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Analysis of component failures dependency influence on system lifetime

Keywords

component failure dependency, reliability characteristics, limit reliability function, transportation system

Abstract

In the paper the results of the reliability investigation of multi-state homogeneous parallel-series systems with independent and dependent components are presented. The multi-state reliability functions of such systems and other reliability characteristics in both cases are determined under the assumption that their components have exponential reliability function. Moreover the asymptotic approach to the reliability evaluation of these systems is also presented and the classes of limit reliability functions for the considered systems in both cases are fixed. Finally, the presented theoretical results are applied to the reliability evaluation of the shipyard rope transportation system. The comparison of the multi-state exact and limit reliability functions of the considered transportation system under the assumption that its components are independent and under the assumption that its components have failure dependency is performed and illustrated graphically.

1. Introduction

The paper is concentrated on the reliability analysis of large multi-state parallel-series systems with dependent and independent failures of components. A parallel-series system with dependent components is considered as a system of a number of parallel subsystems linked in series, each of them composed of components with dependent failures. The system is failed if all components in at least one of its subsystem are failed. In the reliability analysis of their parallel subsystems, it seems natural to assume that the failures of one or several of their components may cause the reliability characteristics of their un-failed components worsening. In such systems the increased load caused by one or several of its components' failures may cause the increase of the failure rates of remaining un-failed components. The rules of load sharing between remaining not failed components that are the rules of their failure rates increase may be different. In the paper it is assumed that the load is distributed equally among all un-failed components of considered parallel systems and subsystems. This means that the failure rates of these components are changing in an analogical way that is the failure rates are increasing with the same level.

Some results on limit reliability functions of two-state parallel-series systems with equal load sharing among components of parallel subsystems in a case when the number of components in these subsystems is large were obtained by Smith [6]-[7]. Other results on limit reliability functions of two-state parallel-series systems in a case of the parallel-series system structure's shape when the number of parallel subsystems is large were fixed by Harlow and Smith [8]. Some partial results on limit reliability functions in the first of these two cases for multi-state parallel-series systems can be found in my recent publications as well [1]-[2].

2. Reliability of multi-state systems

Taking into account the importance of the safety effectiveness of large systems it seems reasonable to consider the multi-state approach in their reliability analysis. The assumption that the systems are composed of multi-state components with reliability state degrading in time without repair gives the possibility for more precise analysis of their reliability and safety effectiveness. This assumption allows us to distinguish a system reliability critical state to exceed which is either dangerous for the environment or does not assure the required level of effectiveness of this system exploitation. Then, an important system reliability characteristic is the time to the moment of exceeding the system reliability critical state and its distribution, which is called the system risk function. This distribution is strictly related to the system multi-state reliability function that is a basic characteristic of the multi-state system.

One of basic multi-state reliability structures with components degrading in time is parallel-series system. In the multi-state reliability analysis to define parallel-series systems with degrading components we assume that:

- E_{ij}, $i = 1,2,\ldots,k_n$, $j = 1,2,\ldots,l_i$, $k_n, l_1, l_2, \ldots, l_{k_n}$, $n \in N$, are components of a system,
- all components and a system under consideration have the state set $\{0,1,...,z\}$, $z \ge 1$,
- the reliability states are ordered, the state 0 is the worst and the state z is the best,
- the component and the system reliability states degrade with time t without repair,
- $T_{ij}(u)$, $i = 1,2,\ldots,k_n$, $j = 1,2,\ldots,l_n$, $k_n, l_1, l_2, \ldots, l_{k_n}$, $n \in N$, are independent random variables representing the lifetimes of components Eij in the state subset $\{u, u+1,...,z\}$, while they were in the state z at the moment $t = 0$,
- $T(u)$ is a random variable representing the lifetime of a system in the reliability state subset $\{u, u+1,...,z\}$, while it was in the reliability state z at the moment $t = 0$,
- $e_{ij}(t)$ are components E_{ij} states at the moment t, $t \in < 0, \infty)$,
- $s(t)$ is the system reliability state at the moment t, $t \in < 0, \infty)$.

Under above assumptions we introduce the following definition of multi-state reliability function of a component.

Definition 1. A vector

$$R_{ij}(t,\cdot) = [R_{ij}(t,0), R_{ij}(t,1), ..., R_{ij}(t,z)],\ t \in (-\infty, \infty),$$

where

$$R_{ij}(t,u) = P(e_{ij}(t) \ge u \mid e_{ij}(0) = z) = P(T_{ij}(u) > t), \qquad (1)$$

for $t \in < 0, \infty)$, $u = 0,1,...,z$, $i = 1,2,...,k_n$, $j = 1,2,...,l_i$, is the probability that the component E_{ij} is in the reliability state subset $\{u, u+1,...,z\}$ at the moment t, $t \in < 0, \infty)$, while it was in the reliability state z at the moment $t = 0$, is called the multi-state reliability function of a component E_{ij}.

It is clear from *Definition 1*, that $R_{ij}(t,0) = 1$.

Definition 2. A vector

$$\overline{\boldsymbol{R}}_{k_n,l_n}(t,\cdot) = [\overline{\boldsymbol{R}}_{k_n,l_n}(t,0), \overline{\boldsymbol{R}}_{k_n,l_n}(t,1), \ldots, \overline{\boldsymbol{R}}_{k_n,l_n}(t,z)]$$

where

$$\overline{\boldsymbol{R}}_{k_n,l_n}(t,u) = P(s(t) \ge u \mid s(0) = z) = P(T(u) > t) \qquad (2)$$

for $t \in < 0, \infty)$, $u = 0,1,...,z$, is the probability that the system is in the reliability state subset $\{u, u+1,...,z\}$ at the moment t, $t \in < 0, \infty)$, while it was in the reliability state z at the moment $t = 0$, is called the multi-state reliability function of a system.

