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Optimization of Operation and Safety of Baltic Port and Shipping Critical Infrastructure Network with Considering Climate-Weather Change Influence – Minimizing Critical Infrastructure Network Operation Cost

Keywords

critical infrastructure network, operation process, climate-weather change process, operation cost, optimization, port, shipping

Abstract
The paper is devoted the optimization of operation process and minimization of operation cost for Baltic Port and Shipping Critical Infrastructure Network (BPSCIN) at variable operation conditions related to the climate-weather change. For this network, the optimal transient probabilities that minimize the mean value of the total operation costs are found. Finally, cost analysis of BPSCIN operation impacted by climate-weather change is presented in case the BPSCIN is non-repairable and in case it is repairable after exceeding its critical safety state.

1. Introduction

Various types of costs are associated with each business activity. The highest one are related to operation process in complex technical systems. Therefore, it is important that the system is optimal and works reliable as long as possible. Achieving this goal is possible based on research in reliability modelling [Kołowrocki, 2014] with considering of the impact of its operation process [Soszyńska, 2007]. The well-known results for safety and operation process of complex technical systems are given in [Kołowrocki, 2014], [Kołowrocki, Soszyńska-Budny, 2011], [Soszyńska, 2007]. The equally important problem is to ensure the safety of complex technical systems including its operation process. One of the approach to investigate the complex technical systems safety including its operation process is the usage of the semi-Markov process models [Grabski, 2015], [Kołowrocki, Soszyńska-Budny, 2011], [Soszyńska, 2007]. Very often, this complex systems are important for societies' daily functioning. Therefore are considered as the critical infrastructures or the critical infrastructure networks.

Due to the progressing climate change, it has become a significant problem to consider their impact on the safety of critical infrastructures. The operation process related to the climate-weather change can be described in the same way as operation process [Kołowrocki et al., 2017a].These two processes have influence to safety of critical infrastructures and critical infrastructure networks. This influence can be described by the general safety model of the multistate critical infrastructure network changing its safety structure and its components safety parameters during variable operation process [Kołowrocki, Soszyńska-Budny, 2011, 2014, 2012a-b] and at different climate-weather states of the critical infrastructure operating area [Kołowrocki, et al., 2017b-c].

The main aim of the paper is the optimization of operation and safety lifetime of Baltic Port and Shipping Critical Infrastructure Network (BPSCIN) defined in [Guze, Kołowrocki, 2017a,b], it is possible to find the optimal values of limit transient probabilities of BPSCIN operation process related to the climate-weather change, to minimize the mean value of the BPSCIN operation cost. Furthermore, the BPSCIN optimal critical operation cost function

and the optimal moment when its operation cost exceeds a permitted level can be found.
It is done based on the combination of the models of operation process and climate-weather change process and the linear programming [Klabjan, Adelman, 2006], [Kołowrocki, Soszyńska-Budny, 2011].

2. Baltic Port and Shipping Critical Infrastructure Network Operation Process Related to Climate-Weather Change Process

Taking into account the results obtained in [Kołowrocki, et al., 2017c], we consider the impact of the operation process related to the climate-weather change process $ZC(t)$, $t \in <0,\infty)$, in a various way at this process states zc_{bl}, $\upsilon = 1,2,3$, $b = 1^{(\upsilon)},2^{(\upsilon)},...,v^{(\upsilon)}$, $l = 1^{(\upsilon)},2^{(\upsilon)},...,w^{(\upsilon)}$, on Baltic Port and Shipping Critical Infrastructure Network. Furthermore, according to results given in [Kołowrocki, et al., 2017c], we assume that the changes of the states of operation process related to the climate-weather change process $ZC(t)$, $t \in <0,\infty)$, at BPSCIN operating area have an influence on its safety structure and on the safety of its assets A_i, $i = 1,2,...,n^{(\upsilon)}$, $\upsilon = 1,2,3$, as well.
Taking into account results in [Kołowrocki, et al., 2017b-c], we assume, that Baltic Port and Shipping Critical Infrastructure Network during its operation process is taking $V = v^{(1)} \cdot v^{(2)} \cdot v^{(3)}, V \in N$, different operation states $z_1, z_2, \ldots, z_V$. Moreover, we define the BPSCIN operation process $Z(t)$, $t \in <0,+\infty)$, with discrete operation states from the set $\{z_1, z_2, \ldots, z_V\}$. In the next step, we make the assumption that we have either calculated analytically or evaluated approximately by experts the vector of limit values of transient probabilities of the Baltic Port and Shipping Critical Infrastructure Network operation process $Z(t)$ at the particular operation states z_b, $\upsilon = 1,2,3$, $b = 1^{(\upsilon)},2^{(\upsilon)},...,v^{(\upsilon)}$.
We also assume that the climate-weather change process $C(t)$, $t \in <0,+\infty)$, at the BPSCIN operating area is taking $W = w^{(1)} \cdot w^{(2)} \cdot w^{(3)}, W \in N$, different climate-weather states $c_1, c_2, \ldots, c_W$ and its is defined in [Kołowrocki, et al., 2017c]. The vector of limit values of transient probabilities of the climate-weather change process $C(t)$ at the particular climate-weather states c_l, $\upsilon = 1,2,3$, $l = 1^{(\upsilon)},2^{(\upsilon)},...,w^{(\upsilon)}$, is defined similarly to [Kołowrocki, et al., 2017c].

