

Kirchhoff's laws from Tellegen's Theorem

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Summary: It is well known that Kirchhoff's voltage (current) law and Tellegen's theorem ($\mathbf{v}^T \mathbf{i} = 0$) imply Kirchhoff's current (voltage) law where \mathbf{v} and \mathbf{i} refer to the same graph. Careful analysis shows that these statements are not applicable to practical networks unless \mathbf{v} and \mathbf{i} refer to different networks having the same graph. They are rewritten and proved to take care of this.

Key words:

Kirchhoff's laws, Tellegen's theorem, circuit analysis, graphs and electrical networks, network theorems

1. INTRODUCTION

Let N_a and N_b be two networks with same incidence matrix. There is a one one correspondence between their vertices and elements. \mathbf{v}_a (\mathbf{v}_b) be the vector of voltages of all elements of N_a (N_b). \mathbf{i}_a and \mathbf{i}_b be their current vectors. Let voltage and current directions be same in all elements. Then Tellegen's theorem [1,2,3] states that

$$\mathbf{v}_a^T \mathbf{i}_a = 0 \tag{1}$$

$$\mathbf{v}_b^T \mathbf{i}_b = 0 \tag{2}$$

$$\mathbf{v}_a^T \mathbf{i}_b = 0 \tag{3}$$

$$\mathbf{v}_b^T \mathbf{i}_a = 0 \tag{4}$$

They can be combined into a single equation $\mathbf{v}^T \mathbf{i} = 0$ where \mathbf{v} and \mathbf{i} refer to the same directed graph. But they may or may not belong to same network [1]. Penfield et al [2] studied this theorem extensively and suggested many applications in electrical networks. It is well known that any two of Kirchhoff's laws and Tellegen's theorem imply the other. More precisely the following statements are widely accepted [1,2,4]:

- i. Tellegen's theorem is a consequence of Kirchhoff's laws.
- ii. If, for **all** \mathbf{v} satisfying KVL, $\mathbf{v}^T \mathbf{i} = 0$, then \mathbf{i} satisfies KCL.
- iii. If, for **all** \mathbf{i} satisfying KCL, $\mathbf{v}^T \mathbf{i} = 0$, then \mathbf{v} satisfies KVL.

Recently Willems [4] studied these statements for active and reactive components of currents and voltages of power networks. It is the purpose of this paper to clarify an important aspect of statements (ii) and (iii). \mathbf{v} and \mathbf{i} in these statements refer to the same graph. This is enough for statement (i). But this is not enough for statements (ii) and (iii). Following the experience of Tellegen's theorem one should not take \mathbf{v} and \mathbf{i} from same as well as different networks. Similarly one should not think that it is arbitrary or immaterial. **They must be taken from different networks** i.e. only equations (3) and (4) must be used. Equations (1) and (2) should not be used. Otherwise the conclusions of the statements will not be true except for pathological cases. This aspect is discussed in Section II. These statements and their proofs are re-phrased to reflect this change. This is done in Section III.

2. ANALYSIS OF EXISTING RESULTS

First let us derive KCL from KVL and Tellegen's equation. Theorem 1 given below deals with this.

Theorem 1 [1]: Let \mathbf{v} (\mathbf{i}) be the vector of voltages (currents) of all edges of a directed graph G . Let $\mathbf{v}^T \mathbf{i} = 0$ for **all** \mathbf{v} satisfying KVL. Then \mathbf{i} satisfies KCL.

The statement of the theorem is silent on the networks from which \mathbf{v} and \mathbf{i} are taken. The proof refers to the graph only. It does not refer to any network and so one tends to think that it is immaterial. But this is not true from a practical point of view. It is important to understand that \mathbf{v} and \mathbf{i} must belong to different networks to make this theorem meaningful. The following example highlights this and other problems that arise if \mathbf{v} and \mathbf{i} belong to the same network.