If

$$p(t,\cdot) = [p(t,0), p(t,1), \ldots, p(t,z)] \text{ for } t \in < 0, \infty),$$

where

$$p(t,u) = P(s(t) = u \mid s(0) = z)$$

for $t \in < 0, \infty)$, $u = 0,1,\ldots,z$, is the probability that the system is in the state u at the moment t, $t \in < 0, \infty)$, while it was in the state z at the moment $t = 0$, then

$$\overline{\boldsymbol{R}}_{k_n,l_n}(t,0) = 1,\ \overline{\boldsymbol{R}}_{k_n,l_n}(t,z) = p(t,z),\ t \in < 0, \infty), \qquad (3)$$

and

$$p(t,u) = \overline{\boldsymbol{R}}_{k_n,l_n}(t,u) - \overline{\boldsymbol{R}}_{k_n,l_n}(t,u+1) \qquad (4)$$

for $t \in < 0, \infty)$, $u = 0,1,\ldots,z-1$.

Moreover, if

$$\overline{\boldsymbol{R}}^{(m)}_{k_n,l_n}(t,u) = 1 \text{ for } t \le 0,\ u = 1,\ldots,z,$$

then the mean lifetime of the system in the state subset $\{u, u+1,\ldots,z\}$ is

$$M(u) = \int_0^\infty \overline{\boldsymbol{R}}_{k_n,l_n}(t,u)dt,\ u = 1,\ldots,z, \qquad (5)$$

and the standard deviation of the system sojourn time in the state subset $\{u, u+1,\ldots,z\}$ is

$$\sigma(u) = \sqrt{N(u) - [M(u)]^2},\ u = 1,\ldots,z, \qquad (6)$$

where

$$N(u) = 2\int_0^\infty t\overline{\boldsymbol{R}}_{k_n,l_n}(t,u)dt,\ u = 1,\ldots,z. \qquad (7)$$

Besides

$$\overline{M}(u) = \int_0^\infty p(t,u)dt, \ u = 1,\dots,z, \tag{8}$$

is the mean lifetime of the system in the state *u*, while the integrals (5), (6) and (7) are convergent. Then, according to (3)-(5) and (7), we get the following relationship

$$\overline{M}(u) = M(u) - M(u+1), \ u = 1,2,\dots,z-1,$$

$$\overline{M}(z) = M(z). \tag{9}$$

Definition 3. A probability

$$\boldsymbol{r}(t) = P(s(t) < r \mid s(0) = z) = P(T(r) \le t),$$

that the system at the moment t, $t \in <0,\infty)$, is in the subset of states worse than the critical state r, $r \in \{1,2,\dots,z\}$, while it was In the state z at the moment $t = 0$ is called a risk function of the multi-state system.

Under this definition, from (1), we have

$$\boldsymbol{r}(t) = 1 - P(s(t) \ge r \mid s(0) = z) = 1 - \overline{\boldsymbol{R}}_{k_n,l_n}(t,r), \tag{10}$$

for $t \in (-\infty,\infty)$ and moreover, if τ is the moment when the risk exceeds a permitted level δ, $\delta \in <0,1>$, then

$$\tau = \boldsymbol{r}^{-1}(\delta), \tag{11}$$

where $\boldsymbol{r}^{-1}(t)$, if exists, is the inverse function of the risk function $\boldsymbol{r}(t)$.

2.1. Multi-state parallel-series systems

Definition 4. A multi-state system is called parallel-series if its lifetime $T(u)$ in the state subset $\{u,u+1,...,z\}$ is given by

$$T(u) = \min_{1 \le i \le k_n}\{\max_{1 \le j \le l_i}\{T_{ij}(u)\}\}, \ u = 1,2,\dots,z.$$

Definition 5. A multi-state parallel-series system is called homogeneous if its component lifetimes $T_{ij}(u)$ in the state subset have an identical distribution function

$$F(t,u) = P(T_{ij}(u) \le t), \ u = 1,2,\dots,z, \ i = 1,2,\dots,k_n, \ j = 1,2,\dots,l_i,$$

i.e. if its components E_{ij} have the same reliability function

$$R(t,u) = 1 - F(t,u), \ u = 1,2,\dots,z.$$

Definition 6. A multi-state parallel-series system is called regular if

$$l_1 = l_2 = \dots = l_{k_n} = l_n, \ l_n \in N,$$

i.e. if the numbers of components in its parallel subsystems are equal.

2.1.1. Parallel-series systems with independent components

The results presented below can be found in [4].

Proposition 1. If in a homogeneous regular multi-state parallel-series system

(i) components failure independently,

(ii) components have reliability functions $R(t)$,

then the multi-state system reliability function is given by the formula

$$\overline{\boldsymbol{R}}_{k_n,l_n}(t,\cdot) = [1, \overline{\boldsymbol{R}}_{k_n,l_n}(t,1),\dots,\overline{\boldsymbol{R}}_{k_n,l_n}(t,z)],$$

where

$$\overline{\boldsymbol{R}}_{k_n,l_n}(t,u) = [1-[1-R(t)]^{l_n}]^{k_n} \text{ for } t \in (-\infty,\infty), \ u = 1,\dots,z,$$

where k_n is the number of its parallel subsystems linked in series and l_n is the number of components in each subsystem.

In the case when components of a system have exponential reliability functions i.e.

$$R(t,\cdot) = [1, R(t,1),\dots,R(t,z)], \tag{12}$$

where

$$R(t,u) = 1 \text{ for } t < 0,$$

$$R(t,u) = \exp[-\lambda(u)t] \text{ for } t \ge 0, \ \lambda(u) > 0, \tag{13}$$

then the multi-state system reliability function takes form

$$\overline{\boldsymbol{R}}_{k_n,l_n}(t,\cdot) = [1, \overline{\boldsymbol{R}}_{k_n,l_n}(t,1),\dots,\overline{\boldsymbol{R}}_{k_n,l_n}(t,z)], \tag{14}$$

where

$$\overline{\boldsymbol{R}}_{k_n,l_n}(t,u) = 1 \text{ for } t < 0,$$

$$\overline{\boldsymbol{R}}_{k_n,l_n}(t,u) = [1-[1-\exp[-\lambda(u)t]]^{l_n}]^{k_n} \quad (15)$$

for $t \geq 0,\ u = 1,\ldots,z.$

2.1.2. Parallel-series systems with dependent component failures

A multi-state parallel-series system is in the reliability state subset $\{u, u+1,...,z\}$ if all of its parallel subsystems are in this state subset. Taking into account this fact, a multi-state parallel-series system with dependent components is considered as a system of linked independently in series multi-state parallel subsystems composed of components with failure dependency. In each of these subsystems we assume the following model of failure dependency. After getting out of the reliability state subset $\{u, u+1,...,z\}$ $v, v = 0,1,2,\ldots,l_i - 1,$ of components in a subsystem the increased load is shared equally among others so as their load increase with the same scale. Then their reliability is getting worse so that the mean values of the *i*-th, $i = 1,2,\ldots,k_n$, subsystem component lifetimes $T_{ij}{}'(u)$ in the state subset $\{u, u+1,...,z\}$ are of the form

