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predictive and reactive scheduling,
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PREDICTIVE SCHEDULING BASED ON KNOWLEDGE ACQUIRED FROM FREQUENCY OF MACHINE WORK DISTURBANCES

During execution of a schedule some uncertain events may take place for example: resources may become unavailable, machine can be broken. Uncertainty should be included in the process of job scheduling. In the paper the problem to generate a workable, proactive baseline schedule under production constraints and unexpected event occurrence conditions is considered. The proactive baseline schedule protects against anticipated occurrences that may appear during the schedule execution. The machine breaking time is searched and the information is used to generate a robust schedule. In the paper the method of data acquisition basing on probability theory is proposed. The time of machine breaking is acquired from historical data of frequency of machine failure. A numerical example of building a hypothesis H:{the cumulative distribution function of the failure time is a normal distribution}, verification of the hypothesis, and predictive scheduling is presented. The normal distribution is proposed to describe failure time of machine as it gives consideration to a gradual wear process of the machine. The paper is proposition of improving simulation systems such as the Enterprise Dynamics or Taylor and scheduling systems such as Knowledge based Rescheduling System and Multi Objective Immune Scheduling Algorithm.

1. INTRODUCTION

The baseline schedule involves allocating jobs to constrained resources using some measures to evaluate the performance of a solution. From the baseline schedule some crucial information are read: peak and low capacity, requirement periods for material procurement and ability to meet deadlines of jobs execution. During execution of the schedule some uncertain events may take place such as: jobs may take more or less time than previously estimated, resources may become unavailable, machine can be broken, material and rough products suppliers may deliver overdue, jobs priority may change, some jobs may be phased out from production and some may be introduced into production, due dates may be modified. Uncertainty should be included in the process of job scheduling. The problem is to

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generate a workable, proactive baseline schedule under production constraints and unexpected event occurrence conditions. The proactive baseline schedule protects against anticipated occurrences that may appear during the schedule execution.

The problem is to generate the workable, proactive baseline schedule based on knowledge acquired from historical data about frequency of machine work disturbances. Making an assumption that data acquisition, feedback and quick response are crucial in real time, computer aided scheduling and control systems are searched. The proactive scheduling should be based on predictable disturbances using on-line data processing and uncertain disturbances using historical data processing. In the paper theory of probability is used to acquire data about mean time to failure of machine MTTF (Mean Time To Failure) from a sample of machine failure frequency.

After an unexpected event appears the schedule becomes unfeasible and rescheduling interval occurs. The more changes in the initial schedule are the less robustness the schedule has. The solution robustness of the schedule does not depend on the rescheduling interval (the schedule nervousness) as much as a stability of the schedule. After the unexpected event occurs newly generated schedule should be similar to the initial one. According to [1], the schedule is stable if the proper amount of resources can be ordered if booked in advance based on the initial schedule and a single disruption occurs during schedule execution. The solution robust schedule is basic to identify capacity requirement periods, plan external activities such as tools and materials procurement, preventive maintenance, fulfill due dates requirements, effective resource utilization.

The schedule is quality robust if a value used to evaluate the schedule, such as makespan, lateness/tardiness deviation, number of tardy jobs, resource utility, is under the given threshold. Taking into account disturbances previously mentioned, the quality robustness of the schedule means the insensitivity of the schedule to disturbance that results in affecting the value of criterion used to evaluate quality of the schedule [2].

Three problems connected with the predictive scheduling are: data acquisition, the solution robust scheduling and quality robust scheduling. The paper is proposition of improving simulation systems such as the Enterprise Dynamic and Taylor because of main shortcomes:

Jobs are scheduled according to priority rules such as: FIFO, LIFO, Random, Sort by label ascending (for example job with lowest duration time), Sort by label descending, user can predefine which location in a queue has a new job. No optimization in order to reach the optimal schedule is done taking into account criteria such as Makespan, Total Tardiness, Machine utilisation. No multi criteria optimization is performed.

The Enterprise Dynamics and Taylor are simulation systems but there is no knowledge acquisition. Systems generate predictive schedules for data introduced by a decision maker. For each machine the decision maker can define MTTF using various distributions but there is no possibility to adjust the distribution to the data and to estimate its parameters. The decision maker defines parameters.

After the disturbance has occurred new simulation with new input parameters and new constrains is performed. There are no parameters used to evaluate the solution and quality robust schedules.