According to these assumptions about the Baltic Port and Shipping Critical Infrastructure Network operation process $Z(t)$, $t \in <0,+\infty)$, and the climate-weather change process $C(t)$, we introduce the Baltic Port and Shipping Critical Infrastructure Network joint process of operation process and climate-weather change process called the joint Baltic Port and Shipping Critical Infrastructure Network operation process related to climate-weather change marked by $ZC(t)$, $t \in <0,+\infty)$, and we assume that it can take $V \cdot W, V, W \in N$, different operation states related to the climate-weather change $zc_{11}, zc_{12}, \ldots, zc_{VW}$. Next, we make the assumption that the Baltic Port and Shipping Critical Infrastructure Network operation process related to climate-weather change $ZC(t)$, at the moment $t \in <0,+\infty)$, is at the state zc_{bl}, $\upsilon = 1,2,3$, $b = 1^{(\upsilon)},2^{(\upsilon)},...,v^{(\upsilon)}$, $l = 1^{(\upsilon)},2^{(\upsilon)},...,w^{(\upsilon)}$, if and only if at that moment, the operation process $Z(t)$ is at the operation states z_b, $\upsilon = 1,2,3$, $b = 1^{(\upsilon)},2^{(\upsilon)},...,v^{(\upsilon)}$, and the climate-weather change process $C(t)$ is at the climate-weather state c_l, $\upsilon = 1,2,3$, $l = 1^{(\upsilon)},2^{(\upsilon)},...,w^{(\upsilon)}$, what we express as follows:

$$(ZC(t) = zc_{bl}) \Leftrightarrow (Z(t) = z_b \cap C(t) = c_l),$$
$$t \in <0,+\infty),\ \upsilon = 1,2,3,\ b = 1^{(\upsilon)},2^{(\upsilon)},...,v^{(\upsilon)},$$
$$l = 1^{(\upsilon)},2^{(\upsilon)},...,w^{(\upsilon)}. \quad (1)$$

The transient probabilities of the BPSCIN operation process related to climate-weather change $ZC(t)$ at the operation states zc_{bl}, $b = 1,2,...,v$, $l = 1,2,...,w$, are defined in [Kołowrocki, et al., 2017c]:

$$pq_{bl}(t) = P(ZC(t) = zc_{bl}),\ t \in <0,+\infty),\ \upsilon = 1,2,3,$$
$$b = 1^{(\upsilon)},2^{(\upsilon)},...,v^{(\upsilon)},\ l = 1^{(\upsilon)},2^{(\upsilon)},...,w^{(\upsilon)}. \quad (2)$$

Hence the limit values of the transient probabilities of the joint Baltic Port and Shipping Critical Infrastructure Network operation process related to climate-weather change $ZC(t)$ at the operation states zc_{bl}, $\upsilon = 1,2,3$, $b = 1^{(\upsilon)},2^{(\upsilon)},...,v^{(\upsilon)}$, $l = 1^{(\upsilon)},2^{(\upsilon)},...,w^{(\upsilon)}$, are given as

$$pq_{bl} = \lim_{t \to \infty} pq_{bl}(t),\ \upsilon = 1,2,3,$$
$$b = 1^{(\upsilon)},2^{(\upsilon)},...,v^{(\upsilon)},\ l = 1^{(\upsilon)},2^{(\upsilon)},...,w^{(\upsilon)}, \quad (3)$$

and under assumption that the processes $Z(t)$ and $C(t)$ are independent, they can be found from [Kołowrocki, et al., 2017b-c] according to formula

$$pq_{bl} = p_b q_l,\ \upsilon = 1,2,3,\ b = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., v^{(\upsilon)},\ l = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., w^{(\upsilon)}, \quad (4)$$

where p_b, $\upsilon = 1,2,3$, $b = 1,2,...,v^{\upsilon}$, are the limit transient probabilities of the operation process *Z*(*t*) at the particular operation states z_b, $\upsilon = 1,2,3$, $b = 1,2,...,v^{\upsilon}$, and q_l, $\upsilon = 1,2,3$, $l = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., w^{(\upsilon)}$, are the limit transient probabilities of the climate-weather change process *C*(*t*) at the particular climate-weather startes c_l, $\upsilon = 1,2,3$, $l = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., w^{(\upsilon)}$.

Other interesting characteristics of the joint Baltic Port and Shipping Critical Infrastructure Network operation process $ZC_{bl}(t)$ are its total sojourn times $\hat{\theta}\hat{C}_{bl}$, $\upsilon = 1,2,3$, $b = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., v^{(\upsilon)}$, $l = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., w^{(\upsilon)}$, at the particular operation states zc_{bl}, $\upsilon = 1,2,3$, $b = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., v^{(\upsilon)}$, $l = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., w^{(\upsilon)}$, during the fixed sufficiently large Baltic Port and Shipping Critical Infrastructure Network operation time θ. They have approximately normal distributions with the expected values given by

$$\hat{M}\hat{N}_{bl} = E[\hat{\theta}\hat{C}_{bl}] = pq_{bl}\theta, \quad (5)$$

where pq_{bl}, $\upsilon = 1,2,3$, $b = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., v^{(\upsilon)}$, $l = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., w^{(\upsilon)}$, are defined by (3) and given by (4) in the case the processes *Z*(*t*) and *C*(*t*) are independent.

