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QUALITATIVE DESCRIPTION OF METAL ORE DEPOSITS PARAMETERS BASED ON SELECTED FUZZY LOGIC OPERATORS ON THE EXAMPLE OF A KGHM POLISH COPPER S.A. COPPER-SILVER MINE

JAKOŚCIOWY OPIS PARAMETRÓW ZŁOŻA RUD METALI W OPARCIU O WYBRANE OPERATORY LOGICZNE ZBIORÓW ROZMYTYCH NA PRZYKŁADZIE KOPALNI RUD MIEDZIOWO-SREBROWYCH W KGHM „POLSKA MIEDŹ” S.A.

The basis for a mineral deposit delimitation is a qualitative and quantitative assessment of deposit parameters, qualifying a deposit as an economically valuable object. A conventional approach to the mineral deposit recognition and a deposit detailed parameters qualification in the initial stages of development work in the KGHM were presented in the paper. The goals of such recognition were defined, which through a gradual detailed expansion, resulting from the information inflow, allows for the construction of a more complete decision-making model. The description of the deposit parameters proposed in the article in the context of fuzzy logic, enables a presentation of imprecise statements and data, which may be a complement to a traditional description. Selected non-adjustable and adjustable *s*-norm and *t*-norm operators were demonstrated. Operators effects were tested for selected ore quality parameters (copper content and deposit thickness) by constructing adequate membership functions. In a practical application, the use of chosen fuzzy logic operators is proposed for the assessment of the qualitative parameters of copper-silver ore in the exploitation blocks for one of the mines belonging to KGHM Polish Copper S.A. The considerations have been extended by including the possibility of using compensation operators.

Keywords: ore deposit parameters, mineral deposit management, fuzzy sets, logical operator

Podstawą delimitowania złoża jest ocena parametrów jakościowych oraz ilościowych, kwalifikująca twór przyrodniczy jako obiekt o znaczeniu gospodarczym. W artykule przybliżono konwencjonalne podejście do rozpoznania serii złożowej i szczegółowych parametrów złoża, realizowane w kopalniach KGHM Polska Miedź S.A. w trakcie wykonywania tzw. robót przygotowawczych. Określono celowość takiego rozpoznania, które poprzez stopniowe uszczegółowianie, wynikające z napływu informacji, pozwala na konstrukcję bardziej kompletnego modelu decyzyjnego. Dopełnieniem tego tradycyjnego

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opisu może być zaproponowana w artykule deskrypcja parametrów złoża w ujęciu logiki rozmytej, umożliwiająca przedstawienie nieprecyzyjnych stwierdzeń i danych. Zademonstrowano wykorzystanie nastawialnych i nienastawialnych operatorów mnogościowych *s*-normy i *t*-normy a działanie operatorów przetestowano dla wybranych parametrów jakościowych rudy (zawartości miedzi oraz miąższości złoża) konstruując adekwatne funkcje przynależności. W praktycznym zastosowaniu przetestowano wybrane operatory logiki rozmytej do oceny parametrów jakościowych złoża rud miedziowo-srebrowych w blokach eksploatacyjnych dla jednej z kopalń KGHM Polska Miedź S.A. Rozważania poszerzono o możliwość zastosowania operatorów kompensacyjnych.

Słowa kluczowe: ruda miedzi, jakość złoża, zbiory rozmyte, operator logiczny

1. Introduction

Estimation of mineral deposit parameters aimed at the most faithful representation of actual deposit size is a difficult task. Evaluation of the parameters of the deposit – resource volume, quality of the mineral, position in space – is, irrespective of the method used, usually characterised by a certain degree of inaccuracy. This is due both to variation in the deposit itself and to the degree to which the deposit has been explored.

Certain fuzzy categories operate in the deposit description. Mining geologists use descriptive terms, e.g. large/small deposit, rich/poor ore, high/low series thickness, high/low metal content, and good/bad enrichment indicators. Quantitative quantifiers usually enable a precise description of a specific deposit parameter, but this is usually based on point observation; a need exists for the use of such observation regarding the part of the deposit which cannot be sampled directly. The issue of deposit description and its grade qualification is debatable and difficult to assess clearly. For example, the evaluation 'rich ore' does not specify the limits of metal content at which a given part is considered rich. In relation to porphyry copper deposits, should the content be 0.5 or 0.7% Cu, or perhaps more? Easy and unequivocal conversion of qualitative to quantitative terms is not usually possible. The use of classical set theory, using a membership of 0 (does not belong) or 1 (belongs), is simply ineffective here. More helpful is fuzzy reasoning based on estimation of membership in a particular set within a certain range. Usually, this involves the interval [0, 1], where '0' means exclusion from the set, while '1' signals full membership. Between these two extremes is a field of partial membership, which can be successfully used in the description of deposit features.

Certainly, deposit parameters, both overall and at a particular point, are usually, as mentioned above, quantitative measures, e.g. deposit thickness (m) or content of useful components (%). Evaluation of these parameters is the basis for spatial deposit orientation and ultimately boils down to calculation of mineral resources and/or useful component. It will also be recalled that, apart from quantitative terms, descriptive characterisations are also used. Such characterisations are sometimes carried out very crudely, based on individual feelings and observations.

In the geological and mining field, set theory and fuzzy logic tools have been used occasionally. Use of a fuzzy approach in various geology branches has been presented by Demmicco's and Klir's (2003) in their book. Possible applications of fuzzy reasoning in basic geology are indicated, i. a. for: stratigraphic modeling, earthquake research, or reef growth. List of applications in the mining field is wider. Bandopadhyay (1987) indicates the potential for the use of fuzzy logic in the selection of an appropriate system of operation. Kesimal & Bascetin (2002) have authored a study using applications of the fuzzy approach to multi-criteria decision-making in mining practice. Whittle and Borzorgebrahimi (2004) tested the set theory for a hybrid pits model, where the pits model, derived from Lerchs-Grossmann algorithm and conditional simulation were linked. Such approach reduce uncertainty at the early stages of mine development, where the data related to orebody characteristic are usually presented imprecisely. Kawalec (2005, 2008) creates the concept of fuzzy model of an ultimate lignite pit also with the use of the Lerchs-Grossmann algorithm. A similar concept was developed by Niedbał (2014) in relation to the open pit mine for the unspecified porphyry copper ore deposit. Yujić et al. (2011) present a deterministic fuzzy linear optimisation model of mine production for various mining systems. Mine operations planning has been presented by Brzychczy et. al (2014) in case of coal mine and by Gligoric et al. (2015) in relation to lead-zinc ore mine. Pham (1997) uses fuzzy set tools in a quality assessment of iron ore. Tutmez (2007) applies uncertainty-oriented fuzzy methodology to the estimation of an ore reserve. Elmas & Sahin (2013) present reasoning about barite ore grades based on a fuzzy neural network. The use of fuzzy numbers and fuzzy modelling for quantitative evaluation of solid mineral resources is the subject of scientific investigations by Bárdossy et al. (2003) and Tutmez et al. (2007). Similar considerations concerning evaluation of a resource base have been attempted by Luo & Dimitrakopoulos (2003) for tin skarn deposits, Tahmasebi and Hezarkhani (2010) in relation to porphyry copper deposits, and Tutmez & Dag (2007) in the context of lignite coal deposits. Bárdossy & Fodor (2005) identify the importance of fuzzy arithmetic in assessing the completeness and usefulness of exploratory work.