$$E[T_{ij}{}'(u)] = E[T_{ij}(u)] - \frac{v}{l_i}E[T_{ij}(u)] = \frac{l_i - v}{l_i}E[T_{ij}(u)],$$

$$j = 1,2,\ldots,l_i,\ v = 0,1,2,\ldots,l_i - 1,\ u = 1,2,\ldots,z. \quad (16)$$

Then the rates of getting out from the reliability state subset $\{u, u+1,...,z\}$ of remaining components of the *i*-th, $i = 1,2,\ldots,k_n$, multi-state subsystem are given by

$$\lambda^{(v)}(u) = \frac{l_i - v}{l_i}\lambda(u),\ v = 0,1,2,\ldots,l_i - 1, \quad (17)$$

$$u = 1,2,\ldots,z.$$

Proposition 2. If in a homogeneous regular multi-state parallel-series system

(i) components failure in dependent way according to (16),

(ii) components have exponential reliability functions given by (12)-(13),

then the multi-state system reliability function is given by the formula

$$\overline{\boldsymbol{R}}_{k_n,l_n}(t,\cdot) = [1, \overline{\boldsymbol{R}}_{k_n,l_n}(t,1),\ldots,\overline{\boldsymbol{R}}_{k_n,l_n}(t,z)], \quad (18)$$

where

$$\overline{\boldsymbol{R}}_{k_n,l_n}(t,u) = 1 \text{ for } t < 0,$$

$$\overline{\boldsymbol{R}}_{k_n,l_n}(t,u) = \left[\sum_{v=0}^{l_n-1}\frac{(l_n\lambda(u)t)^v}{v!}\exp[-l_n\lambda(u)t]\right]^{k_n}$$

for $t \geq 0,\ u = 1,\ldots,z.$ (19)

3. Asymptotic approach

The investigation of exact reliability functions of large systems often leads to complicated formulae. Thus, from practical point of view, the asymptotic approach to large systems reliability evaluation is very important. The suggested method allows us to obtain formulae that simplify optimising calculations.

In the paper in the asymptotic approach to the reliability evaluation of multi-state parallel-series systems the linear standardization of their lifetimes in the reliability state subsets is used. This approach relies on an investigation of limit distribution of a standardized random variable $(T(u) - b_n(u))/a_n(u)$, $u = 1,2,\ldots,z$, where $T(u)$ is the system lifetime in the state subset $\{u, u+1,\ldots,z\}$ and $a_n(u) > 0$, $b_n(u) \in (-\infty,\infty)$, are called normalizing constants. For that reason, we assume the following definition [4].

Definition 7. A reliability function

$$\boldsymbol{\mathfrak{R}}(t,\cdot) = [1, \boldsymbol{\mathfrak{R}}(t,1),\ldots,\boldsymbol{\mathfrak{R}}(t,z)],\ t \in (-\infty,\infty),$$

is called a multi-sate limit reliability function of a system with reliability function

$$\overline{\boldsymbol{R}}_{k_n,l_n}(t,\cdot) = [1, \overline{\boldsymbol{R}}_{k_n,l_n}(t,1),\ldots,\overline{\boldsymbol{R}}_{k_n,l_n}(t,z)],$$

if there exist normalizing constants $a_n(u) > 0$, $b_n(u) \in (-\infty,\infty)$ such as

$$\lim_{n\to\infty}\overline{\boldsymbol{R}}_{k_n,l_n}(a_n(u)t + b_n(u),u) = \boldsymbol{\mathfrak{R}}(t,u) \text{ for } t \in C_{\mathfrak{R}(u)},$$

where $C_{\mathfrak{R}(u)}$ is the set of continuity points of $\boldsymbol{\mathfrak{R}}(t,u),\ u = 1,2,\ldots,z.$

From *Definition 7* it follows that the knowledge the limit reliability function of a system $\boldsymbol{\mathfrak{R}}(t,\cdot)$ for sufficiently large *n* allows us to estimate the multi-state reliability function of a system $\overline{\boldsymbol{R}}_{k_n,l_n}(t,\cdot)$ according the following approximate formula

$$\overline{\boldsymbol{R}}_{k_n,l_n}(t,\cdot) \cong \boldsymbol{\mathfrak{R}}((t-b_n(u))/a_n(u),\cdot), \tag{20}$$

i.e.

$$[1, \overline{\boldsymbol{R}}_{k_n,l_n}(t,1),\ldots,\overline{\boldsymbol{R}}_{k_n,l_n}(t,z)]$$

$$\cong [1, \boldsymbol{\mathfrak{R}}(\frac{t-b_n(1)}{a_n(1)},1),\ldots,\boldsymbol{\mathfrak{R}}(\frac{t-b_n(z)}{a_n(z)},z)],\ t \in (-\infty,\infty).$$

3.1. Reliability evaluation of large parallel-series systems with independent components

In this point the possibilities of multi-state asymptotic approach to the reliability evaluation of large parallel-series systems are presented. There are formulated two propositions that allow us to find multi-state limit reliability functions of homogeneous parallel-series systems with independent components under the assumption that they have exponential reliability functions.

Proposition 3. If in a homogeneous regular multi-state parallel-series system

(i) components failure independently,

(ii) components have exponential reliability functions given by (12)-(13),

(iii) $k_n = n,\ l_n - c\log n >> s,\ c > 0,\ s > 0,$

(iv) $a_n(u) = \frac{1}{\lambda(u)\log n},\ b_n(u) = \frac{1}{\lambda(u)}\log\frac{l_n}{\log n},\ u = 1,2,\ldots,z,$

then

$$\overline{\boldsymbol{\mathfrak{R}}}_3(t,\cdot) = [1, \overline{\boldsymbol{\mathfrak{R}}}_3(t,1),\ldots,\overline{\boldsymbol{\mathfrak{R}}}_3(t,z)],\ t \in (-\infty,\infty), \tag{21}$$

where

$$\overline{\boldsymbol{\mathfrak{R}}}_3(t,u) = \exp[-\exp t] \text{ for } u = 1,\ldots,z, \tag{22}$$

is the multi-state limit reliability function of considered system.