3. Optimization of Operation and Safety of BPSCIN

3.1. BPSCIN Operation Cost Related to Climate-Weather Change Process

We may introduce the instantaneous operation cost of the Baltic Port and Shipping Critical Infrastructure Network impacted by the operation process $ZC_{bl}(t)$, $t \in <0,\infty)$, related to the climate-weather change process in the form of vector

$$\boldsymbol{K}^4(t,\cdot) = [1,\ \boldsymbol{K}^4(t,1),\ ...,\ \boldsymbol{K}^4(t,z)],\ t \in <0,\infty), \quad (6)$$

with the coordinates given by

$$\boldsymbol{K}^4(t,u) \cong \sum_{b=1}^{v^{(\upsilon)}} \sum_{l=1}^{w^{(\upsilon)}} pq_{bl} [\boldsymbol{K}^4(t,u)]^{(bl)} \text{ for } t \geq 0,\ u = 1,2,...,z,$$

where pq_{bl}, $\upsilon = 1,2,3$, $b = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., v^{(\upsilon)}$, $l = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., w^{(\upsilon)}$, are the limit transient probabilities at the states zc_{bl}, $\upsilon = 1,2,3$, $b = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., v^{(\upsilon)}$, $l = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., w^{(\upsilon)}$, of the operation process ZC(t), $t \in <0,\infty)$, related to the climate-weather change defined by (3) and $[\boldsymbol{K}^4(t,u)]^{(bl)}$, $u = 1,2,...,z$, $\upsilon = 1,2,3$, $b = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., v^{(\upsilon)}$, $l = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., w^{(\upsilon)}$, are the coordinates of BPSCIN conditional instantaneous operation costs in the safety state subsets $\{u, u+1, ..., z\}$, $u = 1,2,...,z$, impacted by the operation process *ZC*(*t*), $t \in <0,\infty)$, related to the climate-weather change process at the states zc_{bl}, $\upsilon = 1,2,3$, $b = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., v^{(\upsilon)}$, $l = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., w^{(\upsilon)}$, defined in the form of the vector

$$[\boldsymbol{K}^4(t,\cdot)]^{(bl)} = [1,\ [\boldsymbol{K}^4(t,1)]^{(bl)},\ ...,\ [\boldsymbol{K}^4(t,z)]^{(bl)}],\ t \in <0,\infty),\ \upsilon = 1,2,3,\ b = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., v^{(\upsilon)},\ l = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., w^{(\upsilon)}. \quad (8)$$

The dependency (7) can also be clearly expressed in the linear equation for the mean value of the Baltic Port and Shipping Critical Infrastructure Network total unconditional operation costs in the safety state subsets $\{u, u+1, ..., z\}$, $u = 1,2,...,z$,

$$\overline{\boldsymbol{K}}^4(u) \cong \sum_{b=1}^{v^{(\upsilon)}} \sum_{l=1}^{w^{(\upsilon)}} pq_{bl} [\overline{\boldsymbol{K}}^4(u)]^{(bl)},\ u = 1,2,...,z, \quad (9)$$

where pq_{bl}, $\upsilon = 1,2,3$, $b = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., v^{(\upsilon)}$, $l = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., w^{(\upsilon)}$, are the limit transient probabilities at the states zc_{bl}, $\upsilon = 1,2,3$, $b = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., v^{(\upsilon)}$, $l = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., w^{(\upsilon)}$, of the operation process ZC(t), $t \in <0,\infty)$, related to the climate-weather change defined by (3) and $[\overline{\boldsymbol{K}}^4(u)]^{(bl)}$, $u = 1,2,...,z$, $\upsilon = 1,2,3$, $b = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., v^{(\upsilon)}$, $l = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., w^{(\upsilon)}$, are the mean values of the Baltic Port and Shipping Critical Infrastructure Network total conditional instantaneous operation costs in the safety state subsets $\{u, u+1, ..., z\}$, $u = 1,2,...,z$, at the operation states zc_{bl}, $\upsilon = 1,2,3$, $b = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., v^{(\upsilon)}$, $l = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., w^{(\upsilon)}$, defined by

$$[\overline{\boldsymbol{K}}^4(u)]^{(bl)} = \int_0^{[\mu^4(u)]^{(bl)}} [\boldsymbol{K}^4(t,u)]^{(bl)} dt,$$
$$u=1,2,...,z,\ \upsilon=1,2,3,\ b=1^{(\upsilon)},2^{(\upsilon)},...,v^{(\upsilon)},$$
$$l=1^{(\upsilon)},2^{(\upsilon)},...,w^{(\upsilon)}, \qquad (10)$$

where $[\mu^4(r)]^{(bl)}$, $u=1,2,...,z$, $\upsilon=1,2,3$, $b=1^{(\upsilon)},2^{(\upsilon)},...,v^{(\upsilon)}$, $l=1^{(\upsilon)},2^{(\upsilon)},...,w^{(\upsilon)}$, are the mean values of the Baltic Port and Shipping Critical Infrastructure Network conditional lifetimes $[T^4(u)]^{(bl)}$, $u=1,2,...,z$, $\upsilon=1,2,3$, $b=1^{(\upsilon)},2^{(\upsilon)},...,v^{(\upsilon)}$, $l=1^{(\upsilon)},2^{(\upsilon)},...,w^{(\upsilon)}$, in the safety state subset $\{u,u+1,...,z\}$ at the Baltic Port and Shipping Critical Infrastructure Network operating process related to the climate-weather change state zc_{bl}, $\upsilon=1,2,3$, $b=1^{(\upsilon)},2^{(\upsilon)},...,v^{(\upsilon)}$, $l=1^{(\upsilon)},2^{(\upsilon)},...,w^{(\upsilon)}$, given according to (9) and [Kołowrocki K., et al., 2017b-c] by the following way