Papers by Bárdossy et al. (1988, 1990) in the field of geostatistics, related to shortages of exploration data, can also be useful for geological-mining applications. Such gaps occur frequently in geological and mining practice and can be compensated with the use of fuzzy logic tools.

2. Evaluation of deposit grade parameters in the activities of KGHM mines

The copper-silver ores of the Fore-Sudetic Monocline (SW Poland) are among the largest deposits in the world. They are classified among strata bound within the Zechstein sedimentary series. These objects, which are not homogeneous, include Permian sandstones, bitumen-carbonaceous marly shales, dolomites or marls of Central Europe, Carboniferous Bunter Sandstone in Kazakhstan, Precambrian metamorphic rocks in Siberia or Brazil, Precambrian metamorphosed sandstones and shales and dolomites of the Copperbelt in central Africa, and sandstones of assorted ages in the US and Canada. Deposits in the Fore-Sudetic Monocline, which show great continuity and adherence to the same geological layers, are divided into mining areas within which operations are being carried out. The boundaries of these separated deposits are contractual and run independently of lithological boundaries or gaps in mineralisation; their horizontal course is determined partially according to administrative reasons and deposit tectonics. Extraction of minerals is being carried out by three mines: Lubin, Rudna and Polkowice-Sieroszowice.

The deposit is characterised by imprecise boundaries determined by the breakeven grade of equivalent copper (Cu_e) in the deposit profile according to the viability criteria of 0.5% for balance resources (Fig. 1). In the contour thus described it occupies an area of over 40 km and is slightly more than 10 km wide, covering an area of over 400 km^2. After the dip marking the boundary of the deposit, regardless of Cu_e content, the agreed limit of available technical support at a depth of 1250 meters has been adopted.

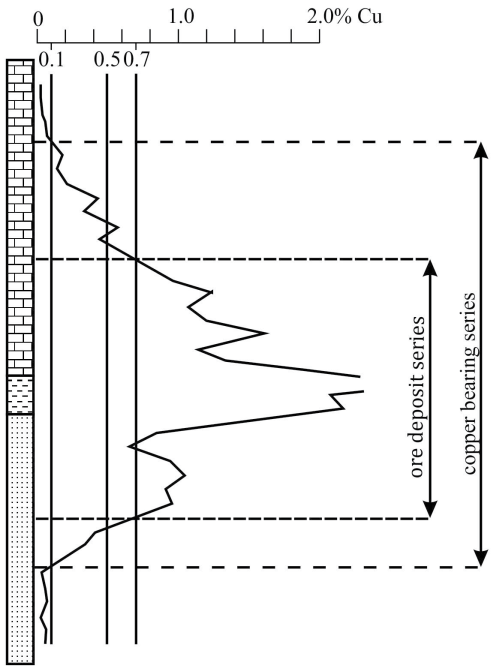

Fig. 1. Heterogeneity of copper content in the deposit profile (according to Nieć, 1997, updated)

In the vertical profile the zone of balanced copper content includes the following lithostratigraphic links (from top):

- dolomite 0-4 m
- shale (Kupferschiefer) 0-1.2 m
- sandstone 0.5-20 m.

The largest area is covered by a sandstone series. Depending on its form, the thickness of the deposit varies from 0.4 to 26 m. The lowest value (about 1 m) is reached where the deposit occurs only in copper shale and clay dolomite underlain by red sandstone, as well as in marginal zones of the deposit area. If ore mineralisation includes all three types of rock (sandstone, shale, and dolomite) and the deposit has a regular form, its thickness is typically 3-6 meters, increasing to more than 10 m within the elevation of the sandstone roof. The maximum thickness found was 26 m (Nieć, 1997). However, over such a wide range, low and medium thicknesses dominate. The minimum thickness occurs in the south-western area in the Polkowice and Sieroszowice deposits, where the ore mineralisation is confined essentially to Kupferschiefer and clay dolomite. Deposit thickness in this zone is negligible and usually does not exceed 2 m, and over a considerable deposit area is only 0.4-1 m (Nieć & Piestrzyński, 1996). The mineralisation is discontinuous and creates lens-nest forms. In the case of full profile mineralisation the deposit form becomes more complex.

The main components of useful ores are copper and silver. As mentioned above, isolines of 0.5% Cu content define the boundaries of the deposit. The copper content in the ore depends on the lithological type of ore and distance between the mineralisation zone and the sandstone roof. Among the lithological types of ore, the highest metal contents are recorded within the ore shale (Kupferschiefer), where they vary from 2-3% to more than 10%. The remaining lithological types of ore mineralisation are several times lower and the average content in sandstone and carbonate rocks is in the range of 1-3% copper. Silver content is much more varied, in the range 30-70 g/Mg. Again, the richest is Kupferschiefer, in which silver concentrations reach hundreds of grams per ton. The thickness of Kupferschiefer has a critical influence on the average content of copper and silver.

The Polkowice and Sieroszowice deposits, where the relative share of Kupferschiefer is higher than in other mines despite the generally lower total deposit thickness, result in high average contents of copper.

Mines using development galleries lead to the identification of ore series. The primary purpose of development works is evaluation of geological-mining conditions as well as of the value of minerals so as to enable profitable excavation. With regard to geological and mining law and other requirements, the mining entrepreneur is obliged to conduct so-called rational mineral deposit management. The degree to which this task is carried out is verified by mining geological surveys, along with the appropriate mining supervisory organs. There is a great and fully justified necessity for the mine operator to acquire the necessary geological information to carry out 'safe' mining work in the following areas: delimitation of the roof and bottom of the deposit, identification of geological-mining conditions (faults, the main directions of fracture, coverage of barren zones, geotechnical parameters of surrounding rocks, potential natural hazards, etc.). Along with geological information related to ore quality parameters, it is possible to qualify particular parts of the deposit for economically justified operation.