Example 1: Consider the simple network shown in Fig. 1. We will assume that KVL and Tellegen's equation hold in this network and examine whether KCL is satisfied or not under different conditions.

- i. $v_1 = 5, i_1 = -4, v_2 = 2, i_2 = 7, v_3 = 3$ and $i_3 = 2$ satisfy KVL ($v_1 = v_2 + v_3$), component equations ($v_1 = 5, v_2 = \frac{2}{7} i_2, v_3 = \frac{3}{2} i_3$) and the Tellegen's equation ($v_1 i_1 + v_2 i_2 + v_3 i_3 = 0$). It is clear that the currents do not satisfy KCL. But Th. 1 is not violated because we considered only one set of values of voltages satisfying KVL whereas the theorem requires us to consider **all** \mathbf{v} . Now consider a slightly different situation. Let $i_1 = -4, i_2 = 7$ and $i_3 = 2$. KCL is violated. Tellegen's equation gives $-4v_1 + 7v_2 + 2v_3 = 0$. Using KVL we get $3v_2 - 2v_3 = 0$. This is an additional constraint on voltages. It does not allow us to choose **all** \mathbf{v} satisfying KVL. Hence Th. 1 is not applicable. On the other hand take $i_1 = -2, i_2 = 2$ and $i_3 = 2$. These currents satisfy KCL. Tellegen's equation gives $-2v_1 + 2v_2 + 2v_3 = 0$ which is same as KVL. Therefore there is no additional constraint on voltages and Th. 1 is applicable.
- ii. Take $v_1 = E$ where E is any arbitrary value. Then $v_3 = E - \frac{2}{7} i_2 = \frac{3}{2} i_3$ from KVL and component equations. This gives

$$i_3 = \frac{2}{21} (7E - 2i_2) \tag{5}$$

From $\mathbf{v}^T \mathbf{i} = 0, E i_1 + \frac{2}{7} i_2^2 + \frac{3}{2} i_3^2 = 0$. Substitute for i_3 from equation (5) and simplify. We get

$$Ei_1 + \frac{2}{7}i_2^2 + \frac{2}{49 \times 3}(49E^2 + 4i_2^2 - 28Ei_2) = 0 \quad (6)$$

Equations (5) and (6) do not imply the KCLs $i_2 = i_3$ and $i_1 + i_2 = 0$. As E is varied, v_1 , v_2 and v_3 can take on different values satisfying KVL. In spite of that Th. 1 is not violated because the voltages have to satisfy component equations also in addition to KVL. $v_1 = 5$, $i_1 = -4$, $v_2 = 2$, $i_2 = 4$, $v_3 = 3$ and $i_3 = 4$ satisfy KVL and $\mathbf{v}^T \mathbf{i} = 0$ but not component equations, i.e. component equations do not allow voltages to take on all possible values satisfying KVL. But Th. 1 does not allow us to constrain voltages by component equations. Hence the theorem is not useful for practical circuits if \mathbf{v} and \mathbf{i} are taken from the same network intentionally or unintentionally. I.e., the theorem cannot be silent on this.

iii. Let all components in the network of Fig. 1 be ideal voltage sources. Let $v_1 = E_1$, $v_2 = E_2$, $v_3 = E_3$ where E_1 , E_2 and E_3 are arbitrary values. Then using KVL $\mathbf{v}^T \mathbf{i} = 0$ gives $E_2(i_1 + i_2) + E_3(i_1 + i_3) = 0$. Since E_2 and E_3 are arbitrary, $i_1 + i_2 = 0$ and $i_1 + i_3 = 0$. I.e., KCLs are true. In this case component equations do not constrain voltages and hence Th. 1 is true. Thus Th.1 is not true except for pathological networks if \mathbf{v} and \mathbf{i} are from the same network. Note that there is no way to compute these currents even though they satisfy KCL.