$$[\mu^4(u)]^{(bl)} = \int_0^{\infty} [\boldsymbol{S}^4(t,u)]^{(bl)} dt,\ u=1,2,...,z,\ \upsilon=1,2,3,$$
$$b=1^{(\upsilon)},2^{(\upsilon)},...,v^{(\upsilon)},\ l=1^{(\upsilon)},2^{(\upsilon)},...,w^{(\upsilon)}, \qquad (11)$$

and $[\boldsymbol{S}^4(t,u)]^{(bl)}$, $u=1,2,...,z$, $\upsilon=1,2,3$, $b=1^{(\upsilon)},2^{(\upsilon)},...,v^{(\upsilon)}$, $l=1^{(\upsilon)},2^{(\upsilon)},...,w^{(\upsilon)}$, are the coordinates of BPSCIN impacted by the operation process related to the climate-weather change process $ZC(t)$, $t \in < 0,\infty)$, conditional safety functions [Kołowrocki K., et al., 2017b].
Besides, the mean values of the Baltic Port and Shipping Critical Infrastructure Network total conditional instantaneous operation costs in the safety state subsets $\{u,u+1,...,z\}$, $u=1,2,...,z$, at the operation states zc_{bl}, $\upsilon=1,2,3$, $b=1^{(\upsilon)},2^{(\upsilon)},...,v^{(\upsilon)}$, $l=1^{(\upsilon)},2^{(\upsilon)},...,w^{(\upsilon)}$, can be alternatively defined for the Baltic Port and Shipping Critical Infrastructure Network fixed operation time θ by

$$[\overline{\boldsymbol{K}}^4(u)]^{(bl)} = \int_0^{\hat{M}\hat{N}} [\boldsymbol{K}^4(t,u)]^{(bl)} dt,\ u=1,2,...,z,$$
$$\upsilon=1,2,3,\ b=1^{(\upsilon)},2^{(\upsilon)},...,v^{(\upsilon)},$$
$$l=1^{(\upsilon)},2^{(\upsilon)},...,w^{(\upsilon)}, \qquad (12)$$

where $\hat{M}\hat{N}_{bl}$, $\upsilon=1,2,3$, $b=1^{(\upsilon)},2^{(\upsilon)},...,v^{(\upsilon)}$, $l=1^{(\upsilon)},2^{(\upsilon)},...,w^{(\upsilon)}$, are the mean values of the total sojourn times $\hat{\theta}\hat{C}_{bl}$, $\upsilon=1,2,3$, $b=1^{(\upsilon)},2^{(\upsilon)},...,v^{(\upsilon)}$, $l=1^{(\upsilon)},2^{(\upsilon)},...,w^{(\upsilon)}$, at the particular operation states zc_{bl}, $\upsilon=1,2,3$, $b=1^{(\upsilon)},2^{(\upsilon)},...,v^{(\upsilon)}$, $l=1^{(\upsilon)},2^{(\upsilon)},...,w^{(\upsilon)}$, during the fixed sufficiently large Baltic Port and Shipping Critical Infrastructure Network operation time θ, determined by (5).

3.2. Minimization of Baltic Port and Shipping Critical Infrastructure Network Operation Cost Related to Climate-Weather Change

We take into account the linear equation (30 and based on this formula, we can see that the mean value of the Baltic Port and Shipping Critical Infrastructure Network total unconditional operation cost $\overline{\boldsymbol{K}}^4(u)$, $u=1,2,...,z$, is determined by the limit values of transient probabilities pq_{bl}, $\upsilon=1,2,3$, $b=1^{(\upsilon)},2^{(\upsilon)},...,v^{(\upsilon)}$, $l=1^{(\upsilon)},2^{(\upsilon)},...,w^{(\upsilon)}$, of the Baltic Port and Shipping Critical Infrastructure Network operation process at the operation states given by (8.3) and the mean values of its total conditional operation costs $[\overline{\boldsymbol{K}}^4(u)]^{(bl)}$, $u=1,2,...,z$, $\upsilon=1,2,3$, $b=1^{(\upsilon)},2^{(\upsilon)},...,v^{(\upsilon)}$, $l=1^{(\upsilon)},2^{(\upsilon)},...,w^{(\upsilon)}$, at Baltic Port and Shipping Critical Infrastructure Network operating process related to the climate-weather change process zc_{bl}, $\upsilon=1,2,3$, $b=1^{(\upsilon)},2^{(\upsilon)},...,v^{(\upsilon)}$, $l=1^{(\upsilon)},2^{(\upsilon)},...,w^{(\upsilon)}$, given by (9).
Therefore, the BPSCIN total unconditional operation cost optimization approach based on the linear programming [Klabjan, Adelman, 2006], [Kołowrocki, Soszyńska-Budny, 2011] can be proposed. It means, that we may look for the corresponding optimal values $\dot{p}q_{bl}$, $\upsilon=1,2,3$, $b=1^{(\upsilon)},2^{(\upsilon)},...,v^{(\upsilon)}$, $l=1^{(\upsilon)},2^{(\upsilon)},...,w^{(\upsilon)}$, of the limit transient probabilities pq_{bl}, $\upsilon=1,2,3$, $b=1^{(\upsilon)},2^{(\upsilon)},...,v^{(\upsilon)}$, $l=1^{(\upsilon)},2^{(\upsilon)},...,w^{(\upsilon)}$, of Baltic Port and Shipping Critical Infrastructure Network operation process at the operation states zc_{bl}, $\upsilon=1,2,3$, $b=1^{(\upsilon)},2^{(\upsilon)},...,v^{(\upsilon)}$, $l=1^{(\upsilon)},2^{(\upsilon)},...,w^{(\upsilon)}$, to minimize the mean value $\overline{\boldsymbol{K}}^4(u)$, $u=1,2,...,z$, of the Baltic Port and Shipping Critical Infrastructure Network total unconditional operation cost in the safety state subsets $\{u,u+1,...,z\}$, $u=1,2,...,z$, under the assumption that the mean values $[\overline{\boldsymbol{K}}^4(u)]^{(bl)}$, $u=1,2,...,z$, $\upsilon=1,2,3$, $b=1^{(\upsilon)},2^{(\upsilon)},...,v^{(\upsilon)}$, $l=1^{(\upsilon)},2^{(\upsilon)},...,w^{(\upsilon)}$, of the Baltic Port and Shipping Critical Infrastructure Network total conditional operation costs in the safety state subsets $\{u,u+1,...,z\}$, $u=1,2,...,z$, at the operation