In response to the shortcomes following solutions are proposed:

- Using Multi Objective Immune Scheduling Algorithm (MOIA) for optimization [6]. Output of MOIA is an order of jobs reached for given constrains and given criteria can be used as an input data to the simulation systems.
- Working out the predictive system that generates predictive schedule using methods for data acquisition appropriate for the disruption. The method used for the disruption depends on data collection and uncertainties of disturbances. In the paper the method based on probability theory for MTTF estimation is proposed.
- Working out the reactive system that generates both quality and solution robust schedules.

The paper is also a proposition of improving scheduling systems such as Knowledge based Rescheduling System [3] and Multi Objective Immune Scheduling Algorithm [6].

The paper is organized as follows: Section 2 describes the problem of predictive and reactive scheduling and data acquisition based on probability theory. Section 3 describes the methodology of building a hypothesis H:{the cumulative distribution function of the failure time X_i is $F_0(x_i)$} and verification of the hypothesis H. A numerical example of data acquisition and predictive scheduling is presented in Section 4. The paper is summarized in Section 5.

2. PROBLEM FORMULATION

A shop scheduling problem is stated as follows: N jobs, N_j, $j=1,\dots,N$, have to be executed on M machines M_i, $i=1,\dots,M$, each job consists of O operations $O_{j,k}$, $k=1,..,O$, and $O \leq M$, operations are nonpreemptive. Some operations of jobs are predefined to single machines, other can be executed on subset $SM_{i,k}$ of the machine M_i, $sm_{i,k} = \{0,1\}$. $sm_{i,k} = 1$ if the operation k can be executed on machine M_i, otherwise $sm_{i,k} = 0$. It is assumed, that during a scheduling horizon an uncertain disruptions can occur. Machine M_i can work without failures or needs to be repaired.

Repairing time y_i for each machine is predefined. Let X_i define a failure time of machine i, the probability that the machine M_i will be broken at time t, $P_i(X_i \in A_i), A_i \subset R$ and $A_i = \langle a,b \rangle$, a is a start time of the predefined schedule, b is a stop time of the predefined schedule. Given the failure time x_i, the probability that machine M_i will be down at certain time t can be calculated.

Let us assume that we have past data about the failure frequency of the machine M_i. For each machine U_i observations have been done, $X_{i,h}$, $h=1,\dots,U_i$, and $x_{i,1},\dots,x_{i,U_i}$ describe frequencies of machine M_i failures. A cumulative distribution function of the failure time X_i is not known. Basing on a histogram built from historical data of the machine M_i , $x_{i,1},\dots,x_{i,U_i}$ a hypothesis is set:

$$H_i:\{\text{the cumulative distribution function of the failure time } X_i \text{ is } F_0(x_i)\} \quad (1)$$

In order to verify the hypothesis, a test of goodness of fit between the valuated distribution of the sample and the theoretical distribution (1) is realised. Two tests of goodness of fit can be used: chi-square test and Kołogomorow's test [5]. If the distribution function $f(x_i)$of the machine's breaking time X_i and their parameters are known following probabilities can be calculated [4]:

- Probability that machine M_i will be broken at certain time t, $P_i(t)$ as follows:

$$P_i(t) = \int_0^t f\ (x_i)dx \tag{2}$$

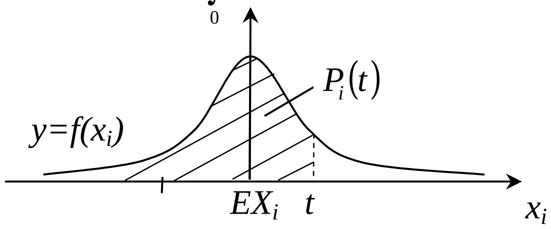

Fig. 1. The Normal distribution

Let us assume that the distribution function $f(x_i)$ of the machine's breaking time X_i is a Normal distribution and the graphical presentation of probability that machine M_i will be broken at certain time t. $P_i(t)$ is presented in Fig.1.

- Probability that the machine M_i will be broken at time from the range $A=\langle a,b\rangle$, $P_i(a \le x_i < b)$ is calculated:

$$P_i(a \le x_i < b) = \int_a^b f(x_i)dx \tag{3}$$

If we have the cumulative distribution function of the machine's repairing time X_i the probability $P_i(a \le x_i < b)$ that the machine Mi will be broken at time from the range $A=\langle a,b\rangle$ is calculated:

$$P_i(a \le x_i < b) = F_i(b) - F_i(a) \tag{4}$$

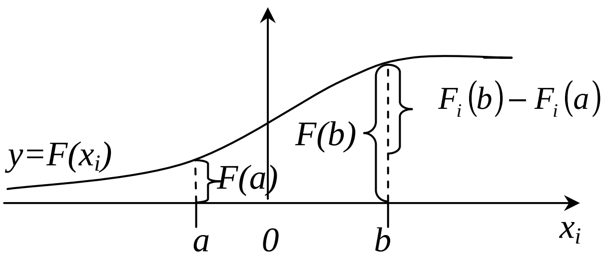

Fig. 2. The cumulative distribution function

Let us assume the cumulative distribution function of the machine's repairing time X_i is the Normal distribution then the graphical presentation of the probability $P_i(a \le x_i < b)$ that the machine M_i will be broken at time from the range $A = \langle a, b \rangle$ is presented in Fig. 2.