iv. Proof of Th.1 and later Th.3 select node voltages arbitrarily to prove KCL. Let us now look at this aspect. Choose node III as reference node. Let $v_{nI} = k$ and $v_{nII} = 0$ where k is any real number. Then $v_1 = k$, $v_2 = k$ and $v_3 = 0$. $i_2 = \frac{7}{2}k$ and $i_3 = 0$ from component equations. $\mathbf{v}^T \mathbf{i} = 0$ gives $i_1 = -\frac{7}{2}k$. Thus KCL is satisfied at node I for all k and the theorem is true. But this is only for specific set of values of currents whereas we want KCL for all values. Thus if we arbitrarily select node voltages, we get KCL. Further, note that KCL is not satisfied at node II.

Next consider the problem of deriving KVL from KCL and Tellegen's equation. Theorem 2 given below deals with this.

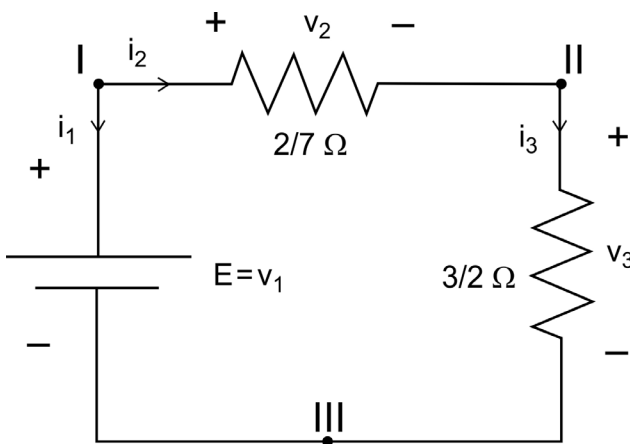


Fig. 1. An example network.

Theorem 2 [1]: Let $\mathbf{v}(\mathbf{i})$ be the vector of voltages (currents) of all edges of a directed graph G . Let $\mathbf{v}^T \mathbf{i} = 0$ for **all** \mathbf{i} satisfying KCL. Then \mathbf{v} satisfies KVL.

As in Th. 1 the statement of this theorem is also silent on the networks from which \mathbf{v} and \mathbf{i} are taken. The proof refers to the graph only. Consequently this theorem also has problems similar to those cited above. Here also \mathbf{v} and \mathbf{i} must belong to different networks to make this theorem meaningful. The following example illustrates the problems that arise if \mathbf{v} and \mathbf{i} belong to the same network.

Example 2: Consider the network shown in Fig. 2. We will assume that KCL and Tellegen's equation hold for this network and examine whether KVL is satisfied or not under different conditions.

- i. $J = 4$, $v_1 = \frac{5}{6}$, $v_2 = 1$, $i_2 = 2$, $v_3 = \frac{2}{3}$, $i_3 = 2$ satisfy the KCL equation ($i_1 + i_2 + i_3 = 0$), component equations ($J = 4$, $i_2 = 2v_2$ and $i_3 = 3v_3$) and the Tellegen's equation ($v_1 i_1 + v_2 i_2 + v_3 i_3 = 0$). But the voltages do not satisfy KVL. Th. 2 is not violated because we have not considered all possible values of currents satisfying KCL. Next consider $v_1 = \frac{5}{6}$, $v_2 = 1$, $v_3 = \frac{2}{3}$. Voltages do not satisfy KVL. Tellegen's equation gives $\frac{5}{6}i_1 + i_2 + \frac{2}{3}i_3 = 0$. This is an additional constraint on currents. This does not allow us to take **all** \mathbf{i} satisfying KCL. Therefore Th. 2 is not applicable. On the other hand take $v_1 = 1$, $v_2 = 1$, $v_3 = 1$. These voltages satisfy KVL and the Tellegen's equation is same as KCL. Therefore there is no additional constraint on currents and Th. 2 is applicable.
- ii. Take $i_1 = -J$ where J is any arbitrary value. Using KCL $i_2 = J - i_1$ and $i_3 = 3v_3$ we get