Proposition 4. If in a homogeneous regular multi-state parallel-series system

(i) components failure independently,

(ii) components have exponential reliability functions given by (12)-(13),

(iii) $k_n \to k,\ k > 0,\ l_n \to \infty,$

(iv) $a_n(u) = \frac{1}{\lambda(u)},\ b_n(u) = \frac{\log l_n}{\lambda(u)},\ u = 1,2,\ldots,z,$

then

$$\overline{\boldsymbol{\mathfrak{R}}}_{10}(t,\cdot) = [1, \overline{\boldsymbol{\mathfrak{R}}}_{10}(t,1),\ldots,\overline{\boldsymbol{\mathfrak{R}}}_{10}(t,z)],\ t \in (-\infty,\infty), \tag{23}$$

where

$$\overline{\boldsymbol{\mathfrak{R}}}_3(t,u) = [1-\exp[-\exp[-t]]]^k \text{ for } u = 1,\ldots,z, \tag{24}$$

is the multi-state limit reliability function of considered system.

3.2. Reliability evaluation of large parallel-series systems with dependent component failures

The proofs of presented below propositions are given in [3].

Proposition 5. If in a homogeneous regular multi-state parallel-series system

(i) components failure in dependent way according to (16),

(ii) components have exponential reliability functions given by (12)-(13),

(iii) $\overline{\boldsymbol{R}}_{k_n,l_n}(t,\cdot)$ is a multi-state system reliability function given by (18)-(19),

(iv) $k_n \to k$ = constant as $n \to \infty$,

(v) $l_n = n,$

(vi) $a_n(u) = \frac{1}{\lambda(u)\sqrt{n}},\ b_n(u) = \frac{1}{\lambda(u)}$ for $u = 1,\ldots,z,$

then

$$\overline{\boldsymbol{\mathfrak{R}}}_2(t,\cdot) = [1, \overline{\boldsymbol{\mathfrak{R}}}_2(t,1),\ldots,\overline{\boldsymbol{\mathfrak{R}}}_2(t,z)],\ t \in (-\infty,\infty), \tag{25}$$

where

$$\overline{\boldsymbol{\mathfrak{R}}}_2(t,u) = [1-F_{N(0,1)}(t,u)]^k \text{ for } u = 1,\ldots,z, \tag{26}$$

and

$$F_{N(0,1)}(t,u) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{t} \exp[-\frac{t^2}{2}]dt, \; t \in (-\infty, \infty),$$

is the multi-state limit reliability function of considered system.
According to *Proposition 5* and by (20) we can get the approximate formula for the exact multi-state reliability function of the considered parallel-series system

$$\overline{\boldsymbol{R}}_{k_n, l_n}(t,\cdot) = [1, \overline{\boldsymbol{R}}_{k_n, l_n}(t,1), \ldots, \overline{\boldsymbol{R}}_{k_n, l_n}(t,z)],$$

where

$$\overline{\boldsymbol{R}}_{k_n, l_n}(t,u) \cong \overline{\mathfrak{R}}_2(\frac{t - b_n(u)}{a_n(u)}, u)$$

$$= [1 - F_{N(0,1)}(\sqrt{n}\lambda(u)t - \sqrt{n}, u)]^k \quad (27)$$

for $t \in (-\infty, \infty), \; u = 1, \ldots, z.$

Proposition 6. If in a homogeneous regular multi-state parallel-series system

(i) components failure in dependent way according to (16),

(ii) components have exponential reliability functions given by (12)-(13),

(iii) $\overline{\boldsymbol{R}}_{k_n, l_n}(t,\cdot)$ is a multi-state system reliability function given by (18)-(19),

(iv) $k_n \to \infty$ as $n \to \infty$,

(v) $l_n = n$,

(vi) $n^{-1/3} \log k_n \to 0$ as $n \to \infty$,

(vii) $a_n(u) = \frac{1}{\lambda(u)\sqrt{2n \log k_n}}$,

$$b_n(u) = \frac{\log(4\pi) + \log\log k_n - 4\log k_n}{\lambda(u)\sqrt{8n \log k_n}} + \frac{1}{\lambda(u)}$$

for $u = 1, 2, \ldots, z$,

then

$$\overline{\mathfrak{R}}_4(t,\cdot) = [1, \overline{\mathfrak{R}}_4(t,1), \ldots, \overline{\mathfrak{R}}_4(t,z)], \; t \in (-\infty, \infty), \quad (28)$$

where

$$\overline{\mathfrak{R}}_4(t,u) = \exp[-\exp t], \; u = 1, 2, \ldots, z, \quad (29)$$

is the multi-state limit reliability function of considered system.
According to *Proposition 6* and by (20) we can get the approximate formula for the exact multi-state reliability function of the considered parallel-series system

$$\overline{\boldsymbol{R}}_{k_n, l_n}(t,\cdot) = [1, \overline{\boldsymbol{R}}_{k_n, l_n}(t,1), \ldots, \overline{\boldsymbol{R}}_{k_n, l_n}(t,z)], \; t \in (-\infty, \infty),$$

where

$$\overline{\boldsymbol{R}}_{k_n, l_n}(t,u) \cong \overline{\mathfrak{R}}_4(\frac{t - b_n(u)}{a_n(u)}, u)$$

$$= \exp[-\exp[(\lambda(u)t - 1)\sqrt{2n \log k_n}$$

$$- \frac{1}{2}\log(4\pi) - \frac{1}{2}\log\log k_n + 2\log k_n]] \quad (30)$$

for $t \in (-\infty, \infty), \; u = 1, 2, \ldots, z.$

4. Application

The obtained theoretical results can be applied to the reliability evaluation of real technical systems that are composed of large number of components with failure dependency. In this part, the ship-rope elevator used in the Naval Shipyard in Gdynia is considered and its reliability analysis is performed. The elevator is composed of a steel platform-carriage moved with 10 rope-hoisting winches fed by separate electric motors. The exact and limit multi-state reliability functions, the mean values and standard deviations of this system lifetimes in the reliability state subsets, the mean values of the system lifetimes in particular reliability states and the system risk function are determined.
In our further analysis we will discuss the reliability of the rope system only. The system under consideration is in order if all its ropes do not fail. Thus we may assume that it is a series system composed of 10 components. Each of the ropes is composed of 22 strands. According to rope reliability data given in their technical certificates and experts' opinions [5] based on the nature of strand failures the following reliability states have been distinguished:
state 3 – a strand is new, without any defects,

state 2 – the number of broken wires in the strand is greater than 0% and less than 25% of all its wires, or corrosion of wires is greater than 0% and less than 25%,
state 1 – the number of broken wires in the strand is greater than or equal to 25% and less than 50% of all its wires, or corrosion of wires is greater than or equal to 25% and less than 50%,
state 0 – otherwise (a strand is failed).