In the initial stages of development work, the geologist possesses only point information, derived from a drilled hole at the initial stage of deposit exploration. These generalised deposit parameters are accepted for a certain parcel, using e.g. the Bołdyriew polygon method (Fig. 2). Depending on the sampling result, such a block is qualified as positive (in economic terms), subeconomic, or barren. Basic deposit parameters defined in this and subsequent stages comprise thickness, copper content, and reserve volume.

Starting from an initial, single observation point, drill hole S-102 (shown in Fig. 2), the data derived therefrom are extrapolated to the entire polygon X1, to which the following initial grade parameters are assigned: deposit thickness – 2.5 m, copper content – 2.87%, yield – 165 kg/m^2. The necessity for detailed geological information requires further geological exploration with the use of heading and development excavations. In the present case, this is represented by a cluster of galleries in a triple arrangement (galleries X-120, X-121, X-122, Fig. 2). The aim of heading and development is the identification of geological forms and structures and natural hazards, the acquisition of geological information related to deposit parameters, and, finally, preparation of initial excavations for mining. An entrepreneur incurring financial expenditure on development work expects results at least similar to the parameters obtained from the drill hole, because this information at this stage is equivalent to the value of the work, which he wishes to confirm. At the moment of the implementation of heading and development work there is, at a geologist's disposal, initially information derived from the channel sample taken in three places, where the development galleries begin, and also from the Bołdyriew polygon. In the Polish classification of mineral resources, such a polygon is estimated to be in the C_1 category, i.e. the resources are initially identified with an accuracy of ±30%. This category corresponds to the Indicated Resources class in CRIRSCO nomenclature. As a result of the further identification of block X1

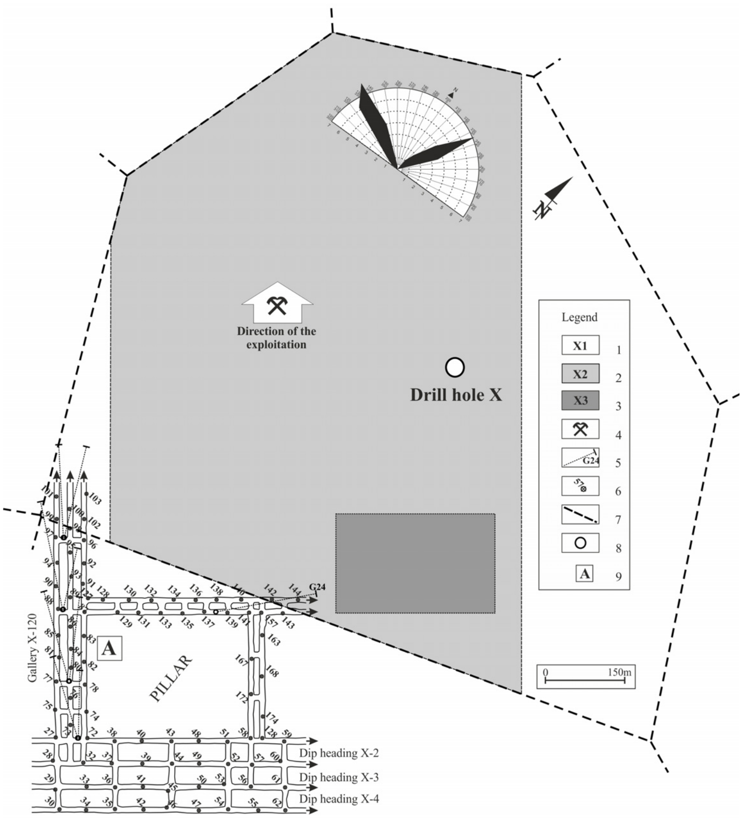

Fig. 2. Part of the deposit covered by a Bołdyriew polygon: 1 – block X1; 2 – block X2; 3 – block X3; 4 – direction of exploitation; 5 – geological drill; 6 – channel sample number; 7 – borders of the Bołdyriew polygon; 8 – surface drill; 9 – part of mining development galleries

within mining works, additional geological information is obtained. In the analysed example there were 1,711 side channel samples (Table 1). The input of new data enabled reclassification of block X1 to X2 with detailed accuracy at levels B (±15% accuracy) and A (±5-10% accuracy). For simplicity, all smaller blocks (Fig. 2) should be considered in the category A+B. This separation corresponds to the proved reserves class in CRIRSCO code. Separation of block X2 corresponds to separation of the operating (exploitation) unit (block). Exploitation blocks may include more than one Bołdyriew polygon, and thus can refer to several master blocks, not only to X1 as was shown in the cited example. Geological blocks refer only to deposit management and are distinguished according to geological criteria. These blocks are not linked to the mining blocks identified for technological and organisational reasons. Fields, blocks, or exploitation floors are designated in order to improve the organisational activities of the mining operation. Returning to the starting point related to estimation of block parameters, 1,090 geological samples were used in the area X2 (Table 1). Within block X2 three further, smaller parts (e.g. X3) were selected, corresponding to the geological block. Grade parameters for block X3 were calculated using 166 geological samples (Table 1).

TABLE 1

List of grade parameters in individual deposit blocks

Block, area [m²]	Lithology	Number of geological samples	Thickness [m]	Copper content Cu [%]	Silver and lead content Ag [g/Mg]/ Pb [%]	Copper yield [kg/m²]	Equivalent copper yield [kg/m²]
Drill hole S-102 1,079,573	average	1	1.96	1.68	25/-	85.48	98.0
	carbonates		1.77	1.74	25/-		
	shale		0.19	1.17	25/-		
Block X1 1,079,573	average	1,711	1.40	3.10	35.51/0.08	112.0	124.8
	carbonates		1.09	2.47	31.47/0.095		
	shale		0.31	5.41	50.32/0.021		
Without stone samples	average	1,554	1.54	3.1	36.38/0.08	123.2	137.6
	carbonates		1.20	2.46	32.38/0.095		
	shale		0.34	5.41	51.1/0.021		
Without sub-economic samples	average	1,511	1.56	3.10	36.81/0.07	124.8	139.6
	carbonates		1.22	2.47	33.41/0.08		
	shale		0.34	5.45	49.45/0.02		
Block X2 745,889	average	1,090	1.44	3.07	39.3/0.09	114.1	128.7
	carbonates		1.14	2.51	34.69/0.111		
	shale		0.31	5.24	57.46/0.019		
Without stone samples	average	980	1.61	3.07	40.79/0.09	127.5	144.5
	carbonates		1.27	2.51	36.22/-		
	shale		0.34	5.24	58.65/0.019		
Without sub-economic samples	average	953	1.64	3.07	42.14/0.07	129.9	147.7
	carbonates		1.30	2.52	38.52/0.087		
	shale		0.34	5.26	56.49/0.015		
Block X3 54,575	average	166	1.79	3.05	21.97/0.01	140.9	151.0
	carbonates		1.48	2.65	20.13/0.006		
	shale		0.31	4.99	31.04/0.011		