- Also, an average time between the machine's failures EX_i from the distribution function of the machine's repairing time X_i is read [4]:

$$EX_i = \int_{-\infty}^{\infty} x_i f(x_i) dx \tag{5}$$

A variance of the machine's repairing time X_i is an average value of the squared deviation of the machine's repairing time X_i from its average value EX_i [4]:

$$D^2 X_i = E\left(X_i^{\ 2}\right) - \left(EX_i\right)^2 \tag{6}$$

The hypothesis setting and verification are performed for W samples $Z_{i,ł,U_i}$, $ł=1,\ldots,W$ for machine M_i and each sample involves U_i observations. For the scheduling horizon $W+1 = \langle a;b \rangle$ from the set of $A_i = \{EX_{i,ł}\}$ and the set of $B_i = \{D^2 X_{i,ł}\}$ using a linear regression a future time between the machine's failures $EX_{i,W+1}$ and a future variance of the machine's failure time $D^2 X_{i,W+1}$ are read. In order to simplify the understanding of predictive scheduling problem the mathematical formulation and numerical example of the problem is presented for one sample.

2.1. PREDICTIVE SCHEDULING

The basic schedules are generated using MOIA [5]. The article is also the proposition of developing MOIA application to predictive scheduling. MOIA is used to give priority rule of jobs to minimize makespan criterion C_{max}. Predictive scheduling consists in placing an operation on the machine from the subset $SM_{i,k}$ if the operation was preliminary assigned (using MOIA) to M_i at the time window (7).

$$\left\langle a + EX_{i,S+1} - D^2 X_{i,S+1},\ a + EX_{i,S+1} + D^2 X_{i,S+1} + y_i \right\rangle \tag{7}$$

The operation is assign to the machine from the subset $SM_{i,k}$ using earliest finishing time of the operation (ETFR rule). If at least two end times of execution of the operation assigned to the machines from the subset $SM_{i,k}$ are equal, the operation is assigned to the machine for which no disturbance is forecasted.

The Flexibility Priority Rule (FPR rule) is in force if ETFR rule cannot be used because there is no solution fulfilling the production route constrains. In that case, for orders of jobs proposed by the MOIA for the operation executed at the time window of machine M_i (7) FPR is computed (8).

The operation $O_{j,k}$ is flexible if it can be executed on at list one machine from the subset $SM_{i,k}$ of the M_i and machine is idle and production routes constrains are fulfilled. The more machines the operation $O_{j,k}$ can be executed on, the higher flexibility priority value of the operation k is.

Let us assume that S_p, $p=1,\ldots,S$ states as a number of best solutions given by MOIA. S by a decision maker is predefined. A flexibility priority value $FPR_{i,p}$ for given machine M_i and given order of operation S_p is counted according to:

$$FPR_{i,p} = \frac{FM_{i,p,k}}{\max FM_{i,p,k}} \tag{8}$$

$$FM_{i,p} = \sum_{i=1}^{M} \{0,1\} \tag{9}$$

where: 1 if operation $O_{j,k}$, can be executed at the time window (7) on machine M_i from the subset $SM_{i,k}$ and machine M_i is idle and the production route constrains are fulfilled, 0 if operation $O_{j,k}$, can be executed on machine M_i from the subset $SM_{i,k}$ but machine M_i is not idle or the production route constrains is not fulfilled, $maxFMi$ – maximal value of $\{FM_{i,p}\}$.

The equation (8) is given taking into account that only one operation can be executed in the time window. Further researches must be given in the subject of predictive scheduling.

The objective of scheduling is to find an order of jobs on machines that minimizes the makespan C_{max},

$$C_{\max} = \max\{C_j \mid j = 1,\ldots,N\} \tag{10}$$

where: C_j is the completion time of job J_j.