We consider the strands as basic components of the system. The system of ropes is in the reliability state subset $\{u, u+1, ..., z\}$, $u = 1,2,3,$ when all of its ropes are in this state subset and each of the ropes is in the reliability state subset $\{u, u+1, ..., z\}$, $u = 1,2,3,$ if at least one of 22 strands is in this state subset. Thus, according to *Definition 6* we conclude that the rope elevator is a regular 4-states parallel-series system composed of k_n = 10 series-linked subsystems (ropes) with l_n = 22 parallel-linked components (strands).
It has been assumed that the strands have exponential reliability functions:

$$R(t,\cdot) = [1, R(t,1), R(t,2), R(t,3)] \text{ for } t \in (-\infty, \infty),$$

$$R(t,u) = 1 \text{ for } t < 0,\ u = 1,2,3,$$

$$R(t,1) = \exp[-0.1613t],\ R(t,2) = \exp[-0.2041t],$$

$$R(t,3) = \exp[-0.2326t] \text{ for } t \geq 0.$$

4.1. The ship-rope elevator as a system with independent components

Discussed in the paper the shipyard rope transportation system under the assumption that its components are independent is also widely described and analysed in [4].
Applying *Proposition 1* the exact multi-state reliability function of the elevator under the assumption that its components are independent is given by the formula

$$\overline{\boldsymbol{R}}_{10,22}(t,\cdot) = [1, \overline{\boldsymbol{R}}_{10,22}(t,1), \overline{\boldsymbol{R}}_{10,22}(t,2), \overline{\boldsymbol{R}}_{10,22}(t,3)], \quad (31)$$

where

$$\overline{\boldsymbol{R}}_{10,22}(t,1) = [1-[1-\exp[-0.1613t]]^{22}]^{10},$$

$$\overline{\boldsymbol{R}}_{10,22}(t,2) = [1-[1-\exp[-0.2041t]]^{22}]^{10},$$

$$\overline{\boldsymbol{R}}_{10,22}(t,3) = [1-[1-\exp[-0.2326t]]^{22}]^{10},$$

for $t \in (-\infty, \infty)$.

The expected values of the elevator lifetimes $T(1)$, $T(2)$, $T(3)$ in the state subsets $\{1,2,3\}$, $\{2,3\}$, $\{3\}$ and their standard deviations counted in years, according to (5)-(7), are:

$$M(1) \cong 13.434,\ M(2) \cong 10.617,\ M(3) \cong 9.316,$$

$$\sigma(1) \cong 2.106,\ \sigma(2) \cong 1.597,\ \sigma(3) \cong 1.360.$$

Hence, from (9), the elevator mean lifetimes in the particular states in years are

$$\overline{M}(1) \cong 2.817,\ \overline{M}(2) \cong 1.301,\ \overline{M}(3) \cong 9.316.$$

Assuming that a critical reliability state of the rope elevator is r = 2, then from (10) its risk function takes the following form

$$\boldsymbol{r}(t) = 1 - \overline{\boldsymbol{R}}_{10,22}(t,2)$$

$$= 1-[1-[1-\exp[-0.2041t]]^{22}]^{10},\ t \in (-\infty, \infty).$$

The moment when the system risk exceeds the permitted level e.g. $\delta = 0.05$, according to (11), is

$$\tau = \boldsymbol{r}^{-1}(\delta) \cong 9 \text{ years and 212 days.}$$

Since the number of parallel subsystems in the system is $k_n = 10$ and the number of components in each subsystem is $l_n = 22$, then taking into account that $l_n = 22 >> \log k_n = \log 10 \cong 2.3$, it seems reasonable to apply in the elevator's reliability evaluation either *Proposition 2* or *Proposition 3*. First applying *Proposition 2* we conclude that multi-state limit reliability function of the elevator is of the form

$$\overline{\boldsymbol{R}}_{10,22}(t,\cdot) = [1, \overline{\boldsymbol{R}}_{10,22}(t,1), \overline{\boldsymbol{R}}_{10,22}(t,2), \overline{\boldsymbol{R}}_{10,22}(t,3)], \quad (32)$$

where

$$\overline{\boldsymbol{R}}_{10,22}(t,1) \cong \exp[-\exp[0.3714t - 5.1969]],$$

$$\overline{\boldsymbol{R}}_{10,22}(t,2) \cong \exp[-\exp[0.4699t - 5.1969]],$$

$$\overline{\boldsymbol{R}}_{10,22}(t,3) \cong \exp[-\exp[0.5356t - 5.1970]],$$

for $t \in (-\infty, \infty)$.

The expected values of the elevator lifetimes $T(1)$, $T(2)$, $T(3)$ in the state subsets $\{1,2,3\}$, $\{2,3\}$, $\{3\}$ and their standard deviations counted in years, according to (5)-(7), are:

$$M(1) \cong 12.453, \ M(2) \cong 9.843, \ M(3) \cong 8.636,$$

$$\sigma(1) \cong 3.199, \ \sigma(2) \cong 2.487, \ \sigma(3) \cong 2.158.$$

Hence, from (9), the elevator mean lifetimes in the particular states in years are

$$\overline{M}(1) \cong 2.610, \ \overline{M}(2) \cong 1.207, \ \overline{M}(3) \cong 8.636.$$