A geologist supervising the progress of mining work, based on his or her own knowledge and experience, adjusts the sampling grid (in accordance with the provisions of the Geological Works Project, licences and applicable normative acts) and the method of controlling the height of the exploitation gate, with the objective of optimal recognition of the ore deposit series. In the case of deposits with low or average variation coefficients, which are usually not possible to quantify at this stage due to the small number of observations, a square grid in the range of 20 to 30 m between samples is assumed. Geological samples are taken directly from the mine face or side walls of excavations galleries using the point method in a linear system. Usually, the sampling grid is adapted to the technical works project of the deposit mine workings. Deposit is excavated with the chamber and pillar systems, and geological samples are taken in every second chamber, and their spatial grid is determined by the dimensions of the technological pillars adopted in the technical project. Such a system allows to obtain a regular sampling grid, which can be concentrated in case of sudden ore declining, barren zone appearing or high variability of the ore deposit. The average waiting time for the results of the sampling assays is three days;

the maximum daily rate of progress of exploitation can range from 3.5 to 7 m, which means that a geologist receives information related to deposit parameters at the moment when he should be getting the next sample. It follows that an erroneous decision by the geologist made as a result of observation of macroscopic ore in the endgate while waiting for the assay results can lead to improper qualification of ore as barren rock or of barren rock as ore. The way to minimise the risk of such mistakes is elimination of human error and entrusting the supervision of the work to an experienced geologist or enforcing more rapid analysis of collected samples.

3. Deposit parameters descriptions using the fuzzy logic concept

Fuzzy logic uses several elementary concepts: linguistic variable, linguistic value, linguistic term set, universe of discourse, fuzzy set, membership function, and grade of membership. The most important of these concepts are function and grade of membership. Contrary to classical set theory, a fuzzy set A is an object comprising non-empty elements of universe X ($A \subseteq X$) whereby each of its elements may be fully included, be a fuzzy member, or be completely excluded (Zadeh, 1965). In mathematical notation, fuzzy set A is a set of ordered pairs $A = \{(x.\mu_A(x)); x \in X\}$, where $\mu_A : X \rightarrow [0.1]$ is called a membership function of fuzzy set A. Taking into account three cases of membership, the following situations are possible:

1. $\mu_A(x) = 1$, the element is fully included in fuzzy set A.
2. $\mu_A(x) = 0$, the element is not included in fuzzy set A.
3. $0 < \mu_A(x) < 1$, the element is a fuzzy member of fuzzy set A.

Using the fuzzy approach. one can define an example of a variable and a linguistic value, e.g. ‘rich copper ore’. Assuming that rich ore should contain at least 1.5% metal, then, according to classical set logic, ore containing less than 1.5% copper would not be considered rich (membership function equals ‘0’). This is not a satisfactory solution because a content of 1.45% copper, being slightly less than 1.5%, thus would likely be considered ‘poor ore’. Where should we draw the line? A satisfactory answer to that question is confusing, due to the conversion from a descriptive to a quantitative measure. In fuzzy logic, metal content in ore of less than 1.5% copper does not rule out its classification in the category ‘rich ore’. According to fuzzy reasoning, all ore is rich, but falls within a different grade of membership, as illustrated in Fig. 3. The grade of membership for ore with metal content of less than 1% copper is 0, while in the range of 1 to 1.5% the grade of membership in the class ‘rich ore’ is defined at greater than zero. In this approach, a hypothetical ore content of 1.3% copper is rich at the grade 0.6. The higher the copper content of the ore, the higher, obviously, the grade of membership.

The qualifications described above can be implemented in a mixed manner, with fuzzy definition of the limit using fuzzy numbers. This number, for the analysed example, is about 1.5% Cu. Again, there is no clear separation of classes (Fig. 4). In this model, the ore content of 1.3% copper is described using the membership function $\mu = 0.3$, and thus lower, which is consistent with intuitive selection of the imprecise determination of the border between sets.

Use of fuzzy sets and their membership functions enables the presentation of imprecise statements and data. With the help of fuzzy operators, data conversion is possible. The fundamental operations used in the context of fuzzy sets are sum (*MAX*), product (*MIN*), and complement.

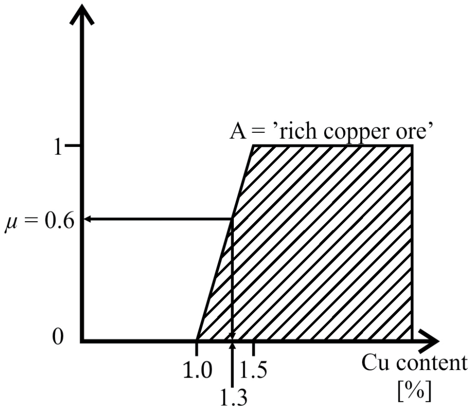

Fig. 3. Ore membership function for the 'rich ore' set category

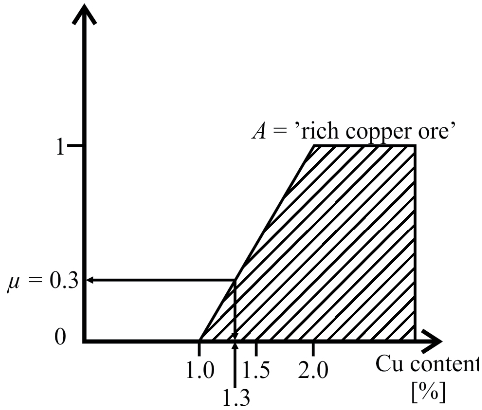

Fig. 4. Ore membership function in the 'rich ore' set category defined using fuzzy numbers

