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GNSS Positioning Error Change-point Detection in GNSS Positioning Performance Modelling

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ABSTRACT: Provision of uninterrupted and robust Positioning, Navigation, and Timing (PNT) services is essential task of Global Navigation Satellite Systems (GNSS) as an enabling technology for numerous technology and socio-economic applications, a cornerstone of the modern civilisation, a public goods, and an essential component of a national infrastructure. GNSS resilience may be accomplished only with complete understanding of the causes of GNSS positioning performance disruptions and degradations, presented in a form of applications- and scenarios-related models. Here the application of change-point detection methods is proposed and demonstrated in a selected scenario of a fast-developing ionospheric storm's impact on GNSS positioning performance, as a novel contribution to forecasting GNSS positioning performance model development and GNSS utilisation risk mitigation.

1 INTRODUCTION

Satellite navigation has become a cornerstone of the modern civilisation, a public goods, and an essential component of a national infrastructure. Growing number of both navigation and non-navigation technology and socio-economic applications (systems and services) rely completely on provision of Global Navigation Satellite Systems (GNSS) Positioning, Navigation, and Timing (PNT) services, thus raising importance of robust GNSS, resilient to sources of GNSS PNT services disruptions and degradations (HM Government Office for Science, 2018). Models of GNSS positioning performance degradations and error sources, based on actual scenarios and experimentally collected data, allows for understanding the causes and mitigation of the effects on GNSS positioning performance in general and in relation to specific applications. GNSS resilience may then be accomplished through various risk containment actions, including error correction modelling, and forecasting GNSS positioning performance degradation for alerts and corrective actions.

Model development process suffers from the complexity of the positioning environment, as described with our Space Weather-GNSS positioning performance coupling model (Filić and Filjar, 2018), (Filić and Filjar, 2019). GNSS resilience has been attempted to be achieved through methodologies, including those that addresses identification of the closest linear descriptor of GNSS positioning errors (Filić and Filjar, 2019), examining GNSS positioning errors dynamics (Lenac, Filić, and Filjar, 2019), and advanced statistical learning approaches applied over massive data sets of potential positioning environment descriptors (Filić and Filjar, 2018). Still, on all attempted approaches and scenarios, the identification of the GNSS positioning performance degradation onset and duration appeared to be the critical issue that raised the model's uncertainty.

Here a change-point detection methodology (Truong, Oudre, and Vaytis, 2019), (Aminikhanghahi and Cook, 2017), (Schroeder, 2016) applied on GNSS positioning error time series is proposed as a valuable contribution to GNSS positioning performance degradation model development. Based on the experimental data and founded on statistical analysis of GNSS positioning error dynamics, its capacity is demonstrated for not only the change-point detection, but also the identification of the intervals with the increased volatility (variance) of GNSS positioning error due to ionospheric conditions.

2 METHODOLOGY

2.1 *Case-study description*

This researcher examined a known case of a fast development of a large ionospheric storm in the period of quiet solar activity, commenced on 17the March, 2015 and known as the 2015 St Patrick's Day Storm. Development of this space weather event, unusual in a period of quiet solar activity, posed as a challenge for the research community addressing the space weather and ionospheric impact on GNSS positioning performance and operation. Space weather and geomagnetic conditions in March, 2015 were properly described with the planetary geomagnetic Kp index, with its dynamics depicted in Figure 1.

Time series of GNSS northing, easting and height positioning errors were derived from the GPS pseudorange observations taken at the Poreč, Croatia reference station during March, 2015 (31 daily data sets), processed using the open-source using a post-processing methodology described elsewhere, for instance (Filić and Filjar, 2018) and (Filić, Filjar, and Routsalainen, 2016). Time series of GNSS northing, easting and height positioning errors were analysed further using the bespoke R-based GNSS statistical analysis framework we developed earlier. Resulting GPS positioning errors time series followed the normal distribution well, and are presented graphically in Figure 2.

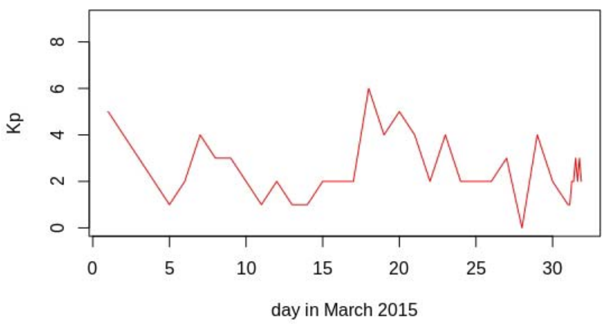

Figure 1. Planetary geomagnetic Kp index dynamics in March, 2015. Based on data provided by (NOAA, 2019).