If a critical reliability state of the rope elevator is $r = 2$, then from (10) its risk function takes the following form

$$\boldsymbol{r}(t) = 1 - \overline{\boldsymbol{R}}_{10,22}(t,2)$$

$$= 1 - \exp[-\exp[0.4699t - 5.1969]], \ t \in (-\infty, \infty).$$

The moment when the system risk exceeds the permitted level e.g. $\delta = 0.05$, according to (11), is

$$\tau = \boldsymbol{r}^{-1}(\delta) \cong 6 \text{ years.}$$

Next applying *Proposition 3* we get

$$\overline{\boldsymbol{R}}_{10,22}(t,\cdot) = [1, \overline{\boldsymbol{R}}_{10,22}(t,1), \overline{\boldsymbol{R}}_{10,22}(t,2), \overline{\boldsymbol{R}}_{10,22}(t,3)], \quad (33)$$

where

$$\overline{\boldsymbol{R}}_{10,22}(t,1) \cong [1 - \exp[-\exp[-0.1613t + 3.0911]]]^{10},$$

$$\overline{\boldsymbol{R}}_{10,22}(t,2) \cong [1 - \exp[-\exp[-0.2041t + 3.0910]]]^{10},$$

$$\overline{\boldsymbol{R}}_{10,22}(t,3) \cong [1 - \exp[-\exp[-0.2326t + 3.0911]]]^{10},$$

for $t \in (-\infty, \infty)$.

The expected values of the elevator lifetimes $T(1)$, $T(2)$, $T(3)$ in the state subsets $\{1,2,3\}$, $\{2,3\}$, $\{3\}$ and their standard deviations counted in years, according to (5)-(7), are:

$$M(1) \cong 13.027, \ M(2) \cong 10.295, \ M(3) \cong 9.034,$$

$$\sigma(1) \cong 2.300, \ \sigma(2) \cong 1.758, \ \sigma(3) \cong 1.506.$$

Hence, from (9), the elevator mean lifetimes in the particular states in years are

$$\overline{M}(1) \cong 2.732, \ \overline{M}(2) \cong 1.261, \ \overline{M}(3) \cong 9.034.$$

If a critical reliability state of the rope elevator is $r = 2$, then from (10) its risk function takes the following form

$$\boldsymbol{r}(t) = 1 - \overline{\boldsymbol{R}}_{10,22}(t,2)$$

$$= 1 - [1 - \exp[-\exp[-0.2041t + 3.0910]]]^{10},$$

for $t \in (-\infty, \infty)$.

The moment when the system risk exceeds the permitted level e.g. $\delta = 0.05$, according to (11), is

$$\tau = \boldsymbol{r}^{-1}(\delta) \cong 8 \text{ years 310 days.}$$

4.2. The ship-rope elevator as a system with dependent component failures

From practical point of view it seems reasonable to consider the shipyard rope transportation system assuming components' dependence. Indeed, while failing some of strands in a rope the load of the remaining not failed ones may be getting larger. Thus, the assumption about dependence of strands is natural and justified.
After considering *Proposition 4* the exact multi-state elevator reliability function is given by the formula

$$\overline{\boldsymbol{R}}_{10,22}(t,\cdot) = [1, \overline{\boldsymbol{R}}_{10,22}(t,1), \overline{\boldsymbol{R}}_{10,22}(t,2), \overline{\boldsymbol{R}}_{10,22}(t,3)], \quad (34)$$

where

$$\overline{\boldsymbol{R}}_{10,22}(t,u) = 1 \text{ for } t < 0, \ u = 1,2,3,$$

$$\overline{\boldsymbol{R}}_{10,22}(t,1) = [\sum_{j=0}^{21} \frac{(3.5486t)^j}{j!} \exp[-3.5486t]]^{10},$$

$$\overline{\boldsymbol{R}}_{10,22}(t,2) = [\sum_{j=0}^{21} \frac{(4.4902t)^j}{j!} \exp[-4.4902t]]^{10},$$

$$\overline{\boldsymbol{R}}_{10,22}(t,3) = [\sum_{j=0}^{21} \frac{(5.1172t)^j}{j!} \exp[-5.1172t]]^{10},$$

for $t \geq 0$.

The expected values of the elevator lifetimes $T(1)$, $T(2)$, $T(3)$ in the state subsets $\{1,2,3\}$, $\{2,3\}$, $\{3\}$ and their standard deviations, according to (5)-(7), in years are:

$$M(1) \cong 4.335, \;\; M(2) \cong 3.426, \;\; M(3) \cong 3.006,$$

$$\sigma(1) \cong 0.773, \;\; \sigma(2) \cong 0.472, \;\; \sigma(3) \cong 0.414.$$

Hence, from (9), the elevator mean lifetimes in the particular states in years are

$$\overline{M}(1) \cong 0.909, \;\; \overline{M}(2) \cong 0.420, \;\; \overline{M}(3) \cong 3.006.$$

If a critical reliability state of the rope elevator is $r = 2$, then from (10) its risk function takes the following form

$$\boldsymbol{r}(t) = 1 - \overline{\boldsymbol{R}}_{10,22}(t,2)$$

$$= \begin{cases} 0, \; t < 0, \\ 1 - [\sum_{j=0}^{21} \frac{(4.4902t)^j}{j!} \exp[-4.4902t]]^{10}, \; t \geq 0. \end{cases}$$

The moment when the system risk exceeds the permitted level e.g. $\delta = 0.05$, according to (11), is

$$\tau = \boldsymbol{r}^{-1}(\delta) \cong 2 \text{ years and 230 days.}$$

In the asymptotic approach to the reliability evaluation of the rope elevator assuming component failure dependency similarly as in the first case we can apply either *Proposition 5* or *Proposition 6*. Applying *Proposition 5* we can find the approximate multi-state reliability function of the rope elevator system.

$$\overline{\boldsymbol{R}}_{10,22}(t,\cdot) = [1, \overline{\boldsymbol{R}}_{10,22}(t,1), \overline{\boldsymbol{R}}_{10,22}(t,2), \overline{\boldsymbol{R}}_{10,22}(t,3)], \quad (35)$$

where

$$\overline{\boldsymbol{R}}_{10,22}(t,1) \cong [1 - F_{N(0,1)}(0.7566t - 4.6904)]^{10},$$

$$\overline{\boldsymbol{R}}_{10,22}(t,2) \cong [1 - F_{N(0,1)}(0.9573t - 4.6904)]^{10},$$

$$\overline{\boldsymbol{R}}_{10,22}(t,3) \cong [1 - F_{N(0,1)}(1.0901t - 4.6904)]^{10},$$

for $t \in (-\infty, \infty)$.