states zc_{bl}, $\upsilon = 1,2,3,$ $b = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., v^{(\upsilon)}$, $l = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., w^{(\upsilon)}$, are fixed.

As a special case of the above formulation of the Baltic Port and Shipping Critical Infrastructure Network total unconditional operation cost optimization problem, if r, $r = 1,2,...,z$, is the Baltic Port and Shipping Critical Infrastructure Network critical safety state, we want to find the optimal values $\dot{p}q_{bl}$, $\upsilon = 1,2,3,$ $b = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., v^{(\upsilon)}$, $l = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., w^{(\upsilon)}$, of the BPSCIN operation process limit transient probabilities pq_{bl}, $\upsilon = 1,2,3,$ $b = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., v^{(\upsilon)}$, $l = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., w^{(\upsilon)}$, at the operation states zc_{bl}, $\upsilon = 1,2,3,$ $b = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., v^{(\upsilon)}$, $l = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., w^{(\upsilon)}$, to minimize the mean value $\overline{\boldsymbol{K}}^4(r)$ of Baltic Port and Shipping Critical Infrastructure Network total unconditional operation cost in the safety state subset $\{r, r+1, ..., z\}$, under the assumption that the mean values $[\boldsymbol{\mu}^4(r)]^{(bl)}$, $\upsilon = 1,2,3,$ $b = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., v^{(\upsilon)}$, $l = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., w^{(\upsilon)}$, of the Baltic Port and Shipping Critical Infrastructure Network total conditional operation costs in the safety state subset $\{r, r+1, ..., z\}$ at the operation states zc_{bl}, $\upsilon = 1,2,3,$ $b = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., v^{(\upsilon)}$, $l = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., w^{(\upsilon)}$, are fixed. To solve this problem, we formulate the optimization problem for Baltic Port and Shipping Critical Infrastrucutre Network as a linear programming model with the objective function given by

$$\overline{\boldsymbol{K}}^4(r) \cong \sum_{b=1}^{v^{(\upsilon)}} \sum_{l=1}^{w^{(\upsilon)}} pq_{bl} [\overline{\boldsymbol{K}}^4(r)]^{(bl)}, \qquad (13)$$

for a fixed $r \in \{1,2,...,z\}$ and with the following bound constraints

$$\breve{p}q_{bl} \le pq_{bl} \le \hat{p}q_{bl}, \ \upsilon = 1,2,3, \ b = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., v^{(\upsilon)}, \ l = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., w^{(\upsilon)}, \qquad (14)$$

$$\sum_{b=1}^{v^{(\upsilon)}} \sum_{l=1}^{w^{(\upsilon)}} pq_{bl} = 1, \qquad (15)$$

where

$$[\overline{\boldsymbol{K}}^4(r)]^{(bl)}, \ [\overline{\boldsymbol{K}}^4(r)]^{(bl)} \ge 0, \ \upsilon = 1,2,3, \ b = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., v^{(\upsilon)}, \ l = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., w^{(\upsilon)}, \qquad (16)$$

are fixed mean values of BPSCIN conditional lifetimes in the safety state subset $\{r, r+1, ..., z\}$ and

$\breve{p}q_{bl}$, $0 \le \breve{p}q_{bl} \le 1$ and $\hat{p}q_{bl}$, $0 \le \hat{p}q_{bl} \le 1$,

$\breve{p}q_{bl} \le \hat{p}q_{bl}$, $\upsilon = 1,2,3,$ $b = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., v^{(\upsilon)}$, $l = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., w^{(\upsilon)}$,

are lower and upper bounds of the transient probabilities pq_{bl}, $\upsilon = 1,2,3,$ $b = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., v^{(\upsilon)}$, $l = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., w^{(\upsilon)}$, respectively.

Now, we can obtain the optimal solution of the formulated by (13)-(17) the linear programming problem using the procedure presented in [Kołowrocki, et al., 2017c].

It means, that we can find the optimal values $\dot{p}q_{bl}$, $\upsilon = 1,2,3,$ $b = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., v^{(\upsilon)}$, $l = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., w^{(\upsilon)}$, of the transient probabilities pq_{bl}, $\upsilon = 1,2,3,$ $b = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., v^{(\upsilon)}$, $l = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., w^{(\upsilon)}$, that minimize the mean value of the Baltic Port and Shipping Critical Infrastructure Network unconditional operation cost in the safety state subset $\{r, r+1, ..., z\}$, defined by the linear form (13), giving its minimum value in the following form

$$\dot{\overline{\boldsymbol{K}}}^4(r) \cong \sum_{b=1}^{v^{(\upsilon)}} \sum_{l=1}^{w^{(\upsilon)}} \dot{p}q_{bl} [\overline{\boldsymbol{K}}^4(r)]^{(bl)} \qquad (18)$$

for a fixed $r \in \{1,2,...,z\}$.