2.2 *Change detection methods*

(Gustafsson, 2000) defined a change detection as a decision of inconsistency of the observed data with the nominal model. (Killick and Eckley, 2014) narrowed the definition of change detection to identification of the instance of time (the point in time series of data, dubbed *change-point*), when (where) statistical properties before and after that time (point) differ.

Definition 1: Let the time series of observed data is given, as in (1).

$$x(t) = (x_1, x_2, \ldots, x_n) \tag{1}$$

The change-point is defined as a time-series sample x_τ, taken at the instant of time τ, exists if statistical properties of the sub-set $x(t) = (x_1, x_2, \ldots, x_\tau)$ of original data differs from statistical properties of the remaining sub-set $(x_{\tau+1}, x_{\tau+2}, \ldots, x_n)$. A change-point may be defined either as a global (a single change-point in the whole set of observations), or a local (an element of a set of multiple change-point in the set of observations).

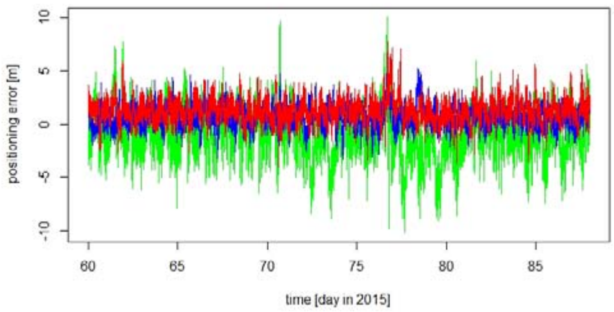

Figure 2 Time series of GPS northing, easting, and height positioning errors, derived from experimental observations taken at the Poreč, Croatia reference station (data provided by (Sonel, 2019) network)

A common experience would tell that a sufficiently large set of observations very probably comprises a number of change-points. Their identification may be understood as an optimisation problem that may be defined as follows.

Let assume a set of observations $x(t) = x_1, x_2, \ldots, x_n$ comprises k change-points, with defined as $cp = (cp_1, cp_2, \ldots, cp_k)$, with the cp_1 as the first ($cp_1 = x_1$), and the cp_k being the last ($cp_k = x_n$) element of the original time series. A single change-point may be detected/defined using the likelihood ratio test (2)

$$LR = \max_{\tau}\left(l(x_{1:\tau}) + l(x_{\tau-1:n}) - l(x_{1:n})\right) \tag{2}$$

where l is the likelihood function applied on the data set specified. A change-point is inferred as detected if the condition $LR > \lambda$ is met, with λ being a suitably selected penalty parameter. The time of change (identification of a change-point in the time series) may be then inferred as given in (3).

$$\tau = \arg\max\left(l(x_{1:\tau}) + l(x_{\tau-1:n}) - l(x_{1:n})\right) \tag{3}$$

The procedure may be generalised for multiple change-point detection using problem formulation as given in (4), with additional modelling flexibility provided by introduction of penalty function *f(k)* related to the number of change-points, and penalty constant λ (Truong, Oudre, and Vaytis, 2019).

$$\min_{k,\tau}\left\{\sum_{i=1}^{k}\left[-l\left(x_{(\tau_{i-1}+1):\tau_i}\right)\right]+\lambda f\left(k\right)\right\} \tag{4}$$

Solution of the optimisation problem (4) depends on the expected number of change-points, complexity of the process that generated time series, and the required computational performance (Killick, 2016), (Chandola and Vatsavai, 2011). Change-point detection methods utilises various approaches and constraints (Truong, Oudre, and Vaytis, 2019). Among them, one can distinguish: (i) At Most One Change (AMOC), a method suitable for a single change-point detection, (ii) Binary Segmentation, developed by (Scott and Knott, 1974), approximate in its accuracy, but computationally efficient, (iii) Segment Neighbourhood, exact in its accuracy, but computationally demanding, and (iv) Pruned Exact Linear Time (PELT), an exact and computationally efficient method. Optimisation may consider the mean changes in time series only, the variance changes in time series only, or to be concerned with both essential statistical descriptors of a time series.

The methods analysed do not only identify the change-point(s), but also determines the time-series sub-sets of increased variance, usually caused by the very sources of GNSS positioning performance degradations.

The above-described procedures are comprehensibly available as a separate statistical library in the R environment for statistical computing. Named the *changepoint*, it was used in this research, and applied on the GPS northing, easting and height positioning error time series derived from the experimental observations during the case-scenario, as described in Section 2.1.