The expected values of the elevator lifetimes $T(1)$, $T(2)$, $T(3)$ in the state subsets $\{1,2,3\}$, $\{2,3\}$, $\{3\}$ and their standard deviations counted in years, according to (5)-(7), are:

$$M(1) \cong 4.166, \;\; M(2) \cong 3.292, \;\; M(3) \cong 2.891,$$

$$\sigma(1) \cong 0.776, \;\; \sigma(2) \cong 0.613, \;\; \sigma(3) \cong 0.538.$$

Hence, from (9), the elevator mean lifetimes in the particular states in years are

$$\overline{M}(1) \cong 0.873, \;\; \overline{M}(2) \cong 0.401, \;\; \overline{M}(3) \cong 2.891.$$

Assuming that a critical reliability state of the rope elevator is $r = 2$, then from (10) its risk function takes the following form

$$\boldsymbol{r}(t) = 1 - \overline{\boldsymbol{R}}_{10,22}(t,2)$$

$$= 1 - [1 - F_{N(0,1)}(0.9573t - 4.6904)]^{10}, \; t \in (-\infty, \infty).$$

The moment when the system risk exceeds the permitted level assuming as before $\delta = 0.05$, according to (11), is

$$\tau = \boldsymbol{r}^{-1}(\delta) \cong 2 \text{ years 80 days.}$$

Now the system multi-state reliability function is estimated from the formula (13) as an application of *Proposition 6*.

$$\overline{\boldsymbol{R}}_{10,22}(t,\cdot) = [1, \overline{\boldsymbol{R}}_{10,22}(t,1), \overline{\boldsymbol{R}}_{10,22}(t,2), \overline{\boldsymbol{R}}_{10,22}(t,3)], \quad (36)$$

where

$$\overline{\boldsymbol{R}}_{10,22}(t,1) = \exp[-\exp[(0.1613t - 1)\sqrt{44\log 10} + \log(50/\sqrt{\pi \log 10})]],$$

$$\overline{\boldsymbol{R}}_{10,22}(t,2) = \exp[-\exp[(0.2041t - 1)\sqrt{44\log 10} + \log(50/\sqrt{\pi \log 10})]],$$

$$\overline{\boldsymbol{R}}_{10,22}(t,3) = \exp[-\exp[(0.2326t - 1)\sqrt{44\log 10} + \log(50/\sqrt{\pi \log 10})]],$$

for $t \in (-\infty, \infty)$.

The expected values of the elevator lifetimes $T(1)$, $T(2)$, $T(3)$ in the state subsets $\{1,2,3\}$, $\{2,3\}$, $\{3\}$ and their standard deviations, according to (5)-(7), in years are:

$$M(1) \cong 4.044, \;\; M(2) \cong 3.196, \;\; M(3) \cong 2.805,$$

$$\sigma(1) \cong 0.787, \ \ \sigma(2) \cong 0.622, \ \ \sigma(3) \cong 0.546.$$

Hence, from (9), the elevator mean lifetimes in the particular states in years are

$$\overline{M}(1) \cong 0.848, \ \ \overline{M}(2) \cong 0.392, \ \ \overline{M}(3) \cong 2.805.$$

If a critical reliability state of the rope elevator is $r = 2$, then from (10) its risk function takes the following form

$$\boldsymbol{r}(t) = 1 - \overline{\boldsymbol{R}}_{10,22}(t,2)$$

$$= 1 - \exp[-\exp[(0.2041t - 1)\sqrt{44 \log 10}$$

$$+ \log(50/\sqrt{\pi \log 10})]] \ \text{for} \ t \in (-\infty, \infty).$$

The moment when the system risk exceeds the permitted level e.g. $\delta = 0.05$, according to (11), is

$$\tau = \boldsymbol{r}^{-1}(\delta) \cong 2 \ \text{years and 11 days.}$$

Comparing the expected values of the elevator lifetimes in the state subset and the elevator mean lifetimes in the particular states in the case when strands failure in dependent and independent way we can conclude that these values are lower in the first case for about 68% percent for exact reliability functions and for about 67% and 69% for approximate reliability functions.
The obtained results illustrate that the increased load of remaining un-failed components causes shortening the lifetime of these components in a significant way. That fact can be interpreted as a decrease of their reliability much faster then for the systems with independent components. Taking into account the presented ship-rope elevator we can notice that the lifetime in the reliability state subset of the elevator under the assumption that strand failure in dependent way is even about 70% shorten then in the case when strands are independent.

5. Conclusion

In the paper the exact reliability analysis and asymptotic approach to the reliability evaluation of homogeneous multi-state parallel-series systems are presented. For these systems the exact and limit reliability functions and other characteristics both in the case when their components are independent and when they are dependent are determined under the assumption that components of systems have exponential reliability functions.

Introduced in the paper the method of reliability evaluation of large systems relies on application of some approximate methods based on classical asymptotic approach to this issue. The obtained results are concerned with typical systems with regular structure. Applied in the paper analytical methods are successful rather for not very complex systems. In this background it seems to be justified the extension of this issue for systems with less regular structures and use of any other reliability analysis methods.