2.2. REACTIVE SCHEDULING

It is assumed, that during the scheduling horizon uncertain disruptions can occur. The disruption has an irreversible effect on C_{max}. $st_{i,j,k}$ is a start time of operation k, of job j on machine M_i. A time the disturbance occurs Dt is the starting point form that operations are rescheduled, $st_{i,j,k} > Dt$. The operation of the job interrupted by the disturbance is finished after the disturbance is eliminated.

Two criteria are optimised in the reactive scheduling: the solution and quality robustness. The solution robustness (the instability) SR of the reactive schedule RS is measured by computing the starting times deviations between operations of the reactive schedule and operations of the previous schedule PS:

$$SR(RS) = \sum_{j=1}^{N} \left| st_{i,j,k}(PS) - st_{i,j,k}(RS) \right| \tag{11}$$

Quality robustness QR is measured as deviation between makespan of RS and makespan of PS:

$$QR(RS) = \sum_{j=1}^{N} \left| C_{\max}(PS) - C_{maz}(RS) \right| \tag{12}$$

After a new disturbance occurs $SR(RS) \Rightarrow SR(PS)$ and $QR(RS) \Rightarrow QR(PS)$ rescheduling procedure is started. The goal is to minimize the SR(RS) and QR(RS).

3. HISTOGRAM BUILDING AND VERIFICATION OF HYPOTHESIS

Procedure of histogram building is based on [5]. $x_{i,1},...,x_{i,U_i}$, $i=1,\dots,M$ is a sample of data of machine M_i failure frequency. A range of X_i in the sample $x_{i,1},...,x_{i,U_i}$ R_i equals:

$$R_i = x_{i,u_i}^{\max} - x_{i,u_i}^{\min} \tag{13}$$

where: $x_{i,u_i}^{\max}$ is the maximal value of the sample of machine M_i, $x_{i,u_i}^{\min}$ is the minimal value of the sample of machine M_i. Values from the sample are grouped into classes – equally length intervals. A number of classes is:

$$k_i = \sqrt{U_i} \tag{14}$$

where: U_i is the number of observations (samples) for machine M_i.
The length of the class is:

$$b_i \approx \frac{R_i}{k_i} \tag{15}$$

b_i value is approximated with an excess. Limit of classes is counted with accuracy to $\frac{1}{2}\alpha_i$, where α_i is the accuracy of evaluated values in the sample and reaches the maximal value from the scope $\langle 0{,}05 - 0{,}1 \rangle$ under the condition that the values $x_{i,1},...,x_{i,U_i}$ divided by α_i give integer results [5]. A bottom limit of first class equals:

$$bx_i^{\min} = x_{i,u_i}^{\min} - \frac{1}{2}\alpha_i \tag{16}$$

A number of values of the sample for machine M_i belonging to *p-th* class is a size of class $n_{i,p}$, $p=1,2,\dots,P$ and $\sum_{p=1}^{s} n_{i,p} = U_i$. A distributive series are described by: a center of the class $\overline{x_{i,p}}$ and the size of class $n_{i,p}$.

The question is: does a cumulative distribution function of the failure time X_i has the normal distribution $N_i(\mu_i, \sigma_i)$?. The normal distribution is used to describe failure time X_i of machine M_i as it gives consideration to a gradual wear process of the machine. Verification of hypothesis H_i:{the cumulative distribution function of the failure time X_i is $F_0(x_i)$ and the distribution of the failure time of machine M_i is $N_i(\mu_i, \sigma_i)$} using Kołomogorow's test is performed [5]:

$$D_{i,p} = \sup_{x_i} \left| S_p\left(g_{i,p}\right) - F_p\left(g_{i,p}\right) \right| \tag{17}$$

where: $D_{i,p}$ is an upper limit from absolute values of differences, $S_p\left(g_{i,p}\right)$ is an empirical cumulative distribution function basing on the ordered limits of classes:

$$g_{(i,1)} \leq g_{(i,2)} \leq \ldots \leq g_{(i,p)} \leq \ldots \leq g_{(i,P)} \tag{18}$$

The empirical cumulative distribution for grouped data (ordered in classes):

$$S_n\left(g_{i,p}\right) = \begin{cases} 0 \;\; if \;\; x < g_{(i,0)} \\ \dfrac{n_{i,1}}{U_i} \;\; if \;\; g_{(i,0)} \leq x < g_{(i,1)} \\ \dfrac{n_{i,1} + n_{i,2}}{U_i} \;\; if \;\; g_{(i,1)} \leq x < g_{(i,2)} \\ \vdots \\ 1 \;\; if \;\; x \geq g_{(i,P)} \end{cases} \tag{19}$$

where: $n_{i,1}, n_{i,2}, \ldots, n_{i,p}, \ldots, n_{i,P}$ are sizes of classes.