The sum of fuzzy sets A and B is the fuzzy set $A \cup B$, with membership function defined as:

$$\Lambda_{x \in X} \mu_{A \cup B}(x) = \max[\mu_A(x),\ \mu_B(x)] \tag{1}$$

The product of fuzzy sets A and B is fuzzy set $A \cap B$, with the membership function defined as:

$$\Lambda_{x \in X} \mu_{A \cap B}(x) = \min[\mu_A(x),\ \mu_B(x)] \tag{2}$$

The complement of fuzzy set A is fuzzy set A', with the membership function defined as:

$$\Lambda_{x \in X} \mu_{A'}(x) = 1 - \mu_A(x) \tag{3}$$

Operations as defined above do not always accurately reflect the intuitive properties of operations on fuzzy sets. For example if $\mu_A(x) \le \mu_B(x)$, the intersection of sets with such membership functions is equal to $\mu_A(x)$, no matter how large the function $\mu_B(x)$ is. To avoid such an inconvenience, the adjustable and non-adjustable operators s-norm (sum) and t-norm (intersection) are used. The operators applied later in the article are presented in Tables 2 and 3.

The calculation of s- and t-norm operators was tested for the description of the ore grade parameters copper content and deposit thickness. Both linguistic variables, 'copper content' and 'deposit thickness', are defined as membership functions in Fig. 5.

Both copper content and deposit thickness are subject to continuous monitoring during ongoing mining operations. Copper content is a significant parameter from the processing point of view, determining the efficiency and economics of the processing. It also affects the quality of the obtained concentrate and influences sales revenue. Deposit thickness is one of the key parameters determining the appropriate mine operating system. Selection of a suitable system translates directly into the costs of the extraction process. Qualification of copper content in the ore and deposit thickness (Fig. 5a) was carried out by a mine geologist (an expert in the field) using three meanings and the membership functions assigned to them. Copper content of the ore of less than 1.3% was considered 'low', about 1.9% 'medium', and more than 1.9% 'high'. Fig. 5a

TABLE 2

Selected non-adjustable s-norm operators

maximum (*MAX*)	$\Lambda_{x\in X}\mu_{A\cup B}(x)=\max[\mu_A(x),\ \mu_B(x)]$
algebraic union	$\Lambda_{x\in X}\mu_{A\cup B}(x)=\mu_A(x)+\mu_B(x)-\mu_A(x)\cdot\mu_B(x)$
Hamacher union	$\Lambda_{x\in X}\mu_{A\cup B}(x)=\dfrac{\mu_A(x)+\mu_B(x)-2\mu_A(x)\cdot\mu_B(x)}{1-\mu_A(x)\cdot\mu_B(x)}$
Einstein union	$\Lambda_{x\in X}\mu_{A\cup B}(x)=\dfrac{\mu_A(x)+\mu_B(x)}{1+\mu_A(x)\cdot\mu_B(x)}$
bounded union	$\Lambda_{x\in X}\mu_{A\cup B}(x)=\dfrac{\mu_A(x)+\mu_B(x)-2\mu_A(x)\cdot\mu_B(x)}{1-\mu_A(x)\cdot\mu_B(x)}$

TABLE 3

Selected non-adjustable and adjustable t-norm operators

minimum (*MIN*)	$\Lambda_{x\in X}\mu_{A\cap B}(x)=\min[\mu_A(x),\ \mu_B(x)]$
intersection (*PROD*)	$\Lambda_{x\in X}\mu_{A\cap B}(x)=\mu_A(x)\cdot\mu_B(x)$
Hamacher product	$\Lambda_{x\in X}\mu_{A\cap B}(x)=\dfrac{\mu_A(x)\cdot\mu_B(x)}{\mu_A(x)+\mu_B(x)-\mu_A(x)\cdot\mu_B(x)}$
Einstein product	$\Lambda_{x\in X}\mu_{A\cap B}(x)=\dfrac{\mu_A(x)\cdot\mu_B(x)}{2-(\mu_A(x)+\mu_B(x)-\mu_A(x)\cdot\mu_B(x))}$
bounded difference	$\Lambda_{x\in X}\mu_{A\cap B}(x)=\max[0.\mu_A(x)+\mu_B(x)-1]$
Dubois-Prade intersection	$\Lambda_{x\in X}\mu_{A\cap B}(x.\alpha)=\dfrac{\mu_A(x)\cdot\mu_B(x)}{\max[\mu_A(x).\mu_A(x).\alpha]},\alpha\in[0.1]$

also shows that a copper content of 1.6% is ($\mu = 0.25$) 'low' and 'medium' to the same degree, and content of 2.1% 'medium' and 'high' to the same degree ($\mu = 0.33$). Similar reasoning applied to the thickness parameter (Fig. 5b) leads to the conclusion that a thickness of 1.33 m is equivalently 'low' and 'medium' at grade $\mu = 0.33$, while a thickness of 3.75 m is both 'medium' and 'high' at membership grade $\mu = 0.5$. Due to the fuzzy delimitation of classes in thickness and copper content it is possible, according to fuzzy logic, to carry out different s- and t-norm operations.

Figure 6 illustrates the selected t-norm operators used to determine the membership function of copper content for the fuzzy set 'low copper content AND medium copper content'. Definition of this set according to classical logic would be impossible.

The *MIN* operator is the most optimistic in relation to the decision criteria. The copper content of 1.6% is equally low and medium here for the grade of membership $\mu = 0.25$. For the remaining operators, lower values of membership function were obtained. This means that the rules of metal content qualification for a set of 'low AND medium copper content' are sharper and require a higher grade of fulfilment for both low and medium copper content in a fuzzy product.