Thus, considering (6), the coordinates of the optimal instantaneous the Baltic Port and Shipping Critical Infrastructure Network operation cost in the form of the vector

$$\dot{\boldsymbol{K}}^4(t,\cdot) = [1, \ \dot{\boldsymbol{K}}^4(t,1), \ ..., \ \dot{\boldsymbol{K}}^4(t,z)], \ t \in <0,\infty),$$

are given by

$$\dot{\boldsymbol{K}}^4(t,u) \cong \sum_{b=1}^{v^{(\upsilon)}} \sum_{l=1}^{w^{(\upsilon)}} \dot{p}q_{bl} [\boldsymbol{K}^4(t,u)]^{(bl)} \ \text{for} \ t \ge 0, \ u = 1,2,...,z, \qquad (19)$$

where $\dot{p}q_{bl}$, $\upsilon = 1,2,3,$ $b = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., v^{(\upsilon)}$, $l = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., w^{(\upsilon)}$, are the optimal limit transient probabilities at the states zc_{bl}, $\upsilon = 1,2,3,$ $b = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., v^{(\upsilon)}$, $l = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., w^{(\upsilon)}$, of the operation process ZC(t), $t \in <0,\infty)$, related to the climate-weather change defined by (6.26) and $[\boldsymbol{K}^4(t,u)]^{(bl)}$, $u = 1,2,...,z,$ $\upsilon = 1,2,3,$ $b = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., v^{(\upsilon)}$, $l = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., w^{(\upsilon)}$, are the coordinates of the Baltic Port and Shipping Critical Infrastructure Network conditional instantaneous operation costs in the safety state subsets $\{u, u+1, ..., z\}$, $u = 1,2,...,z,$ impacted by the operation process $ZC(t)$, $t \in <0,\infty)$, related to the climate-weather change process at the states zc_{bl}, $\upsilon = 1,2,3$, $b = 1^{(\upsilon)}, 2^{(\upsilon)}, \dots, v^{(\upsilon)}$, $l = 1^{(\upsilon)}, 2^{(\upsilon)}, \dots, w^{(\upsilon)}$, defined in the form of the vector

$$[\boldsymbol{K}^4(t,\cdot)]^{(bl)} = [1, [\boldsymbol{K}^4(t,1)]^{(bl)}, \dots, [\boldsymbol{K}^4(t,z)]^{(bl)}],$$
$t \in <0,\infty)$, $\upsilon = 1,2,3$, $b = 1^{(\upsilon)}, 2^{(\upsilon)}, \dots, v^{(\upsilon)}$, $l = 1^{(\upsilon)}, 2^{(\upsilon)}, \dots, w^{(\upsilon)}$.

Replacing in (19) *r* by *u*, we get the expessions for the optimal mean values of the Baltic Port and Shipping Critical Infrastructure Network unconditional operation costs in the safety state subset $\{u, u+1, \dots, z\}$, $u = 1,2,\dots,z$, giving its minimum value in the following form

$$\dot{\bar{\boldsymbol{K}}}^4(u) \cong \sum_{b=1}^{v^{(\upsilon)}} \sum_{l=1}^{w^{(\upsilon)}} \dot{p}q_{bl} [\bar{\boldsymbol{K}}^4(u)]^{(bl)}, \ \{u, u+1, \dots, z\},$$
$$u = 1,2,\dots,z.$$
(20)

The optimal solutions for the mean values of the Baltic Port and Shipping Critical Infrastructure Network unconditional operation costs in the particular safety states are expressed by

$$\dot{\bar{\bar{K}}}^4(u) = \dot{\bar{K}}(u) - \dot{\bar{K}}(u+1), \ u = 1,\dots,z-1,$$
$$\dot{\bar{\bar{K}}}^4(z) = \dot{\bar{K}}(z),$$
(21)

where $\dot{\bar{\boldsymbol{K}}}^4(u)$, $u = 1,2,\dots,z$, are given by (20).
Moreover, if we define the corresponding critical operation cost function by

$$\boldsymbol{K}^4(t) = \boldsymbol{K}^4(t,r) \ \ t \ge 0, \tag{22}$$

and the moment ζ when the Baltic Port and Shipping Critical Infrastructure Network operation cost exceeds a permitted level κ, by

$$\zeta = \boldsymbol{K}^{4^{-1}}(\kappa), \tag{23}$$

where $\boldsymbol{K}^4(t,r)$ is given by (6) for $u = r$ and $\boldsymbol{K}^{4^{-1}}(\kappa)$ is the inverse function of the Baltic Port and Shipping Critical Infrastructure Network cost function $\boldsymbol{K}^4(t)$ given by (22), then the corresponding optimal critical operation cost function is given by

$$\dot{\boldsymbol{K}}^4(t) = \dot{\boldsymbol{K}}^4(t,r) \quad t \ge 0, \tag{24}$$

then the optimal moment $\dot{\zeta}$ when the Baltic Port and Shipping Critical Infrastructure Network operation cost exceeds a permitted level κ, is given by

$$\dot{\zeta} = \dot{\boldsymbol{K}}^{4^{-1}}(\kappa), \tag{25}$$

where $\dot{\boldsymbol{K}}^4(t,r)$ is given by (6.28) in [Kołowrocki et al., 2017b-c] for $u = r$ and $\dot{\boldsymbol{K}}^{4^{-1}}(\kappa)$, if it exists, is the inverse function of the optimal critical operation cost function $\dot{\boldsymbol{K}}^4(t)$ given by (20).

