International Symposium on
HYDROACOUSTICS AND ULTRASONICS
EAA Symposium (formerly 13th FASE Symposium)
Gdańsk-Jurata, 12-16 May 1997

Acoustic streaming generated by ultrasonic transducers, measurements by means of 32 MHz Doppler

A. Nowicki, W. Secomski and J. Wójcik

Department of Ultrasound, Institute of Fundamental Technological Research, Polish Academy of Sciences, Swietokrzyska 21, 00-049 Warsaw, Poland

ABSTRACT

An approximate solution for the streaming velocity generated by flat and weakly focused transducers was derived by directly solving the Dirichlet boundary conditions for the Poisson equation. The theoretical calculations were verified using a purpose-designed 32 MHz pulsed Doppler unit. The applied average acoustic power was changed from 1 μW to 6 mW. The experiments were done on 4 mm diameter flat and focused transducers. The streaming velocity was measured along the ultrasonic beam from 0 to 20 mm. Streaming was induced in a solution of water and corn starch. The experimental results showed that for a given acoustic power the streaming velocity was independent of the starch density in water changed from 0.3 grams to 40 grams of starch in 1 litre of distilled water. For applied acoustic powers, the streaming velocity changed linearly from 0.2 to 40 mm/s. Both, the theoretical solutions for plane and focused waves, and the experimental results were in good agreement.

INTRODUCTION

The acoustic waves which propagate in liquids observe the general laws of hydrodynamics. In a linear medium the dependence between the pressure and the particle velocity is linear; in other words, the acoustic impedance is constant. In a non-linear medium, the impedance varies in time and the acoustic pressure has a constant component and by analogy to electric systems, it may be said that a non-linear medium acts like a pressure rectifier. Waves with finite amplitudes are accompanied by such events as the radiation pressure and streaming.

THEORY

Nyborg [1965] solved the Navier-Stokes equation with an accuracy up to the second order approximations. Wu and Du [1993] developed the theory of deriving the approximate solution to the Nyborg equation.

Adopting relevant assumptions, the authors obtained the Poisson equation as a description of the component v_{2z} of the vector $\mathbf{v} = [v_{2r}, v_{2z}]$ of the streaming velocity:

$$\nabla^2 v_{2z} = \frac{1}{\mu c_o}\frac{\partial I_z}{\partial z} = -\frac{2\alpha}{\mu c_o} I_z \qquad (1)$$

Although Wu and Du [1993] undertook to solve equation (1) from the beginning, the following well-known general solution [Jackson, 1975] includes the relevant boundary conditions for the equation of this type:

$$v_{2z}(\mathbf{x}) = \int_\Omega Q(\mathbf{x}')G(\mathbf{x},\mathbf{x}')d^3x + \frac{1}{4\pi}\int_\Sigma [T(\mathbf{x}',\mathbf{x})]\, d\Sigma \qquad (2)$$

where $Q(\mathbf{x}) = \frac{\alpha}{2\pi\mu c_o} I_z$,

$T(\mathbf{x},\mathbf{x}') = G(\mathbf{x},\mathbf{x}')\partial n' v_{2z}(\mathbf{x}') - v_{2z}(\mathbf{x}')\partial n' G(\mathbf{x},\mathbf{x}')$, $G(\mathbf{x},\mathbf{x}')$ satisfies the equation $\Delta\, G(\mathbf{x},\mathbf{x}') = -4\pi\,\delta(\mathbf{x},\mathbf{x}')$ and is the Green function for the relevant boundary conditions (Dirichlet's or Neuman's) and $\delta(\mathbf{x},\mathbf{x}')$ is the Dirac function, $\mathbf{x}=(x,y,z)$ in Cartesian coordinates and $\mathbf{x}=(r,z)$ in cylindrical coordinates for acoustical field axially symmetrical, z is propagation axis. $\partial n' \equiv \mathbf{n}\nabla' v_{2z}(\mathbf{x}')$ is the component normal to the boundary surface of the gradient of the function v_{2z}, n is the vector unit normal to the surface Σ, Ω denotes 3-dimensional volume bounded by the surface Σ, d^3x is the differential element in Ω. It was assumed that on the boundary Σ for z=0, are given: $v_{2z}(x,y,z=0)=v_{2z}(r,z=0)=v_{2z}(r)=0$, $r=\sqrt{x^2+y^2}$ It means that the Dirichlet boundary problem was considered.