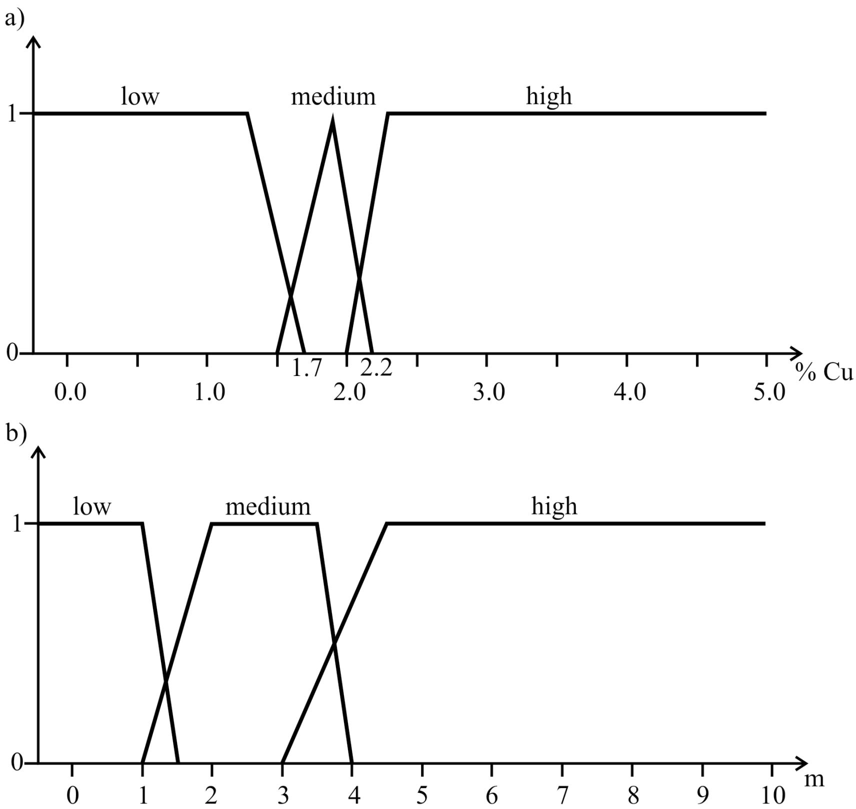

Fig. 5. Significance of the terms; a) low, medium, and high copper content, b) low, medium, and high deposit thickness

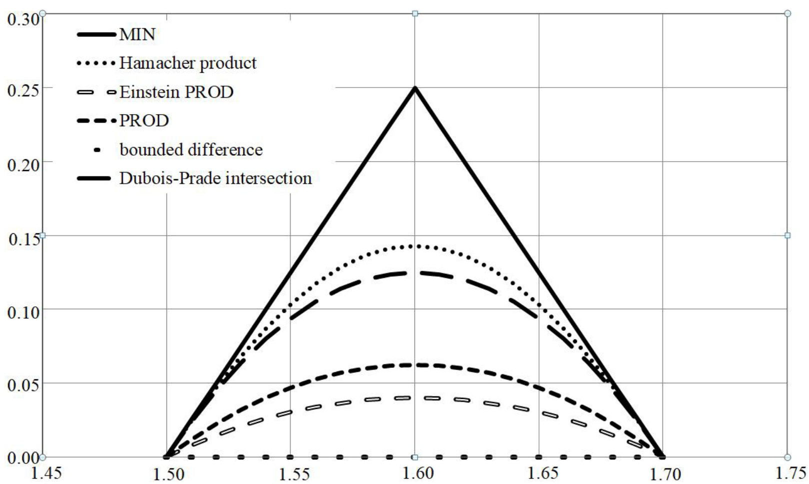

Fig. 6. Membership functions for the fuzzy set 'low AND medium copper content'

An extreme case is represented by a bounded difference operator for which the membership function reaches a constant value $\mu = 0$.

Functioning of *s*-norm operators is presented in Fig. 7, which shows the fuzzy set 'medium OR high deposit thickness'.

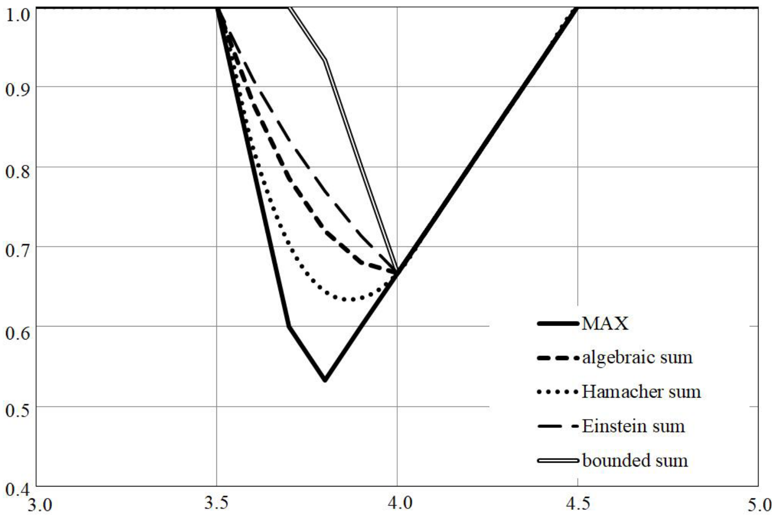

Fig. 7. Membership functions for the fuzzy set 'medium OR high deposit thickness'

In the present approach, based on the union of fuzzy sets, the relationship is the reverse of that of the product. The most optimistic is the bounded-sum operator. For a thickness of 3.8 m it is either medium or high with the grade of membership $\mu = 0.93$, while for the *MAX* operator this grade is $\mu = 0.53$.

In practical use, fuzzy operators were tested for six exploitation blocks in one of the KGHM PC mines. In blocks (corresponding to the X3 block mentioned in Chapter 2) symbolically marked with letters A to F, ore samples were taken from the mine face or side walls in the rhombic system of regular grid with the 24 m parallel interval and 14 m orthogonal interval (Fig. 8).

Average copper content and deposit thickness are compiled in Table 4. Blocks qualified for extraction must be 'rich' in terms of copper content as well as 'thick' in terms of deposit thickness.

TABLE 4

Average deposit parameters in exploitation blocks

Block	Copper content, [%]	Thickness, [m]
A	2.01	4.5
B	2.12	3.6
C	3.11	2.6
D	2.05	3.1
E	2.21	4.2
F	2.25	3.3