A standarised right limit of the class:

$$r_{i,p} = \left(g_{i,p} - \mu_i\right) \tag{20}$$

μ_i is the middle class limit.

Values of cumulative distribution function $F\left(g_{i,p}\right)$ for the distribution $N(0,1)$ of the failure time of machine M_i are read from table 5 [5]. $D_{i,p}^{\max}$ is the maximal value of $D_{i,p}$. If the condition (21) is fulfilled it means that the results of the sample with the significance level α_i does not deny the hypothesis H_i :{ the failure time of machine M_i has the distribution of $N_i(\mu_i, \sigma_i)$}.

$$\sqrt{U_i} D_{i,p}^{\max} < \lambda_{(1-\alpha_i)} \tag{21}$$

4. NUMERICAL EXAMPLE

The machine M_5 failure frequency, is described by the sample, U_5=30. Failure frequency of M_5 is as follows: 5,4; 5,2; 3,8; 4; 5,8; 6,4; 3,1; 6,2; 3,5; 3,9; 4; 4,1; 5,8; 4,3; 4,4; 5,2; 4,5; 4,7; 4,7; 4,8; 4,4; 5,2; 5; 5,9; 5; 5,1; 5,2; 5,3; 5,6; 5,5. The average time between the machine's failures EX_5 is searched. The distribution function of the 5th machine's repairing time X_5 need to be found.
The number of classes equals:

$$k_5 = \sqrt{30} = \lceil 5{,}47 \rceil = 6 \tag{22}$$

The maximal and minimal values of the sample equals:

$$x_{5,6}^{\max} = 6{,}4,\ \ x_{5,7}^{\min} = 3{,}1 \tag{23}$$

The range of X_5 in a sample $x_{5,1}\ldots x_{5,30}$ equals:

$$R_5 = x_{5,6}^{\max} - x_{5,7}^{\min} = 3{,}3 \tag{24}$$

The length of the class is:

$$b_5 \approx \frac{R_5}{k_5} = 3{,}3/6 = 0{,}55 \approx 0{,}6 \tag{25}$$

The accuracy of values in the sample is $\alpha_5 = 0{,}1$, because values from the sample $x_{5,1}\ldots x_{5,30}$ divided by $\alpha_5 = 0{,}1$ give integer results and no bigger value α_i exists. The bottom limit of the first class equals:

$$bx_5^{\min} = x_{5,7}^{\min} - \frac{1}{2}\alpha = 3{,}1 - 0{,}05 = 3{,}05 \tag{26}$$

The distributive series are presented in Table 1. The histogram for the frequency of machine M_5 failure is presented in Fig. 3.

Table 1. The distributive series of the sample

No of class	Class			The distributive series	
				$\overline{x_{i,p}}$	$n_{i,p}$
1	3,05	-	3,65	3,35	3
2	3,65	-	4,25	3,95	5
3	4,25	-	4,85	4,55	7
4	4,85	-	5,45	5,15	8
5	5,45	-	6,05	5,75	5
6	6,05	-	6,65	6,35	2

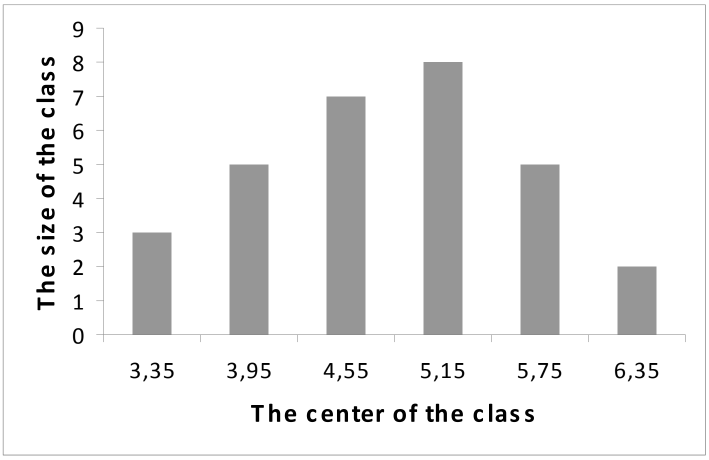

Fig. 3. The histogram of the frequency of machine M_5 failure

The question is what cumulative distribution function describes the failure time of machine M_5? Taking into account sizes of classes $n_{5,p}$ one can assume that the distribution of sizes of classes is approximated to symmetrical distribution. The distribution has one mode with maximal value in one of the middle classes. The hypothesis is H_5:{the distribution of the failure time of machine M_5 is $N_5(\mu_5, \sigma_5)$}.