For the case of a bounded plane wave with a circular section, perpendicular to the propagation axis z, with the radius a, and for a Gaussian beam generated by a circular transducer with the radius a, the following solutions were obtained for the component v_{2z} of the vector of the streaming velocity v along the beam axis, r = 0.

1) Linear (bounded) plane wave

$$v_{2z}(0,z) = \frac{\alpha I_0 a^2}{\mu c_0} \bullet \int_0^\infty e^{-2\alpha a s}\left[\sqrt{1+\left(\frac{z}{a}-s\right)^2} - \sqrt{1+\left(\frac{z}{a}+s\right)^2} + \left(\frac{z}{a}+s\right) - \left|\frac{z}{a}-s\right|\right]ds \qquad (3)$$

2) A weakly focused beam with a Gaussian intensity distribution

$$v_{2z}(0,z) = K\int_0^\infty \sqrt{A(z')}\cdot e^{-2\alpha z'} \left[e^{\frac{2A(z')}{a^2}(z-z')^2}\,\mathrm{erfc}\left[\frac{\sqrt{2A(z')}}{a}(z-z')\right] - e^{\frac{2A(z')}{a^2}(z+z')^2}\,\mathrm{erfc}\left[\frac{\sqrt{2A(z')}}{a}(z+z')\right]\right]dz' \qquad (4)$$

where:

$$K = \frac{\alpha I_0 a^2\sqrt{\pi}}{\mu c_0 B 2\sqrt{2}}, \qquad A(z) \equiv \frac{B\,R\,F(R)}{(z-F(R))^2 + C(R)}$$

$$F(R) \equiv \frac{r_0^2 R}{r_0^2 + (BR)^2}, \quad C(R) \equiv F(R)[R - F(R)]$$

R is the geometrical focal length of the acoustic lens, F(R) is the physical focal length (the position of the maximum of the field intensity distribution), $r_o = (1/2)ka^2$, B is a constant related to the Gaussian beam and is 1 on the transducer surface (z = 0).

In their computer calculations the authors used the numerical approximation of the function erfc(x) [Handbook of Mathematical Functions, 1968]:

$$\mathrm{erfc}(x) = \mathrm{efc}(x)e^{-x^2} + \varepsilon(x),$$

$$|\varepsilon(x)| \le 1.5\cdot 10^{-7}$$

$$\mathrm{efc}(x) \equiv \sum_{n=1}^{5} p_n f(x)^n.\ f(x) = \frac{1}{1+px} \qquad (5)$$

$p=0.3275911$, $p_1=0.254829592$, $p_2=-0.284496736$, $p_3=1.421413741$, $p_4=-1.453152027$, $p_5=1.0614005429$

Substituting approximation (5) into (4), the following expression, convenient for computer calculations is obtained.

$$v_{2z}(0,z) = \frac{\alpha I_0 a\sqrt{\pi}}{2\sqrt{2}\mu c_0 B} \int_0^{z+z_g} \sqrt{A(z')} e^{-2\alpha z'} \left\{ \mathrm{efc}\left[\frac{\sqrt{2A(z')}|z-z'|}{a}\right] - \mathrm{efc}\left[\frac{\sqrt{2A(z')}(z+z')}{a}\right]\right\} dz' \qquad (6)$$

where z_g depends in fact on z and the properties of the function $\sqrt{A(z')}e^{-2\alpha z'}$. In our calculations for $z \leq 3R$ we adopted $z_g = 3R$. For $z \geq 4R$ it is sufficient to adopt $z_g \cong R$. For large z, but ones smaller than $1/\alpha$, z_g may be determined using the relation $z+z_g<\max[1/\alpha, 2R)]$.