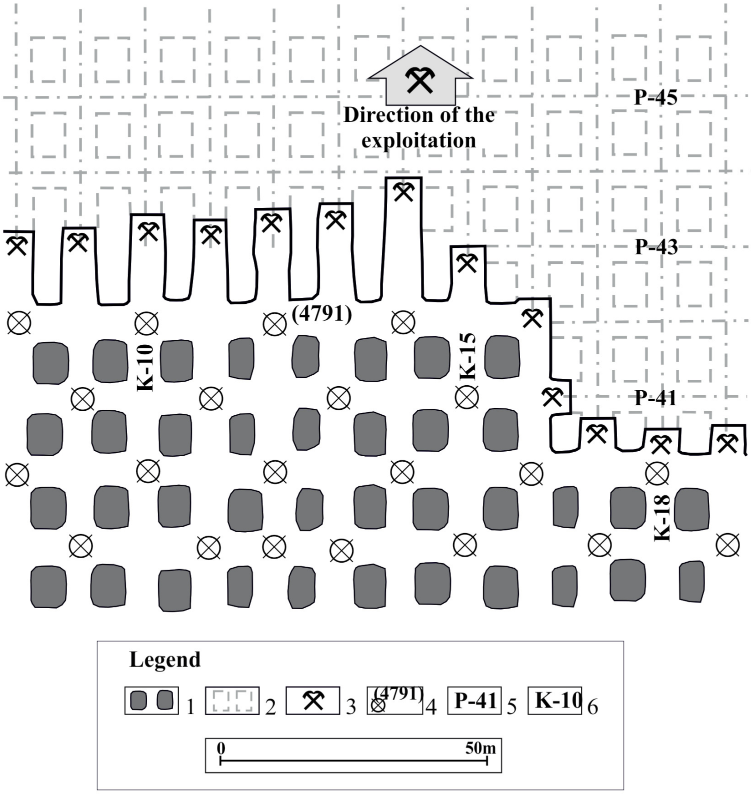

Fig. 8. Fragment of an exploitation block with a scheme of sampling grid; 1 – technological pillars in an exploitation block, 2 – technological pillars in technical project of mine workings, 3 – mine faces, 4 – samples location (exemplary sample number), 5 – technological zones, 5 – chambers

Using the membership function defining high deposit thickness and high copper content (Fig. 7), appropriate grades of membership have been defined (Table 5).

TABLE 5

Grade of membership for deposit parameters in exploitation blocks

Block	Copper content, [%]	Thickness, [m]
A	0.03	1.00
B	0.40	0.40
C	1.00	0.00
D	0.18	0.07
E	0.70	0.80
F	0.83	0.20

Qualification of the exploitation blocks for extraction is based on two *t*-norm fuzzy operators, *MIN* and Einstein product. Ranking of the blocks is summarised in Table 6.

TABLE 6

Exploitation block ranking according to chosen *t*-norm operators

Block	*MIN*	Block	Einstein product
E	0.7000	E	0.5283
B	0.4000	F	0.1461
F	0.2000	B	0.1176
D	0.0700	A	0.0300
A	0.0300	D	0.0067
C	0.0000	C	0.0000

This results in a slightly different classification of exploitation blocks. In the case of the selection of the *MIN* operator, block B occupies the second position, while for the Einstein product operator it occupies the third position. As mentioned above, the *MIN* operator is more optimistic, while for the Einstein product operator it is necessary to meet both requirements simultaneously at a higher grade. An interesting consideration is that, assuming a greater deposit thickness and copper content in the blocks, the result of the *MIN* operator remains unchanged, conditioned by the lower membership function of copper content or deposit thickness. The decision-making process does not always occur in the way thus presumed. Decision-making is usually done under certain environmental conditions, which define the state of the factors influencing the decisions and are independent of the decision-maker. Additionally, the decision-maker may make his or her choice depending on his or her mood, preferences, inclination to incur risks, etc. Analysis of this type of behaviour resulted in the introduction of compensation operators (Zimmermann & Zysno, 1980), which combine the *AND* operator to a greater or lesser extent with the *OR* operator. Zimmermann & Zysno (1980) suggest an *Iγ* operator, defined as the intersection of sets:

$$\mu_A = \left(\prod_{i=1}^{m} \mu_{Ai} \right)^{(1-\gamma)} \left[1 - \prod_{i=1}^{m} \left(1 - \mu_{Ai} \right) \right]^{\gamma} \tag{4}$$

where:

γ — grade of compensation. $0 \le \gamma \le 1$;
μ_A — grade of fulfilment of all premises. $A = A_1 \cap A_2 \cap \ldots \cap A_m$;
μ_{Ai} — grade of fulfilment of component premises.

Compensation operators were calculated for the exploitation blocks, assuming the step range of the grade of compensation. The results are shown in Table 7.

Ranking of blocks which satisfies the condition of high deposit thickness and high copper content differs with respect to the *MIN* operator and replicates the scheme of the Einstein product operator. Application of the compensatory operator better reflects the complexity of the decision-making process, taking into account the influence and importance of both deposit parameters.

Also, in the case of a single exploitation block (e.g. B), a higher deposit thickness would result in a change in the ranking position of the units qualified for extraction (Table 8). With the increasing value of the membership function for deposit thickness, the coefficients indicate the need to position block B in an ever-higher position in the ranking. Similar reasoning can be applied in relation to the other blocks and their parameters.

TABLE 7

Values of compensation operators in exploitation blocks

Block	Grade of compensation			
	0.1	**0.2**	**0.3**	**0.4**
A	0.0426	0.0605	0.0859	0.1220
B	0.1838	0.2111	0.2425	0.2786
C	0.0000	0.0000	0.0000	0.0000
D	0.0169	0.0227	0.0304	0.0408
E	0.5898	0.6211	0.6541	0.6889
F	0.1958	0.2309	0.2723	0.3211

TABLE 8

Values of compensation operator for block B with membership function for copper content $\mu = 0.4$ and variable membership function of thickness

Thickness μ	Grade of compensation					
	0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**
0.4	0.1838	0.2111	0.2425	0.2786	0.3200	0.3676
0.5	0.2267	0.2569	0.2912	0.3301	0.3742	0.4241
0.6	0.2693	0.3022	0.3391	0.3806	0.4271	0.4792
0.7	0.3118	0.3471	0.3865	0.4303	0.4792	0.5335
0.8	0.3541	0.3917	0.4334	0.4796	0.5307	0.5871
0.9	0.3962	0.4362	0.4801	0.5284	0.5817	0.6403
1.0	0.4383	0.4804	0.5265	0.5771	0.6324	0.6931

4. Conclusions

Precise description of reality is a difficult and complex task. In geological mining activity it bears particular importance, because mineral deposits (natural objects) are characterised by considerable uncertainty regarding their real parameters. Full assessment and knowledge of a mineral deposit's form and structure and all of its features is possible, as some experts joke, only when the total resources have been depleted. This, of course, is associated with mine closure and the completion of mining activities. Unfortunately, this is an *ex post* assessment, useless for operational mine activity. Wherever it is necessary to define parameters in a descriptive way, the use of fuzzy set tools appears to be justified and facilitates the characterisation of the object.

