

Enhancement coefficients are determined by formulas

$$K_i = P^i_{ii}C_i^TV_{pi}^{-1}, \qquad K_{ij} = P^i_{ij}C_j^TV_{pj}^{-1}, \tag{10}$$

where P_{ii} is a solution of the Riccati equation

$$\frac{d}{dt}P^i_{ii} = A_iP^i_{ii} + P^i_{ii}A_i^T - P^i_{ii}C_i^TV_{pi}^{-1}C_iP^i_{ii} + D_iW_iD_i^T, \qquad P^i_{ii}(t_o) = P_{oi}, \tag{11a}$$

while P^i_{ij}, $i > j$ satisfies the bilinear equation

$$\frac{d}{dt}P^i_{ij} = A_i^*P^i_{ij} + P^i_{ij}A_j^{*T} + A_{ij}P^i_{ij} + \sum_{k=j+1}^{i-1}A^*_{ik}P^i_{kj}$$

$$A_i^* = A_i - K_iC_i, \qquad P^i_{ij}(t_0) = 0 \quad A^*_{ik} = A_{ik} - K_{ik}C_k \tag{11b}$$

$$A_{ik} = \phi \quad for\ \ i > k+1.$$

It is easy to notice that the equation (9) can be solved independently remembering about the proper order. A designing of the filter equations in a sequential manner should start from the end sub-object.

3. Decentralized algorithm

Phase algorithm is defined in the following form.

Step 1. In the last sub-object we assume that the familiar enforcements in the form of the states of preceding sub-objects results from the connections. The Kalman-Bucy Filter for N sub-object is defined by the last equation in (9), in which there is only one enhancement K_N and x_{N-1}substitutes $\hat{x}_{N-1}$.

Step 2. Next, we consider the sub-object that consists of the filter set above and N-1 sub-object. We create a filter for this system.

Step 3. We adjoin another preceding sub-object to the group of state estimators we obtained. Fig. 2 presents a cascade object with three sub-objects.

Step 4. and next. Further attachment of individual sub-objects occurs when appointing of the group of state estimators for a particular phase has been completed. First sub-object is the last adjoined for which the final architecture of the estimator state set is appointed.

In the estimators group for three sub-objects we can distinguish local filters of enforcements K_1, K_2, *i* K_3. Moreover, there occur other couplings binding signals $y_j - C_j\hat{x}_j$, $j < i$ from the preceding sub-objects and estimators. Enforcements K_{ij}, K_i are defined in a decentralized manner. The estimators group obtained here has been called a sequential filter [8].

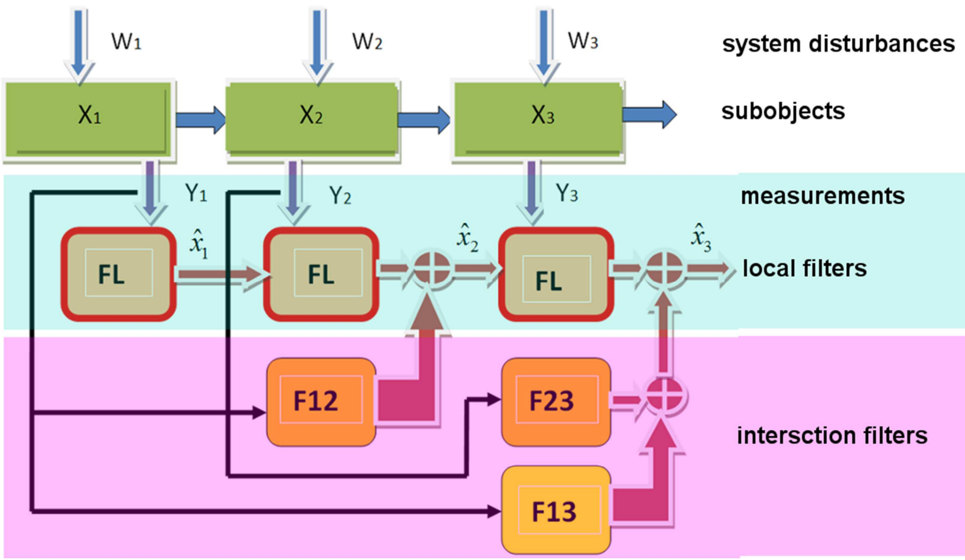

Figure 2. Block diagram of the object with sequential filter

The quality of the presented method is:

- Filter realization requires the knowledge of measurements only from the current objects and the preceding ones.
- Enforcements can be appointed in a sequential manner, solving problems with low dimensionality.
- Local back couplings are calculated based on the solutions of Riccati equations, and cross-linkage based on bilinear solutions.
- The presented method leads to obtaining a structure that corresponds to the classic linear-quadratic problem with the Kalman-Bucy filter.
- Strict proof of how much it is reasonable to separate the design of regulators and filters for cascading objects is an issue in the field of stochastic differential games.

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4. Simulation experiments

Simulation studies were conducted for the object that comprises three identical sub-objects and the same connections between them ($A_{12} = A_{23}$). Such object then was submitted for system disturbances with various set intensity of influence Ws = [1 0; 0 1]. The measurements from the sub-objects had also the same common disturbance covariance V. Values of state matrix in (1b) A_i (i = 1, 2, 3) are as follows: [0.2 0.0; 0.185 0.71]. The influence of the matrix of connections between sub-objects A_{ij} has been investigated on the value of coefficients of filter enforcements K_i, K_{ij} (for W_3 = [6, –4; –4, 2], V = 0.3). Results were shown in Table 1.

Table 1. Filter gains for different connections between the sub-objects (Ws = [6, –4; –4, 2], V = 0.3)

Lp	K_i – local gains	K_{ij}- intersection gains	A_{ij}- intersection matrix
1	–4.3596 2.2538	0.0480 –0.0037	1 0 0 1
2	–4.3596 2.2538	0.2310 $*10^{-5}$ –0.0176 $*10^{-5}$	0.8 0 0 0.8
3	–4.3596 2.2538	0.1483 $*10^{-7}$ –0.0113 $*10^{-7}$	0.7 0 0 0.7
4	–4.3596 2.2538	0.00132 0.00005	0.5 0 0.25 0.5
5	–4.3596 2.2538	0.0451 0.0018	0.5 0 0.3 0.5

Local filter gains have higher values than cross-linkage gains. The state of correlation matrix has a significant influence on the values of cross-linkage gains.

Table 2. Filter enforcements for different covariance of system disturbances

Lp	K_i - local	K_{ij}- intersection gains	A_{ij}- intersection matrix	W_s – covar. sys disturb.
1	–2.9736 1.5121	0.00202 0.00006	0.5 0 0.25 0.5	3 –2 –2 1
2	–2.9736 1.5121	0.16727 –0.0182	0.8 0 0 0.8	3 –2 –2 1
3	–1.1478 1.3545	1.3555 0.0463	0.5 0 0.25 0.5	1.0 –.5 –.5 1.0

Lower values of system disturbance intensity generate lower values of filter enforcements.

Table 3. Filter gains for different covariance of measurements disturbances (W_s = [1.0 –.5; –.5 1.0])

Lp	K_i- local gains	K_{ij}- intersection gains	A_{ij}-intersection matrix	V- covariance measure
1	–0.4845 0.5215	1.0097 0.0618	0.5 0 0.25 0.5	1.2
2	–2.1199 2.6498	2.0910 0.0424	0.5 0 0.25 0.5	0.1

More exact measurements lead to generating of higher coefficients of filter gains, while the growth for the cross-linkages shows lower dynamic of changes. The selected results of simulation studies are presented below.