It was assumed in the calculations that in water $\alpha = 2.53\cdot10^{-16} f^2$ Np/cmHz^2 and $\mu = 0.01$ Poise.

It should be stressed that Wu and Du's and our solutions hardly differ for the plane wave close to the transducer. A more serious problem occurred for numerical attempts to calculate their equation for steaming velocity for focused transducers, for it contains additional terms which have no counterparts in the solution of the Poisson equation obtained using the well-known boundary problem theory for this equation. The solution from equation (6) is convergent; on the other hand, the numerical results are about twice lower than those presented by Wu and Du. To confirm the doubts about the correctness of the solution given by Wu and Du, their results for a continuous wave and the results of the measurements performed by Starritt et al. [1989] were compared.

In the calculations, the following assumptions were made: $P_{tot} = 0.1$ W, $f = 3.5$ MHz, the transducer diameter $\Phi=20$ mm with its focus at depths of 7.3 cm and 11 cm, where the streaming velocity was about 3.4 cm/s. Starritt measured streaming as a function of the distance from the transducer. The power, frequency and diameter of the transducer were the same as those in the calculations made by Wu and Du, but the head was focused at 9.5 cm depth. In the focus the streaming velocity was close to 2 cm/s, about twice lower than that calculated theoretically by Wu and Du, but close to the results from formula (6).

METHOD AND RESULTS

Measuring system:

The streaming was measured using a Doppler ultrasound blood flowmeter. The 32 MHz pulsed Doppler prototype was developed at the Institute of Fundamental Technological Research. The transmitter of the flowmeter generated a 32 MHz burst, lasting 0.5 μs. The repetition frequency was 31.25 KHz.. The flowmeter receiver had a gate with 0.5 μs duration and variable delay. The receiver bandwidth was expanded from the DC up to 4 kHz. The Doppler signal was digitized, stored and next analyzed using the oscilloscope LeCroy 9450A. The signal processing included FFT spectrum analysis and averaging of 100 successive Doppler spectra. The frequency resolution was 0.1 Hz to 1 Hz. The ultrasonic probe contained a lithium niobate transducer, glued to a glass lens or to the flat glass plate. The lens and plate were made from BK-7 optic glass. Three probes were used in the measurements, each containing a transducer with a 4 mm diameter. Two probes were focused at depths of 8 and 12 mm, respectively, whereas the third had no focus. The effective transmitter voltage was measured at the transducer clamps, the electric power was calculated and next the acoustic power was estimated on the basis of Mason's equivalent transducer model. The estimated power was about 1.4 mW for each of the three transducers.

Measurements:

Streaming was measured in a rectangular container filled with a corn starch suspension in distilled water at 20^0 C. Experimentally, it was found that for 1 g of starch per 1 litre of water it is possible to obtain a Doppler signal with a satisfactory signal to noise ratio (>10 dB).

The system consisted of a Doppler flowmeter and an ultrasonic transducer working at 32 MHz, submerged in a container with water. The Doppler signal was analyzed using the oscilloscope LeCroy 9450A with an FFT analysis module. The maximum flow velocity was determined as the maximum frequency below which 90% of the power of the Doppler signal lies. The dependence of the streaming velocity on the radiated acoustic power was measured. It was found that for the power transmitted in the range between 0.001 mW to 6 mW the streaming velocity increased linearly with the power. The correlation coefficient was R=0.9992.

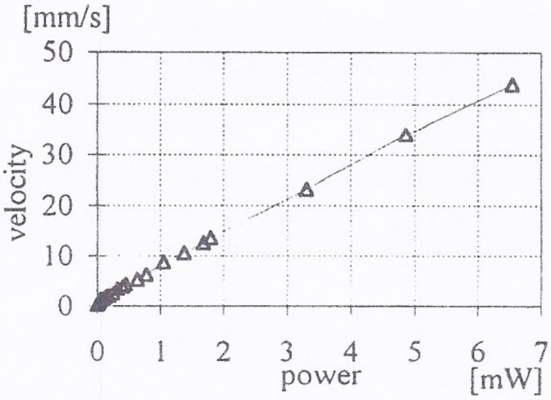

Fig. 1. Streaming velocity in function of the radiated acoustic power; plane 32 MHz transducer.