The considerations presented in this paper were selected from many possible decision-making problems in the operational management of an ore mineral deposit. Some of the proposed approximations of deposit parameters using the membership function may be debatable; nevertheless, the reasoning based on fuzzy logic seems to be useful in and of itself. The fuzzy approach was presented in the context of a mineral deposit description in which a verbal description of deposit parameters was converted into the precise language of mathematics. Additionally, the utility of fuzzy logical operators was demonstrated. Testing of the fuzzy reasoning usefulness was based on simplified examples. Calculations were based on virtual data. However ,the real usability of the fuzzy approach in a qualitative assessment of mineral deposit parameters requires its testing and confirmation in the practice of geological-mining activity. The questions about the density of sampling grid and samples locations seem to be important during a mineral deposit model construction. Information about deposit parameters in an exploitation block comes mainly from side walls of a mine contouring a mine parcel, and the distance between samples may affect the assessment of fuzzy parameters. Perhaps it would be advisable to gather information from inside of the selected block. Mentioned doubts indicate that the fuzzy set theory tools may be useful in addition to, for example, geostatistical methods. The presented outline of the fuzzy logic operators use is only a preliminary step towards the construction of a more complex fuzzy decision-making model. In that model, taking into consideration a number of assumptions and limitations, the final solution would be evaluation of the ore deposit parameters using the fuzzy reasoning.

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References

Bandopadhyay S., 1987. *Fuzzy algorithm for decision making in mining engineering*. *Short Communication*. International Journal of Mining and Geological Engineering **5**, 149-154.

Bárdossy A., Bogardi I., Kelly W.E., 1988. *Imprecise (Fuzzy) Information in Geostatistics*. Mathematical Geology **20**, 4, 287-311.

Bárdossy A., Bogardi I., Kelly W.E., 1990. *Kriging with Imprecise (Fuzzy) Variograms. I: Theory*. Mathematical Geology **22**, 1, 63-79.

Bárdossy G., Fodor J., 2005. *Assessment of the Completeness of Mineral Exploration by the Application of Fuzzy Arithmetic and Prior Information*. Acta Polytechnica Hungarica **2**, 1, 15-31.

Bárdossy G., Szabó R., Varga G., 2003. *A New Method of Resource Estimation for Bauxite and Other Solid Mineral Deposits*. Journal Of Hungarian Geomathematics **1**, 14-26.

Brzychczy E., Kęsek M., Napieraj A., Sukiennik M., 2014. *The Use of Fuzzy Systems in the Designing of Mining Process in Hard Coal Mines*. Archives of Mining Sciences **59**, 3, 741-760.

Demicco R., Klir G., (Eds.) 2003. Fuzzy Logic in Geology. Academic Press, Cambridge USA.

Elmas N., Sahin U., 2013. *Computation of grade values of sediment-hosted barite deposits in north-eastern Isparta (western Turkey)*. Turkish Journal of Earth Sciences **23**, 1-13.

Gligoric Z., Beljic C., Gluscevic B., Cvijovic C., 2015. *Underground Lead-Zinc Mine Production Planning Using Fuzzy Stochastic Inventory Policy*. Archives of Mining Sciences **60**, 1, 73-92.

Kawalec W., 2005. *Koncepcja rozmytego modelu wyrobiska docelowego*. Szkoła Ekonomiki i Zarządzania w Górnictwie 2005. Krynica, 14-16 września 2005. Wydział Górnictwa i Geoinżynierii AGH, Kraków 289-302.

Kawalec W., 2008. *Fuzzy modeling of an Ultimate Pit*. Economic Evaluation and Risk Analysis of Mineral Projects. International Mining Forum, Taylor & Francis, London, 95-106.

Kesimal A., Bascetin A., 2002. *Application of fuzzy multiple attribute decision making in mining operations*. Mineral Resources Engineering **11**, 59-72.

Luo X., Dimitrakopoulos R., 2003. *Data-Driven Fuzzy Analysis in Quantitative Mineral Resource Assessment*. Computers & Geosciences **29**, 3-13.

Nieć M., 1997. *Złoża rud miedzi i srebra*. R. Ney (Ed.), Surowce mineralne Polski. Surowce metaliczne; Miedź, srebro. Wydawnictwo Centrum Podstawowych Problemów Gospodarki Surowcami Mineralnymi i Energią Polskiej Akademii Nauk, Kraków, 9-44.

Nieć M., Piestrzyński A., 1996. *Geologia*. A. Piestrzyński (Ed.), Monografia KGHM „Polska Miedź". CBPM „Cuprum", Lubin, 75-390.

Niedbał M., 2014. *Koncepcja zastosowania teorii zbiorów rozmytych do analizy projektów kopalni odkrywkowej złóż polimetalicznych*. Zeszyty Naukowe Instytutu Gospodarki Surowcami Mineralnymi i Energią Polskiej Akademii Nauk **86**, 55-68.

Pham T.D., 1997. *Grade Estimation Using Fuzzy-Set Algorithms*. Mathematical Geology **29**, 2, 291-305.

Tahmasebi P., Hezarkhani A., 2010. *Application of Adaptive Neuro-Fuzzy Inference System for Grade Estimation; Case Study, Sarcheshmeh Porphyry Copper Deposit, Kerman, Iran*. Australian Journal of Basic and Applied Sciences **4**, 3, 408-420.

Tutmez B., 2007. *An uncertainty oriented fuzzy methodology for grade estimation*. Computers & Geosciences **33**, 280-288.

Tutmez B., Dag A., 2007. *Use of Fuzzy Logic in Lignite Inventory Estimation. Energy Sources. Part B: Economics*. Planning and Policy **2**, 1, 93-103.

Tutmez B., Tercan A.E., Kaymak U., 2007. *Fuzzy Modeling for Reserve Estimation Based on Spatial Variability*. Mathematical Geology **39**, 1, 87-111.

Whittle D., Borzorgebrahimi A., 2004. *Hybrid pits – Linking conditional simulation and Lerchs-Grossman through set theory*. Orebody Modelling and Strategic Mine Planning. 22-24 November 2004, Perth 399-404.

Yujić S., Miljanović I., Kuzmanović M., Bartulović Z., Gajić G., Lazić P., 2011. *The Deterministic Fuzzy Linear Approach in Planning the Production of Mine System with Several Open Pits*. Archives of Mining Sciences **56**, 3, 489-497.

Zadeh L., 1965. *Fuzzy sets*. Information and Control **8**, 338-353.

Zimmermann H-J., Zysno P., 1980. *Latent connectives in human decision making*. Fuzzy Sets and Systems **4**, 1, 37-51.