The question is that: what values of parameters μ_5, σ_5 are? Let as assume $\mu_5 = 4{,}85$, in the interval $\langle 3{,}05,\ 6{,}65 \rangle$. If the length of the interval equals to 3.6, and there are $30-(3+2)=25$ solutions, 83% solutions will be contained in. Probability of reaching a value from the interval $\langle \mu - 1{,}96\sigma,\ \mu - 1{,}96\sigma \rangle$ equals to 95% in the distribution $N(\mu, \sigma)$. The average value with the number of samples U_5=30, slightly differs from 28,5. As the length of the interval in the distribution $N(\mu, \sigma)$ equals to $3{,}92\sigma$ and the length of the interval $\langle g_{i,p},\ g_{i,p+1} \rangle$ of the researched distribution equals to 0,6, $\sigma_i = 0{,}15$. The hypothesis

$$\text{H}_5\text{:\{ the failure time of machine } M_5 \text{ has the distribution of } N_5(4{,}85{,}0{,}15)\text{\}} \tag{27}$$

has been assumed. Using the Kołmogorow's test the hypothesis is verified. The limits of the classes $g_{i,p}$, the sizes of the classes $n_{i,p}$, the values of $S_p(g_{i,p}), r_{i,p}, F_p(g_{i,p})$ and $|S_p(g_{i,p}) - F_p(g_{i,p})|$ are presented in Table 2.

$$\sqrt{U_5}\, d_{5,6} = \sqrt{30} \cdot 0{,}04843 = 0{,}26528 \tag{28}$$

The quintile of row $1-\alpha_5 = 0{,}9$ limited Kołomogorow's distribution is $\lambda_{(0,9)} = 1{,}224$.
Because $0{,}26528 < 1{,}224$, the results of the sample with the significance level $\alpha_5 = 0{,}1$ do not deny the hypothesis H$_5$.

Table 2. Data used for hypothesis H_5 verification

P	$g_{i,p}$	$n_{i,p}$	$S_p(g_{i,p})$	$r_{i,p}$	$F_p(g_{i,p})$	$\lvert S_p(g_{i,p})-F_p(g_{i,p})\rvert$
1	- 3,05	0	0	-1,8	0,0359	0,0359
2	3,05 - 3,65	3	0,1	-1,2	0,1151	0,0151
3	3,65 - 4,25	5	0,267	-0,6	0,2743	0,00763
4	4,25 - 4,85	7	0,5	0	0,5	0
5	4,85 - 5,45	8	0,767	0,6	0,7257	0,040967
6	5,45 - 6,05	5	0,93	1,2	0,8849	0,04843= $D_{5,6}^{\max}$
7	6,05 - 6,65	2	1	1,8	0,9641	0,0359

In order to generate the predictive schedule the information of the average time between the machine M_5 failure is needed $MTTF_5 = EX_5$. $MTTF_5$ from the distribution function of the machine's repairing time $N_5(4{,}85{,}0{,}15)$ is read:

$$f\,(x_5)=\frac{1}{\sigma_5\sqrt{2\pi}}e^{-(x_5-\mu_5)^2/\,2\sigma_5^{\,2}} \tag{29}$$

where: $\mu_5 = 4{,}85, \sigma_5 = 0{,}15$

Density of distribution $N_5(4{,}85{,}0{,}15)$ for the frequency of machine M_5 failure is presented in Fig. 4. In the distribution $N(\mu,\sigma)$ [4]:

$$EX=\mu \tag{30}$$

$$D^2X=\sigma^2 \tag{31}$$

The average time between the machine's M_5 failures EX_5

$$EX_5=\mu_5=4.85 \tag{32}$$

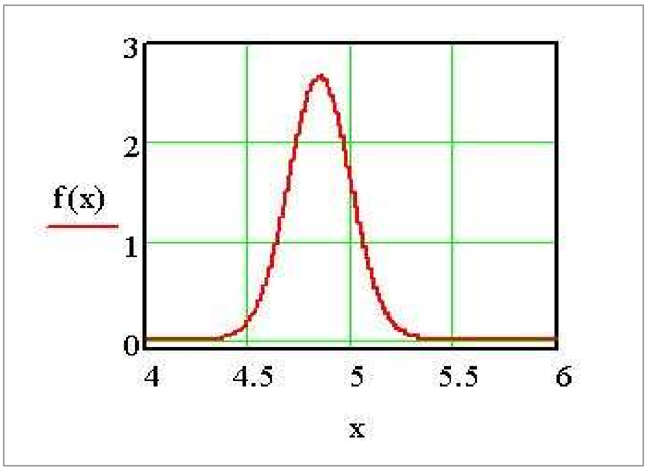

Fig . 4. The density of distribution $N_5(4{,}85{,}0{,}15)$ for the frequency of machine M_5 failure