The streaming velocities were experimentally measured for three transducers. Fig.2 shows the streaming velocity for a plane transducer. Fig.3 shows the streaming velocities for transducers focused at 8 mm and 12 mm depth, respectively.

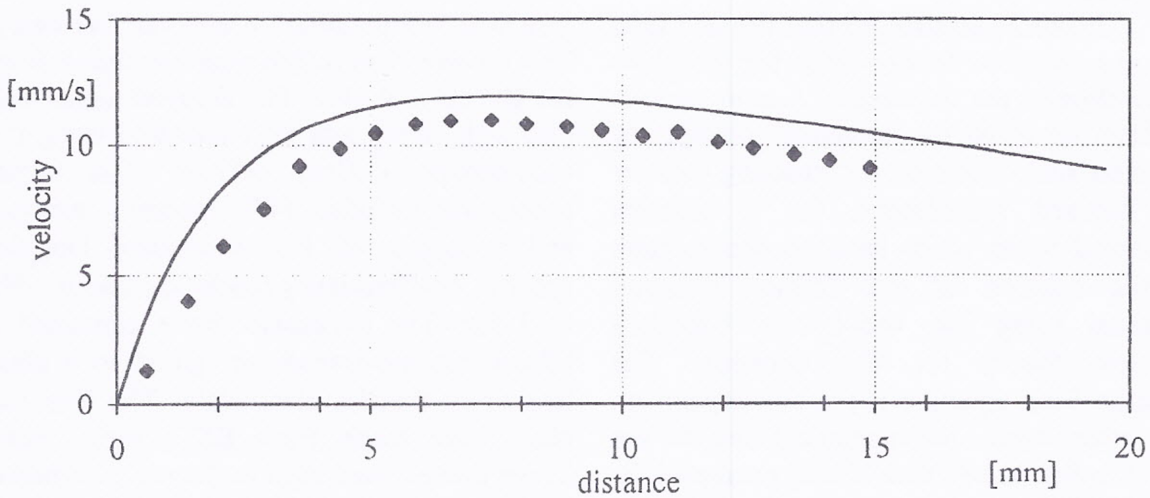

Fig. 2. Streaming velocity vs. distance, plane 32 MHz transducer. The radius of the transducer is equal to 4 mm. The solid line - theoretical calculations, points - experimental results.