$$v_3 = \frac{1}{3}(J - 2v_2) \quad (7)$$

$$\mathbf{v}^T \mathbf{i} = -Jv_1 + 2v_2 \frac{J}{2} + 3v_3 \frac{J}{3} \quad \text{gives}$$

$$-Jv_1 + 2v_2 \frac{J}{2} + \frac{1}{3}(J - 2v_2)^2 = 0 \quad (8)$$

Equations (7) and (8) do not imply the KVL equations $v_1 = v_2 = v_3$. Since J is arbitrary the currents can take on many values satisfying KCL. But Th. 2 is not violated because currents cannot take on all possible values because of component equations. Thus Th. 2 does not allow the component equations to constrain currents. Hence this theorem cannot be applied to practical networks if \mathbf{v} and \mathbf{i} are taken from the same network intentionally or unintentionally. I.e., the theorem cannot be silent on this.

- iii. Let all components in the network of Fig. 2 be ideal current sources. Let $i_1 = J_1$, $i_2 = J_2$, $i_3 = J_3$ where J_1 , J_2 and J_3 are arbitrary values. Then using KCL $\mathbf{v}^T \mathbf{i} = 0$ gives $J_2(-v_1 + v_2) + J_3(-v_1 + v_3) = 0$. Since J_2 and J_3 are arbitrary, $v_1 = v_2$ and $v_1 = v_3$. I.e., KVLs are true. In this case component equations do not constrain currents and hence Th. 2 is true. Thus Th. 2 is true if all elements are ideal cur-

rent sources if v and i are from the same network. But these networks have no practical value. Note that there is no way to compute these voltages although they satisfy KVL.

- iv. Proof of Th. 2 and later Th. 5 select currents arbitrarily to prove KVL. Let us now look at this aspect. Choose $i_1 = -k$ and $i_3 = 0$ where k is any real number. Then $i_2 = k$ from KCL. $v_2 = k/2$ and $v_3 = 0$ from component equations. $\mathbf{v}^T \mathbf{i} = 0$ gives $v_1 = k/2$. Therefore v_1 and v_2 satisfy KVL for all k . I.e., Th. 2 is satisfied for v_1 and v_2 . But this is only for specific set of values whereas we want KVL for all values. Thus if we arbitrarily choose currents we get KVL. But it is limited to specific values only. Further, note that KVL is not satisfied for v_2 and v_3 .

3. MODIFIED THEOREMS

In this section Theorems 1 and 2 are stated and proved more clearly so that they are realistic. Let G be the directed graph of the given network N . Let G_a and G_b be two identical copies of G . They may be derived from two identical copies of N . We will rewrite these theorems and their proofs using G_a and G_b . \mathbf{v}_a (\mathbf{v}_b) are the vectors of voltages of all edges of G_a (G_b). \mathbf{i}_a and \mathbf{i}_b are their current vectors. Since each network has one graph only, it is clear that G_a and G_b refer to two physically different networks.

Theorem 3: Let $\mathbf{v}^T \mathbf{i}_a = 0$ for all \mathbf{v}_b satisfying KVL. Then \mathbf{i}_a satisfies KCL.

Proof: Consider the node transformation $\mathbf{v}_b = \mathbf{A}^T \mathbf{v}_{nb}$. It follows from this that taking all \mathbf{v}_b satisfying KVL in G_b , implies taking all values of \mathbf{v}_{nb} without any constraints. From Tellegen's equation we have

$$\mathbf{v}^T \mathbf{i}_a = \mathbf{v}^T \mathbf{i}_{nb} \mathbf{A} \mathbf{i}_a = 0 \quad (9)$$

Since \mathbf{v}_{nb} and \mathbf{i}_a belong to different graphs (networks), choosing \mathbf{v}_{nb} has no effect on \mathbf{i}_a .