A variance of the machine's M_5 repairing time X_5 is:

$$D^2 X_5 = 0.0225 \tag{33}$$

Let us assume there are $W=10$ samples of machine M_5 frequency failure. According to data from first sample $ł=1$, the average time between the machine M_5 failures equals $EX_{5,1} = 4.85$, and the variance of the machine M_5 failures time X_5 equals $D^2 X_{5,1} = 0{,}0225$. Let us assume the sets of $A_5 = \{4.85, 5, 4.75, 4.67, 4.8, 5, 4.8, 5.2, 5.1, 4.9\}$ and $B_5 = \{0.0225, 0.01, 0.0225, 0.0256, 0.0225, 0.01, 0.0225, 0.0256, 0.0196\}$. The average time between the machine M_5 failures $EX_{5,W+1}$ for the scheduling horizon W+1, and the variance of the machine M_5 failures time $D^2 X_{5,W+1}$ for the scheduling horizon $W+1$ are read form linear regressions (Fig. 5 and Fig. 6).

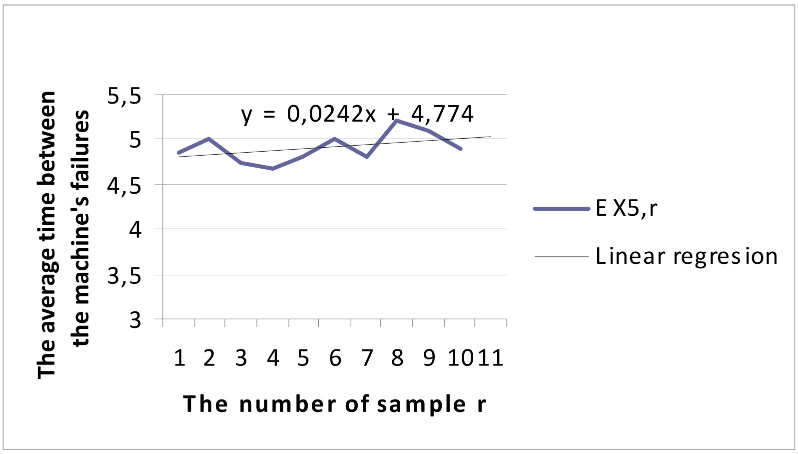

Fig. 5. The average time between the machine's M_5 failures

Let us assume that the scheduling horizon is $\langle 0;16 \rangle$ and repairing time y_5 for machine M_5 is $y_5=3.5$. Because of $EX_{5,11} = 5.0402$ and $D^2 X_{5,W+1} = 0.0222$, the time window when machine M_5 is probable to be broken equals $\langle 5.018, 8.5624 \rangle$. The predictive scheduling problem consists in assigning three jobs described by Matrix of Production Routes MP (34), Matrix of Processing Time MT (35) into five machines, with information that machine M_5 can be not available at the time window $\langle 5.018, 8.5624 \rangle$. The subset S$M_{5,k}$ (36) of the M_5 describes machines on which operations can be executed in case of M_5 failure.

$$MP = \begin{bmatrix} 1,0,3,2,4 \\ 4,3,1,2,5 \\ 1,5,3,4,2 \end{bmatrix},\ MT = \begin{bmatrix} 2,0,3,2,1 \\ 2,2,3,2,3 \\ 2,5,1,2,2 \end{bmatrix},\ SM_{5,k} = \begin{bmatrix} 0,1,1,0,0 \\ 1,0,1,1,0 \\ 0,0,1,1,0 \end{bmatrix} \tag{34,35,36}$$