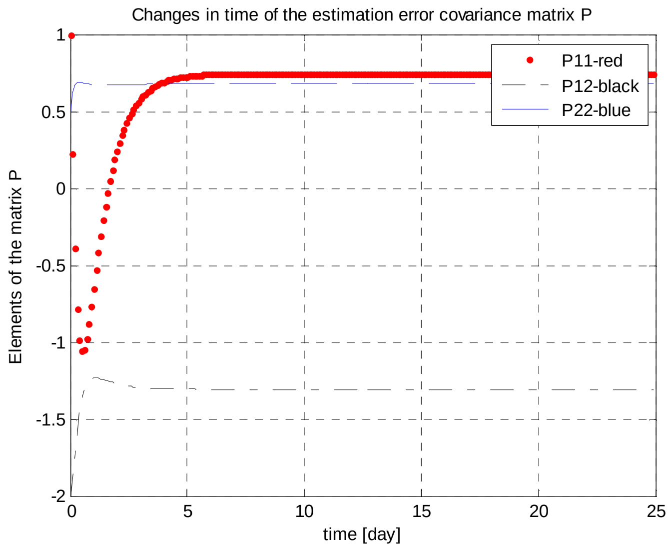

Figure 3. The course of elements of estimation error covariance matrix (for covariance matrix system disturbances $W = [6 \quad -4; -4 \quad 2]$)

For the courses from Fig. 1 we obtained the coefficients of local gains with values: $K_{FL} = [-1.9959; 0.9957]$.

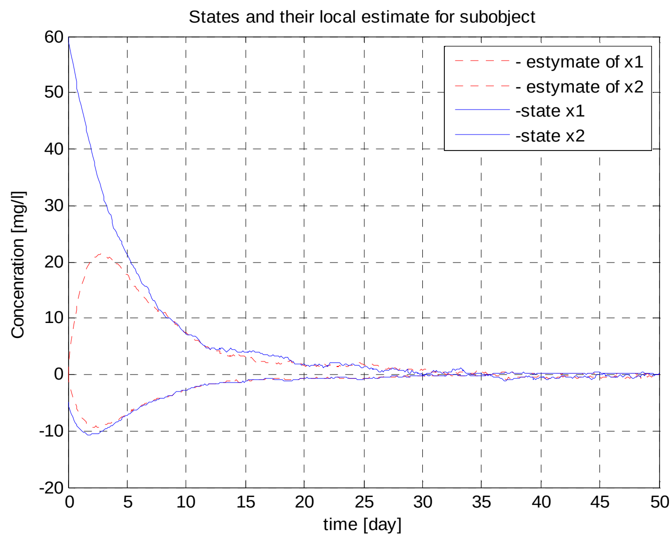

Figure 4. The course of the state coordinate x_1 and its estimate

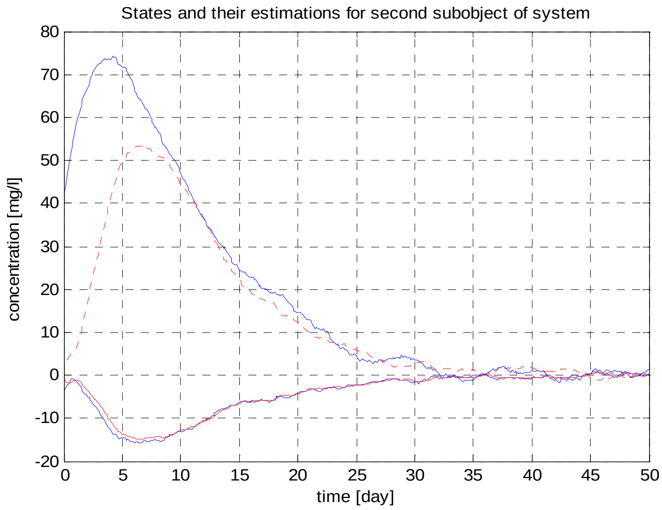

Figure 5. The course of the estimate of state (from local and sequential filter)