Taking $v_{nb1} = 1, v_{nb2} = v_{nb3} = \dots = 0$ we get KCL at node I for all \mathbf{i}_a from equation (9). Similarly we get KCLs at other nodes also. This proves the theorem.

Corollary 4: Let KVL and Tellegen's equations be universally true (i.e., KVL is true for all networks and Tellegen's equations hold for all pairs of networks hav-

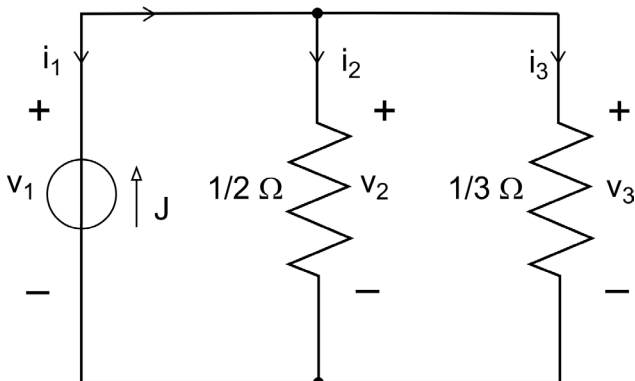


Fig. 2. An example network.

ing same graph) then KCL is also universally true.

Proof: Let N be any network to which we want to prove KCL. Let G be its graph. Let G_a and G_b be two copies of G . Applying Th. 3 KCL is true for N . But N is arbitrary. Hence KCL is true for all networks. b

Example 3: Let us now look at the observations of Example 1 using this knowledge. N_a is the network given in Fig.1. Take another copy of this network and call it N_b .

- Let $i_{a1} = -4, i_{a2} = 7$ and $i_{a3} = 2$. KCL is violated in N_a . Tellegen's equation gives $-4v_{b1} + 7v_{b2} + 2v_{b3} = 0$. Using KVL we get $3v_{b2} - 2v_{b3} = 0$. This is an additional constraint on voltages. It does not allow us to choose all \mathbf{v} satisfying KVL in N_b . Hence Th. 3 is not applicable. If we take $i_{a1} = -4, i_{a2} = 2$ and $i_{a3} = 2$ KCL is satisfied. Tellegen's equation gives $-4v_{b1} + 2v_{b2} + 2v_{b3} = 0$. This satisfies KVL and hence this is not an additional constraint. Hence Th. 3 is applicable.
- Since choosing voltages in N_b (G_b) does not affect currents in N_a (G_a) the problem in observation (ii) of Example 1 does not arise now.
- Even if all the elements are ideal voltage sources, Th. 3 is satisfied. This can be easily verified. I.e., Th.1 can afford to be silent on the networks of \mathbf{v} and \mathbf{i} for such networks.
- Take node III as the reference node in N_b . Take $v_{nbI} = 1$ and $v_{nbII} = 0$. Then $v_{b1} = v_{b2} = 1$ and $v_{b3} = 0$. $\mathbf{v}^T \mathbf{i}_a = (1)i_{a1} + (1)i_{a2} + (0)i_{a3} = 0$. This gives KCL at node I of N_a for all possible currents. This does not constrain KCL at node II. That has to be determined separately. We get KCL at node II by taking $v_{nbI} = 0$ and $v_{nbII} = 1$ whereas KCL is not satisfied at node II in observation (iv) of Example 1. Thus the problems of Example 1 do not exist anymore.

Theorem 5: Let $\mathbf{v}^T \mathbf{i}_b = 0$ for all \mathbf{i}_b satisfying KCL. Then \mathbf{v}_a satisfies KVL.