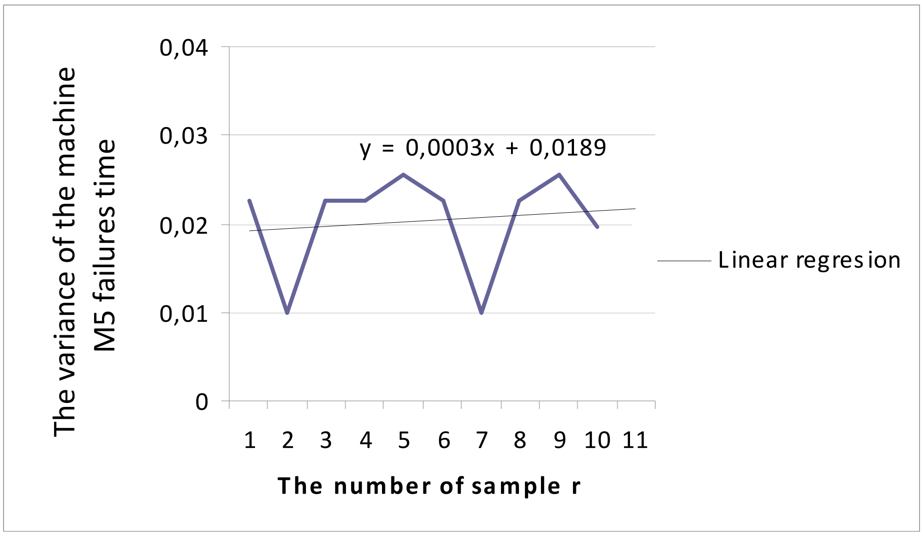

Fig. 6. The variance of the machine M_5 failures time $D^2 X_{5,W+1}$

Let us consider the order of jobs [J_2, J_1, J_3] proposed by the MOIA. C_{max} of the basic schedule equals 14. The predictive schedule is generated using ETFR rule for the operation $O_{3,5}$ that was preliminary assigned to M_5 at time window $\langle 5.018, 8.5624 \rangle$. The operation $O_{3,5}$ can also be executed on machine M_3 (A) or M_4 (B) as the condition that the machines are idle is fulfilled. According to the production route constrains the 3rd operation of J_3 can not be executed on M_4. To generate predictive schedule ETFR rule is used for the 3rd operation taking into account two possibilities M_3 and M_5. As end times of execution of the operation are equally the operation is assigned to the machine for which no disturbance is forecasted.

The predictive schedule is presented in Fig. 7.

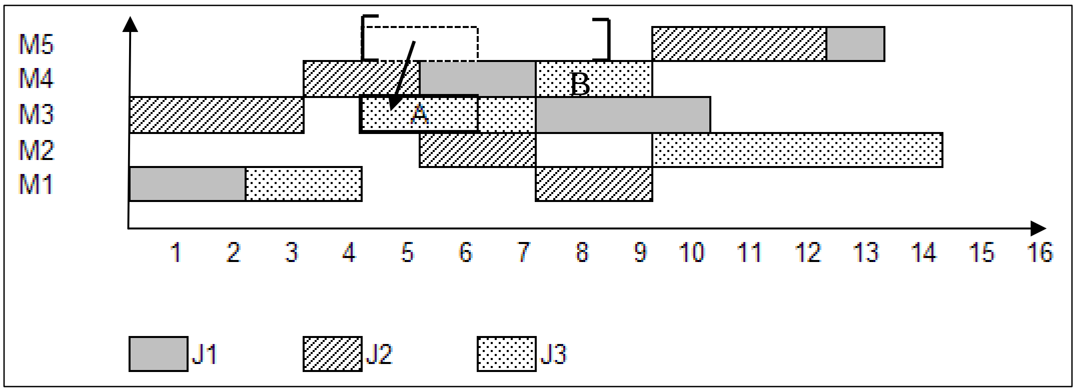

Fig. 7. The Gantt chart for the predictive scheduling problem

The rule FPR is in force if ETFR rule cannot be used because no machine is idle or because of the production route constrains. Further researchers should be given to minimize C_{max} criterion.

5. SUMMARY

In the paper the problem to generate a workable, proactive baseline schedule under production constraints and unexpected event occurrence conditions is considered. The proactive baseline schedule protects against anticipated occurrences that may appear during the schedule execution. The time of machine breaking is searched and the information is used to generate the robust schedule. In the paper the method of data acquisition basing on probability theory is proposed. A numerical example of building a hypothesis H:{the cumulative distribution function of the failure time is the normal distribution}, verification of the hypothesis, and predictive scheduling is presented.

The paper is proposition of improving simulation systems such as the Enterprise Dynamics or Taylor and scheduling systems such as Knowledge based Rescheduling System or Multi Objective Immune Scheduling Algorithm.

In the future work, a model of machine failures will be considered in the successive failure-free times with Weibull distributions and followed by exponentially distributed times of repairs. The goal is to work out the predictive scheduling system reflecting a production system and nature of disturbances able to estimate unknown parameters of the system.

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