3.3. Cost Analysis of Baltic Port and Shipping Critical Infrastructure Network Operation Impacted by Climate-Weather Change

We consider the Baltic Port and Shipping Critical Infrastructure Network as the multistate critical infrastructure network, consisted of n components in its operation process $ZC(t)$, $t \in <0,\infty)$, related to climate-weather change. Furthermore, we assume that the operation costs of the Baltic Port and Shipping Critical Infrastructure Network single basic components at the operation state zc_{bl}, $\upsilon = 1,2,3$, $b = 1^{(\upsilon)}, 2^{(\upsilon)}, \dots, v^{(\upsilon)}$, $l = 1^{(\upsilon)}, 2^{(\upsilon)}, \dots, w^{(\upsilon)}$, during its operation time θ, $\theta \ge 0$, amount $[K_i(\theta)]^{(bl)}$, $\upsilon = 1,2,3$, $b = 1^{(\upsilon)}, 2^{(\upsilon)}, \dots, v^{(\upsilon)}$, $l = 1^{(\upsilon)}, 2^{(\upsilon)}, \dots, w^{(\upsilon)}$, $i = 1,2,\dots,n$.

We take into three cases, when the Baltic Port and Shipping Critical Infrastructure Network is:

1. non-reparaible,
2. repariable with ignored time of renovation,
3. repariable with non-ignored time of renovation.

Firstly, we suppose that the Baltic Port and Shipping Critical Infrastructure Network is non-repairable and during the operation time θ, $\theta \ge 0$, it has not exceeded the critical safety state r. In this case, the total cost of the non-repairable BPSCIN during the operation time θ, $\theta \ge 0$, is given by

$$K(\theta) \cong \sum_{b=1}^{v^{(\upsilon)}} \sum_{l=1}^{w^{(\upsilon)}} pq_{bl} \sum_{i=1}^{n} [K_i(\theta)]^{(bl)}, \ \theta \ge 0, \tag{26}$$

were pq_{bl}, $\upsilon = 1,2,3$, $b = 1^{(\upsilon)}, 2^{(\upsilon)}, \dots, v^{(\upsilon)}$, $l = 1^{(\upsilon)}, 2^{(\upsilon)}, \dots, w^{(\upsilon)}$, are transient probabilities defined by (3).

Next, we consider case when we additionally assume that Baltic Port and Shipping Critical Infrastructure Network is repairable after exceeding the critical safety state r and its renovation time is ignored and the cost of its single renovation is constant and equal to K_{ign}.

In this case, the total operation cost of the repairable Baltic Port and Shipping Critical Infrastructure Network with ignored its renovation time during the operation time θ, $\theta \geq 0$, amounts

$$K_{ign}(\theta) \cong \sum_{b=1}^{v^{(\upsilon)}} \sum_{l=1}^{w^{(\upsilon)}} pq_{bl} \sum_{i=1}^{n} [K_i\ (\theta)]^{(bl)} + K_{ign} H(\theta, r),$$
$$\theta \geq 0, \tag{27}$$

were pq_{bl}, $\upsilon = 1,2,3$, $b = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., v^{(\upsilon)}$, $l = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., w^{(\upsilon)}$, are transient probabilities defined by (3) and $H(\theta, r)$ is the mean value of the number of exceeding the critical safety state r by the Baltic Port and Shipping Critical Infrastructure Network operating at the variable conditions during the operation time θ defined by (3.58) in [Kołowrocki, Soszyńska-Budny, 2011] and in [Guze, Kołowrocki, 2017].

Now, we assume that Baltic Port and Shipping Critical Infrastructure Network is repairable after exceeding the critical safety state r and its renewal time is non-ignored and have distribution function with the mean value $\mu_0(r)$ and the standard deviation $\sigma_0(r)$ and the cost of its single renovation is K_{n-ign}.

In this case, the total operation cost of the repairable BPSCIN with not ignored its renovation time during the operation time θ, $\theta \geq 0$, amounts

$$K_{n-ign}(\theta) \cong \sum_{b=1}^{v^{(\upsilon)}} \sum_{l=1}^{w^{(\upsilon)}} pq_{bl} \sum_{i=1}^{n} [K_i\ (\theta)]^{(bl)} + K_{n-ign} \overline{\overline{H}}(\theta, r),$$
$$\theta \geq 0,$$
(28)

were pq_{bl}, $\upsilon = 1,2,3$, $b = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., v^{(\upsilon)}$, $l = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., w^{(\upsilon)}$, are transient probabilities defined by (3) and $\overline{\overline{H}}(\theta, r)$ is the mean value of the number of renovations of Baltic Port and Shipping Critical Infrastructure Network operating at the variable conditions during the operation time θ defined with accordance to (3.92) in [Kołowrocki, Soszyńska-Budny, 2011] and to [Guze, Kołowrocki, 2017].

The particular expressions for the mean values $H(\theta, r)$ and $\overline{\overline{H}}(\theta, r)$ for the repairable Baltic Port and Shipping Critical Infrastructure Network with ignored and non-ignored renovation times respectively defined by (3.58) and (3.92) in [Kołowrocki, Soszyńska-Budny, 2011], are determined in Chapter 3 in [Kołowrocki, Soszyńska-Budny, 2011] for typical multistate repairable critical infrastructure operating at the variable operation conditions and in [Guze, Kołowrocki, 2017].