Proof: Consider the mesh transformation $\mathbf{i}_b = \mathbf{B}^T \mathbf{i}_{mb}$ where \mathbf{i}_{mb} is the vector of all independent mesh currents in G_b (In general \mathbf{i}_{mb} is the vector of all link currents). It follows from this equation that choosing all \mathbf{i}_b in G_b satisfying KCL implies choosing all values of \mathbf{i}_{mb} without any constraints. From Tellegen's equation we have

$$\mathbf{v}^T \mathbf{i}_b = (\mathbf{B} \mathbf{v}_a)^T \mathbf{i}_{mb} = 0 \quad (10)$$

Since \mathbf{i}_{mb} and \mathbf{v}_a belong to different graphs (networks), choosing \mathbf{i}_{mb} has no effect on \mathbf{v}_a . Taking $i_{mb1} = 1, i_{mb2} = i_{mb3} = \dots = 0$ we get KVL for the first loop from equation (10) for all \mathbf{v}_a . Similarly we get KVLs for other loops also. This proves the theorem.

Corollary 6: Let KCL and Tellegen's equations be universally true (i.e., KCL is true for all networks and Tellegen's equations hold for all pairs of networks having same graph) then KVL is also universally true.

Proof: Let N be any network to which we want to prove KVL. Let G be its graph. Let G_a and G_b be two copies of G . Applying Th. 5 KVL is true for N . But N is arbitrary. Hence KVL is true for all networks.

Example 4: Let us now look at the observations of Example 2 using this knowledge. N_a is the network given in Fig. 2. Take another copy of this network and call it N_b .

- i. Consider $v_{a1} = 5/6$, $v_{a2} = 1$, $v_{a3} = 2/3$. Voltages do not satisfy KVL in N_a . Tellegen's equation gives $5/6 i_{b1} + i_{b2} + 2/3 i_{b3} = 0$. This is an additional constraint on currents. This does not allow us to take **all** i_b satisfying KCL in N_b . Therefore Th. 5 is not applicable. Now take $v_{a1} = 1$, $v_{a2} = 1$, $v_{a3} = 1$. Voltages satisfy KVL in N_a . Tellegen's equation is same as KCL. Thus there is no additional constraint on currents. Hence Th. 5 is applicable.
- ii. Since choosing currents in N_b does not affect voltages in N_a the problem in observation (ii) of Example 2 does not arise now.
- iii. Even if all the elements are ideal current sources, Th. 5 is satisfied. This can be easily verified. I.e., Th. 2 can afford to be silent on the networks of v and i for such networks.
- iv. Choose $i_{b1} = 1$ and $i_{b3} = 0$. Then $i_{b2} = -1$ from KCL. $v^T_a i_b =$ gives $v_{a1} = v_{a2}$. I.e., KVL holds for loop 1 in N_a . We get KVL $v_{a2} = v_{a3}$ by taking $i_{b1} = 0$ and $i_{b3} = 1$ whereas KVL is not satisfied for this loop in observation (iv) of Example 2.

Thus the problems of Example 2 do not exist anymore.

Remarks:

1. It is clear from the proofs of Theorems 3 and 5 that (i) the networks to which v and i belong cannot be arbitrarily decided and (ii) v and i don't have to be chosen from the same network in addition to choosing them from different networks.
2. Tellegen's theorem refers to one graph only whereas Ths. 3 and 5 refer to two graphs. It is interesting to note that Tellegen's theorem can also be stated using two different graphs. We can state Tellegen's equation as $v^T i = 0$ where v and i refer to two separate but identical graphs. Equations (3) and (4) follow from this. When N_b is a copy of N_a both of them have same voltages and currents. So equations (3) and (4) give equations (1) and (2). Thus all four Tellegen's equations can be put as a single equation using two graphs also. Further this allows us to put all three statements mentioned in the Introduction in a common frame work by choosing v and i from different graphs always.

4. CONCLUSIONS

It is well known that KVL (KCL) together with Tellegen's equation implies KCL (KVL). It is shown here that voltages and currents must be taken from different networks to achieve this. More precisely they need two separate but identical graphs. We can do this even for Tellegen's theorem.