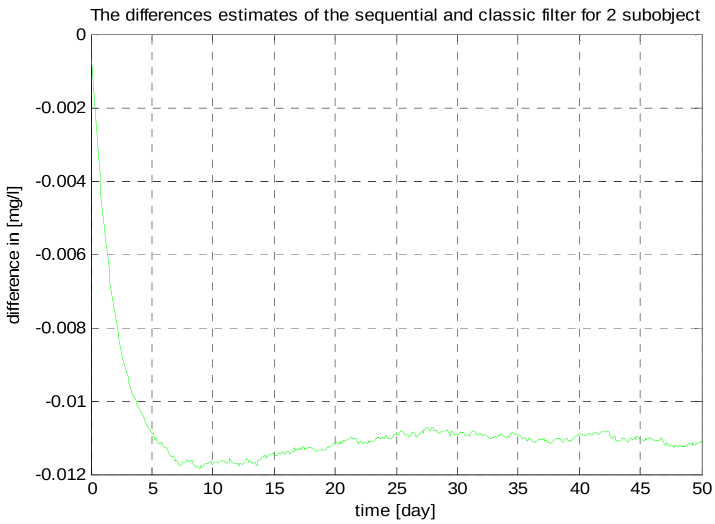

Figure 6. The course of the difference of estimates for sub-object 2 (from local and sequential filter)

The investigation covered simulation courses of the filtering process of object state using a group of local and sequential filters. Biggest differences between state and estimate occur at the beginning of the filtering process, and it results from setting zero initial estimates. However the course of state estimates is relatively quick in keeping the change character of state changes and estimation errors decrease.

Gains of filter are closely dependent on the character of disturbance the increase in measurement noise intensity reduce them. Cross-linkage gains for segments further than the first-third are relatively low and are practically insignificant. The influence on the values of cross-linkage gains also depend on the elements and configuration of matrix of connections between sub-objects, that is the growth of values generate higher filter gains.

5. Conclusion

The simulation studies and the solution discussed in the paper that allows determining the enforcement of sequential filter in a decentralized manner present a general approach. However specific complex structures of objects need to be considered. In this article a sequential filter has been obtained, that comprises local estimators that realize estimations with local enforcements (optimal local filters) and corrections that result from the fact that the complexity of the problem of estimation error minimization has been taken into consideration. Those additional corrections of estimates that come from the corrections calculated from the bilinear equations improve the quality of estimation process in relation to the local considerations. It needs to be emphasized that first the Riccati equations (connected with sub-objects) should be solved, and next bilinear equations be calculated, on the basis of which cross-linkage enforcements are calculated. Dimensionality of both issues comes down to the size of individual sub-objects. In such a way a decentralized method of gains determination in filtering system of objects with complex structure is obtained. This is a hierarchical way of solving estimation questions for objects with high dimensionality. A feature of the presented method is that filter realization requires all sub-objects to be measured, not necessarily the one which represent whole states vectors. It can be assumed that the idea of consideration has a hierarchical character in a way. The gains of this filter can be appointed in a sequential manner, solving problems with low dimensionality. It is necessary to notice that the structure of gain matrix for the whole object is a lower triangular, despite the occurrence of zero elements that correspond with them in the state matrix A (except from diagonal and neighboring ones below).

A digital simulation of the object state filtering process has been conducted with the use of the group of sequential filters. The biggest differences between state and estimate occur at the beginning of filtering process, and it results from the approved initial zero estimates. However the course of state estimates is relatively quick in keeping the change character of state changes and estimation errors decrease. Coefficients are closely dependent on the character of disturbance, the increase in measurement noise intensity reduce them. Cross-linkage gains for segments further than the first-third are relatively low and are practically insignificant. The influence on the values of cross-linkage gains also depend on the elements and configuration of matrix of connections between sub-objects, that is the growth of values generate higher filter gains. The analysis of the results of filtering process simulations for the cases when we use a group of unrelated local filters and a group of sequential filters fails to show any significant differences. However, a trace of covariance matrix for the latter case is minimal, of 1÷2%, smaller than in the former. Such small differences, in the process of practical realization, entitle to consider alternative exclusion of the connection, only when it would lead to a significant reduction of the costs of control system.

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