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Appendix

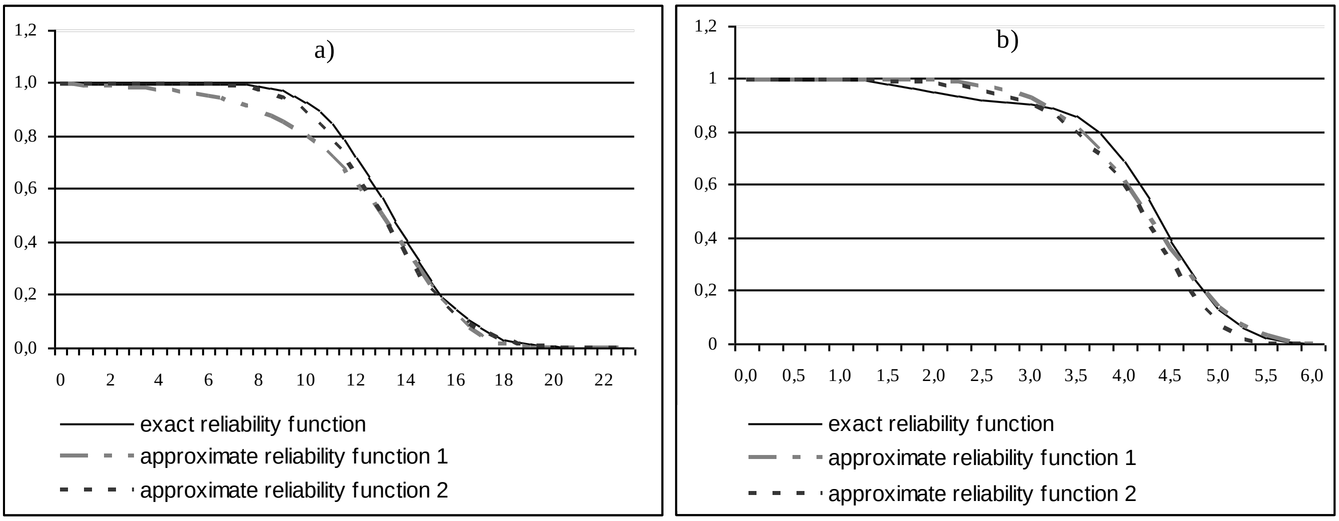

Figure 1. Graphs of the rope elevator exact and approximate reliability functions in the state subset $u \geq 1$
a) in the case when components are independent b) in the case when components fail dependently

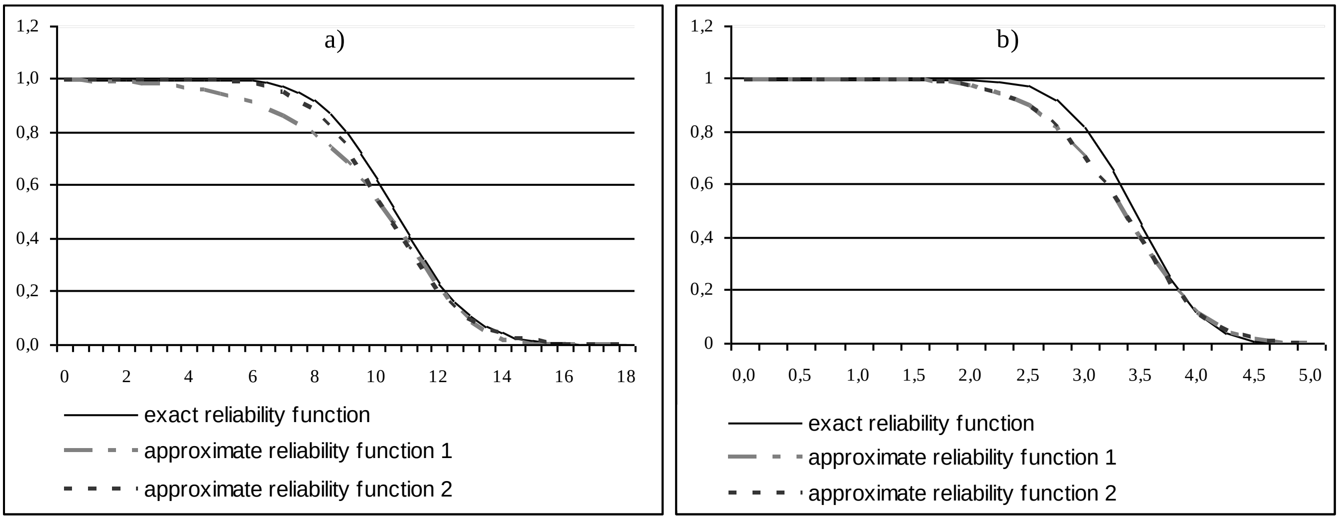

Figure 2. Graphs of the rope elevator exact and approximate reliability functions in the state subset $u \geq 2$
a) in the case when components are independent b) in the case when components fail dependently

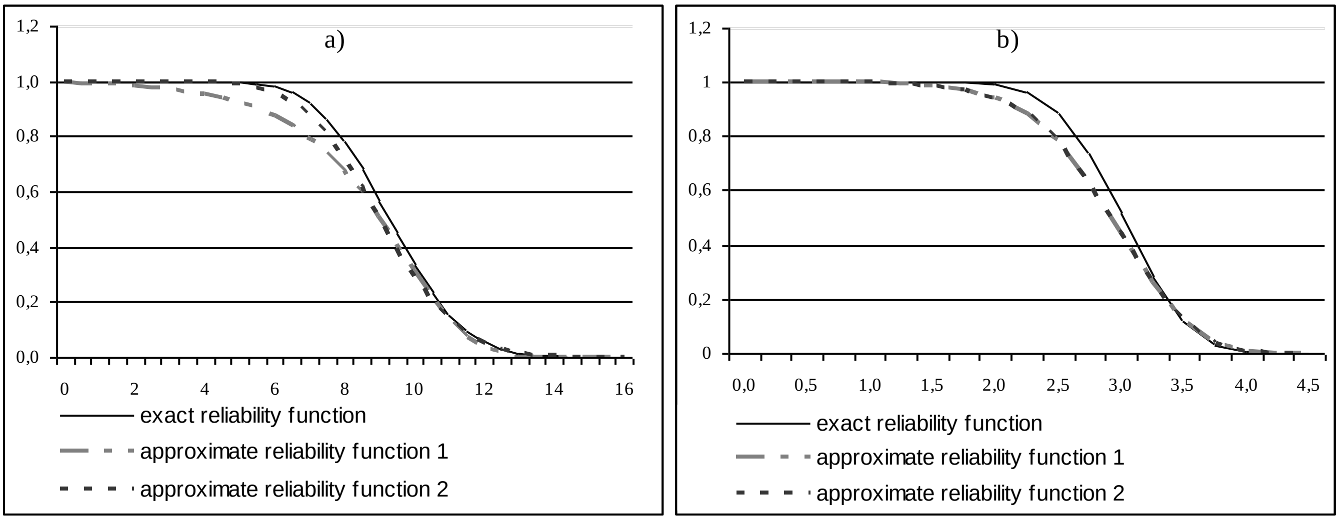

Figure 3. Graphs of the rope elevator exact and approximate reliability functions in the state subset $u = 3$
a) in the case when components are independent b) in the case when components fail dependently