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DISCUSSION

(Both the Author and the Reviewer authorized publication of their discussion)

Reviewer's comments 1:

There are no novel ideas in the paper. Tellegen's theorem depicts only the interconnection properties of the circuit. It does not contain a clear physical meaning. Components of different vectors of voltages and currents those are sufficient to the Tellegen's theorem but not always sufficient to physical equations.

Consider a directed graph G and a vector of voltages $v = [V_1, V_2, \dots, V_n]$, which is sufficient to the KVL for G . The equation of $X_1 * V_1 + X_2 * V_2 + \dots + X_n * V_n = 0$ has innumerable solutions. If one of these solutions is the vector $x = [X_1, X_2, \dots, X_n]$ and if it would be implemented to Tellegen's theorem, then it would be adequate to the theorem, but it would be adequate to the KCL only for special case. Willems noted this feature for the case of active currents in the mentioned paper. Hence Tellegen's theorem is correct and does not need any remarks.

Author's answers:

1. Tellegen's theorem does not contain a clear physical meaning and it is only about interconnection. I don't agree with this view. Tellegen's theorem emphasizes interconnections. But it uses voltages and currents which have to come from somewhere. Wherefrom will they come if there is no associated

network. There is no Tellegen's equation if there are no voltages and currents. Interconnection information is contained in the graph of the network. A directed graph G has edges and vertices. It has no voltages and currents unless the graph is derived from some physical network. So we cannot ignore the underlying network completely. Tellegen's theorem is not about graphs. Tellegen's theorem is about networks. This itself shows that interconnection (graph) alone is not enough. Emphasizing interconnection is merely to highlight its generality. I.e, it applies to any network. This does not mean that there is no network at all. We cannot delink the theorem from networks. Emphasizing interconnection is only to say that any set of quantities satisfying KVL (call them voltages) and any set of quantities satisfying KCL (call them currents) satisfy Tellegen's equation. We may be able to state some Tellegen's equations using fictitious voltages and fictitious currents. But this does not mean that they are always fictitious quantities only. They have to be associated with some network if Tellegen's theorem has to be meaningful. Thus Tellegen's theorem emphasizes interconnection. But it is not about interconnection only. I.e., we cannot keep interconnection only in our minds when we talk of Tellegen's theorem for real life situations.

2. "Consider a directed graph G and a vector of voltages $\mathbf{v} = [V_1, V_2, \dots, V_n]$, which is sufficient to the KVL for G .
The equation of $X_1 * V_1 + X_2 * V_2 + \dots + X_n * V_n = 0$ has innumerable solutions. If one of these solutions is the vector $\mathbf{x} = [X_1, X_2, \dots, X_n]$ and if it would be implemented to Tellegen's theorem, then it would be adequate to the theorem, but it would be adequate to the KCL only for special case. Willems noted this feature for the case of active currents in the mentioned paper."
My arguments are not inconsistent with what Willems said in his paper. As the reviewer pointed out, Willems noted the feature of active currents of power networks and made some interesting observations in his paper. But that is a physical network. It is not just interconnections only. Thus when it comes to reality we cannot ignore networks and harp only graphs.
3. "Hence Tellegen's theorem is correct and does not need any remarks."

First let me clarify that my paper does not say that Tellegen's theorem is wrong. My paper is not on Tellegen's theorem at all. Nor it is about remarks on Tellegen's theorem. It is about KVL (KCL) as a consequence of Tellegen's theorem and KCL (KVL). So I accept Tellegen's theorem and I have no remarks on Tellegen's theorem itself. I did not even say that the existing statements on Kirchhoff's laws from Tellegen's theorem are inherently wrong. I only tried to sharpen the statements. My modified theorems are sharpened versions of existing theorems. This is necessary if we have to use and understand these statements correctly on real life

networks. My paper makes this point amply clear. I believe that this paper is important even if it has no novel ideas because it polishes fundamentals. Since KVL and KCL are fundamental to all electrical engineers, I think that this paper should be of interest to many electrical engineers and not specialists only.