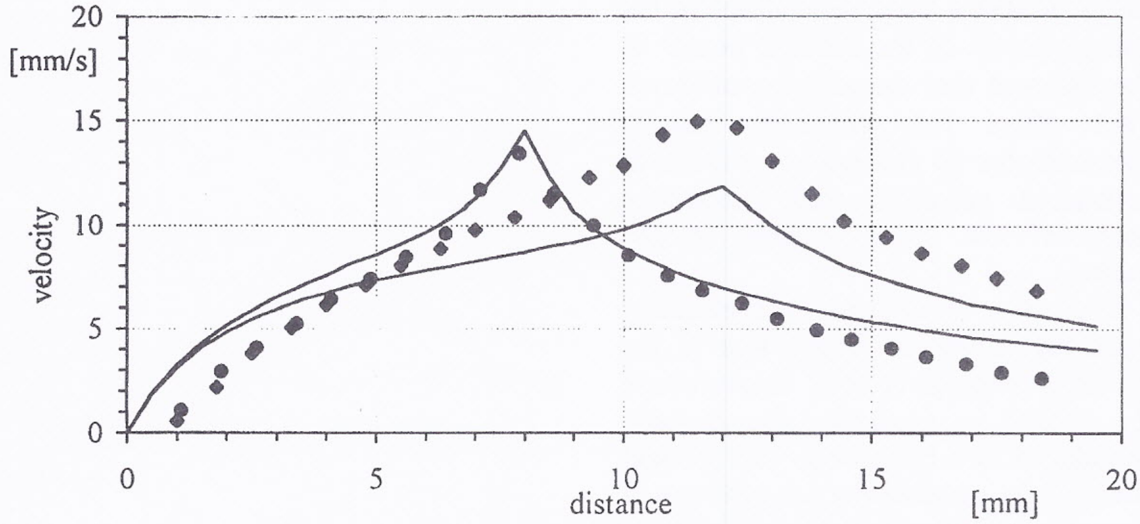

Fig. 3. Streaming vs. distance; focused transducers - focal distance = 8 mm and 12 mm, radius of the transducer is equal to 4 mm. The solid line - theoretical calculations, points - experimental results.

The flow velocity changes were measured for different densities of the starch suspension in water.

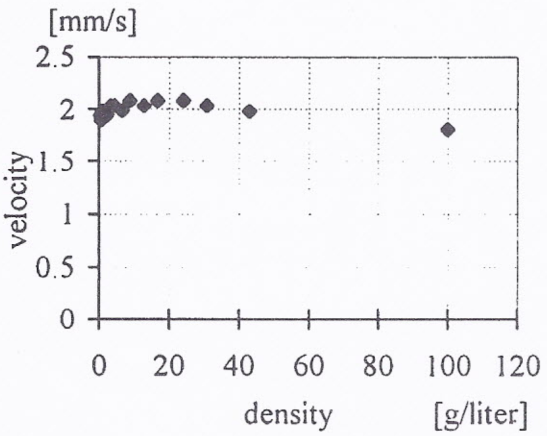

Fig.4. Axial streaming velocity vs. density of starch in water, acoustic power=0.3 mW

It was found that for densities between 0.3 g/l and 40 g/l the streaming velocity hardly varied. The maximum deviation from the mean value was - 5.3%. The stdev/mean ratio= 0.07. For a higher starch concentration than 40 g/l, the streaming velocity fell; it was probably caused by higher viscosity of the suspension.

CONCLUSIONS

The streaming velocities generated by plane and weakly focused ultrasonic heads were calculated. The authors drew on the general assumptions of Wu and Du [1993], who derived the Poisson equation as a description of the axial component of the streaming velocity. In contrast to them, the present authors applied the well-known general solution for the relevant Dirichlet boundary problem for the equation of this type.

The measured velocity changes as a function of depth confirm very well the theoretical calculations for all the three heads. Good quantitative agreement was obtained between the measured streaming velocities and those calculated theoretically. The latter were modified with respect to the solution of the Poisson equation given earlier by Wu and Du. The slight differences are probably caused by the inaccuracy of estimation of the maximum Doppler frequency and the impossibility of accurate measurement of the real acoustic power radiated by transducers.

The measurements for the frequency of 32 MHz indicated a linear dependence of the streaming velocity on the acoustic power. It may be assumed that the addition of starch did not affect the streaming velocity measurements for no changes were found in the measured velocities with percentage variations of the starch content in the suspension.

Given the measurement repeatability and accuracy it may be hoped that in the future the streaming velocity measurement can be applied in evaluating the acoustic power radiated by a transducer. This method may be used to measure the acoustic power of ultrasonographic probes meant for medical diagnostics.

REFERENCES

Abramowitz M. and I.A. Stegun, (eds.), (1968) Handbook of mathematical functions, Dover Publ, New York,

Nyborg, W.I., (1953) Acoustic streaming due to attenuating plane waves, J.Acoust. Soc. Am., 25:68-75.

Nyborg, W.L., (1965) Acoustic streaming, In Mason, W.P., ed., Physical acoustics, IIB, New York: Academic Press, 265-331.

Starritt H.C., Duck F.A., Humphrey V.F., (1989) An experimental investigation of streaming in pulsed diagnostic ultrasound beams, Ultrasound in Med. Biol. 15, 4, 363-373.

Wu J., Du G., (1993) Acoustic streaming generated by focused gaussian beam and finite amplitude toneburst, Ultrasound in Med. Biol., 19, 2: 167-176.