After the optimization of the Baltic Port and Shipping Critical Infrastructure Network operation process related to climate-weather change, the Baltic Port and Shipping Critical Infrastructure Network operation total costs given by (26)-(28) assume their optimal values.

The total optimal cost of the non-repairable Baltic Port and Shipping Critical Infrastructure Network during the operation time θ, $\theta \geq 0$, after its operation process related to climate-weather change optimization is given by

$$\dot{K}(\theta) \cong \sum_{b=1}^{v^{(\upsilon)}} \sum_{l=1}^{w^{(\upsilon)}} \dot{p}q_{bl} \sum_{i=1}^{n} [K_i\ (\theta)]^{(bl)}, \ \ \theta \geq 0, \tag{29}$$

were $\dot{p}q_{bl}$, $\upsilon = 1,2,3$, $b = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., v^{(\upsilon)}$, $l = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., w^{(\upsilon)}$, are optimal transient probabilities defined by (19).

The optimal total operation cost of the repairable Baltic Port and Shipping Critical Infrastructure Network with ignored its renovation time during the operation time θ, $\theta \geq 0$, after its operation process related to climate-weather change optimization amounts from (7.40) as follows

$$\dot{K}_{ign}(\theta) \cong \sum_{b=1}^{v^{(\upsilon)}} \sum_{l=1}^{w^{(\upsilon)}} \dot{p}q_{bl} \sum_{i=1}^{n} [K_i\ (\theta)]^{(bl)} + K_{ign} \dot{H}(\theta, r),$$
$$\theta \geq 0, \tag{30}$$

were $\dot{p}q_{bl}$, $\upsilon = 1,2,3$, $b = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., v^{(\upsilon)}$, $l = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., w^{(\upsilon)}$, are optimal transient probabilities defined by (19) and $\dot{H}(\theta, r)$ is the mean value of the optimal number of exceeding the critical safety state r by the Baltic Port and Shipping Critical Infrastructure Network operating at the variable conditions during the operation time θ defined by (3.29) in [Kołowrocki, Soszyńska-Budny, 2011].

The total optimal operation cost of the repairable Baltic Port and Shipping Critical Infrastructure Network with non-ignored its renovation time during the operation time θ, $\theta \geq 0$, after its operation process related to climate-weather change optimization amounts in the following way

$$K_{n-ign}(\theta) \cong \sum_{b=1}^{v^{(\upsilon)}} \sum_{l=1}^{w^{(\upsilon)}} pq_{bl} \sum_{i=1}^{n} [K_i\ (\theta)]^{(bl)} + K_{n-ign} \overline{\overline{H}}(\theta, r),$$
$\theta \geq 0,$
(31)

were $\dot{p}q_{bl}$, $\upsilon = 1,2,3,$ $b = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., v^{(\upsilon)}$, $l = 1^{(\upsilon)}, 2^{(\upsilon)}, ..., w^{(\upsilon)}$, are optimal transient probabilities defined by (19) and $\dot{\overline{\overline{H}}}(\theta, r)$ is the mean value of the optimal number of renovations of the Baltic Port and Shipping Critical Infrastructure Network operating at the variable operation conditions during the operation time θ defined by (6.37) in [Kołowrocki, Soszyńska-Budny, 2011].

The particular expressions for the optimal mean values $\dot{H}(\theta, r)$ and $\dot{\overline{\overline{H}}}(\theta, r)$ for the repairable the Baltic Port and Shipping Critical Infrastructure Network with ignored and non-ignored renovation times existing in the formulae (6.48) and (6.49), respectively defined by (6.29) in [Kołowrocki, Soszyńska-Budny, 2011] and (6.37) in [Kołowrocki, Soszyńska-Budny, 2011), may be obtain by replacing the transient probabilities pq_{bl} by their optimal values $\dot{p}q_{bl}$ in the expressions for $H(\theta, r)$ and $\overline{\overline{H}}(\theta, r)$ defined by (3.58) and (3.92) in [Kołowrocki, Soszyńska-Budny, 2011], that are determined in Chapter 3 of [Kołowrocki, Soszyńska-Budny, 2011], for typical multistate repaired critical infrastructure operating at the variable operation conditions.

The application of the formulae (26)-(28) and (29)-(31) allow us to compare the costs of the non-repairable and repairable Baltic Port and Shipping Critical Infrastructure Network with ignored and non-ignored times of renovations operating at the variable operation conditions before and after the optimization of their operation processes.

4. Conclusion

In this paper the optimization of operation process and minimization of operation cost for Baltic Port and Shipping Critical Infrastructure Network (BPSCIN) at variable operation conditions related to the climate-weather change have been presented. Especially, the Baltic Port and Shipping Critical Infrastructure Network operation process related to climate-weather change process has been described. Furthermore, the description and analysis of operation cost related to climate-weather change of BPSCIN is given. Based on these results, the following optimal solutions have been determined or estimated for Baltic Port and Shipping Critical Infrastructure:

- the optimal instantaneous operation cost;
- the critical infrastructure operation process related to climate-weather change;
- the total optimal cost of the non-repairable and repairable BPSCIN during the operation time.

The methods used in article are based on results given in [Kołowrocki, et al., 2017c], [Kołowrocki, Soszyńska-Budny, 2017].

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