My paper is about KVL (KCL) from Tellegen's theorem and KCL (KVL) and not Tellegen's theorem. In the first place the voltages and currents must come from physical networks if these statements have to be meaningful. Secondly these statements are not true even for fictitious voltages and fictitious currents unless we assume that they are unrelated. The existing proofs don't say this. But they assume this unknowingly. Tellegen's theorem does not require this whereas KVL / KCL from Tellegen's theorem requires this extra constraint. Component equations in networks do not allow voltages and currents to be unrelated. The whole paper is about this. This constraint is satisfied by taking them from two different networks/graphs. If you insist on interconnection only, I will also state in terms of interconnection only. I will say two different graphs instead of networks. In fact I did this in the paper. But in reality they have to come from two different networks even if they are identical. If you try to prove these statements yourself or read my proofs carefully, you will realize the importance of what I am saying. I am only trying to make the fundamentals more precise.

Reviewer's comments 2:

On the first remark: the citation from the book "Linear and nonlinear circuits" you refer to is "Clearly, Tellegen's theorem depict only the interconnection properties of the circuit or the topology of digraph". I am fully agreed with that statement as there is no point in limiting the theorem with particular physical phenomenon. I agree with you that in order to use Tellegen's theorem the correspondence of voltages and currents to component equations must be examined.

I do not contradistinguish your paper to professor's Williams paper. The example given in the previous review shows that for known set of voltages the given equation has infinite number of solutions. Some of them match with Kirchhoff law for particular directed graph and only one of them has physical meaning. It is a well-known fact and it is considered by characteristics of the Tellegen's theorem related to Kirchhoff law. Professor's Williams paper was mentioned only as an example.

Your paper incorporates an important idea that two voltage and current vectors which meet Kirchhoff's law requirements for the circuit with particular directed graph and therefore Tellegen's theorem may not match to component equations.

In your paper you proposed the way of specifying the definitions of Tellegen's theorem's conclusions related to Kirchhoff law in order to eliminate inaccuracy. However nor theorem itself neither conclusions from it do confirm that voltage and current vectors should match

with component equations. Only the matching of Kirchhoff law with directed graph of particular circuit is stated. Therefore there is no need to additionally logically limit conclusions of the theorem. This is the idea that I tried to explain in my previous review. Purely the conclusions of the theorem were meant. I apologize for my incorrect wording.

I see no reason to include any additional limits to the definitions of the Tellegen's theorem conclusions which refer to Kirchhoff's law. However I agree with you that although the main idea of a paper about two vectors of voltage and current corresponding to Kirchhoff's law for particular directed graph is not new, it could be interesting for readers. Moreover I think that some of the readers may not be familiar enough with that topic. Your paper would be useful for the readers and I recommend it for publication in discussion section of the journal.

Author's answers 2:

1. At graph level we have KVL, KCL and Tellegen's equation. At network level we have KVL, KCL, Tellegen's equation and component equations. I.e., the network has more constraints than the graph. Therefore, a proof given at graph level need not hold at network level. Chua et al proved KVL/KCL at graph level only. They did not examine KVL/KCL at network level. That is why their proofs did not require component equations. But in reality we want KVL and KCL at network level and networks have components. Therefore, we cannot ignore components. We must consider the effect of component equations on the **proofs** of the statements. However, my theorems are also on graphs only. I got rid of the influence of component equations by using two graphs.
2. Chua et al. proved KVL and KCL at graph level. We cannot say that their statements are applicable to networks unless we understand the implications. For example, we need to know whether graph level statements and proofs are true at network level also or not. That is exactly what I did. My paper should be seen as an adaptation/extension/study of graph level statements to understand KVL and KCL at network level given Tellegen's equation. This is important because we have interest in KVL and KCL at network level and not graph level.
3. It is clear from the above explanation that the various ideas present in the paper like discussion concerning component equations and taking two graphs instead of one are necessary. There are no unnecessary "additional limitations" in the paper.