2.3 *Change detection implementation*

This research was conducted using the R environment for statistical computing, with a range of change-point methods deployed in the R library called *changepoint* (Killick, 2016). After the initial exploratory statistical analysis of data, we conducted a through examination of the *changepoint* R library methods, and found the best fit to the problem concerned with the utilisation of the Binary Segmentation change-point detection method (Scott and Knott, 1974) on the optimisation problem involving both the mean and the variance change. The Schwarz (Bayesian) Information Criterion (SIC) (Watanabe, 2013) Penalty Function was selected, and number of expected change-points set to 10. Normal distribution of GPS positioning errors time series was assumed in the change-point detection process, based on the exploratory statistical analysis conducted.

3 CHANGE DETECTION IN GNSS POSITIONING ERROR TIME SERIES

This research addressed the potential of the change-point detection in time series (Truong, Oudre, and Vaytis, 2019) of GNSS positioning errors for the purposes of the GNSS positioning error modelling and mitigation, and potential applications of post-processed GNSS observations.

Using the methodology outlined in Section 2, and by varying parameters of the change-point detection methods in the established R- and *changepoint* library-based framework, optimal change-point detection in GPS northing, easting, and height positioning errors time series was performed. The most suitable change-point detection method was selected and fine-tuned based on the exploratory data analysis (Filić, Filjar, and Ruotsalainen, 2016).

The optimised change-point detection methodology was deployed on the March, 2015 time series, with the results for GPS northing, easting, and height positioning errors depicted in Figures 3, 4, and 5, respectively.

The change-point detection method applied identified correctly the 2015 St Patrick's Storm effects on the GPS positioning performance, as the most prominent degradation throughout March, 2015. Further to this, the method identified periods (time-series sub-sets) of a range of variances, thus determining time-series sub-sets that may be used immediately in development of partial descriptive models.

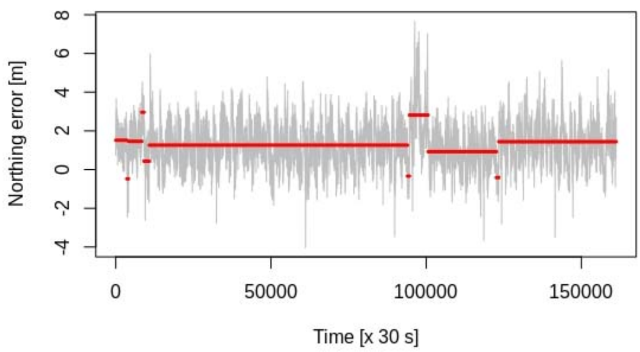

Figure 3. Change-points detected in GPS northing positioning errors time series during March, 2015

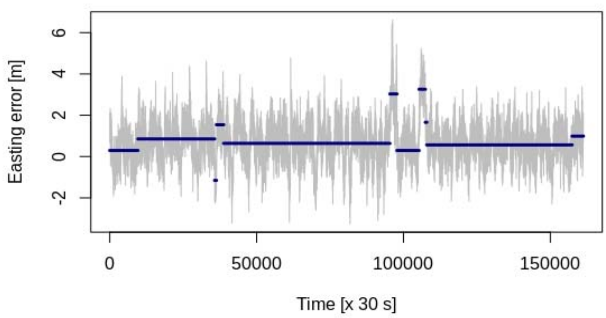

Figure 4. Change-points detected in GPS easting positioning errors time series during March, 2015

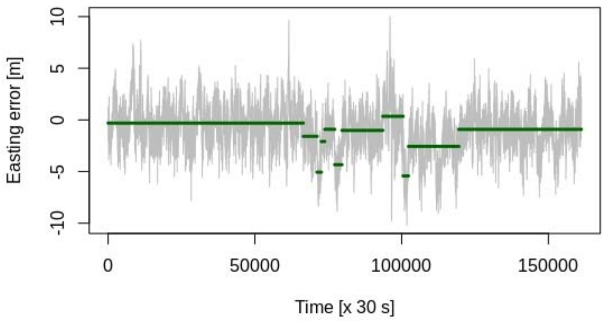

Figure 5. Change-points detected in GPS height positioning errors time series during March, 2015

4 DISCUSSION AND CONCLUSION

Change-point detection is a simple and elegant statistics-based method for time series analysis. Here we argue that it may be utilised for a considerable improvement of GNSS positioning error time-series analysis and a novel contribution in GNSS positioning performance models and forecasts. It allows for the clear and accurate determination of the onset of GNSS positioning performance degradation. The methodology is based on the strict application of statistics on experimental data in a form of time series, and addresses its dynamics only. Comparison with time series of supposed causes of GPS positioning performance degradation confirmed the accuracy of change-point detection.

Traditionally, the onset of a critical GNSS positioning performance event was determined through cross-correlation procedure involving, combining (supposed) cause and (hopefully direct) effect. Regretfully, correlation between the sources of space weather, geomagnetic and ionospheric disturbances, and the GNSS positioning performance degradation (error dynamics) is frequently weak, as the result of a complex and non-linear relationship between descriptors of space weather, geomagnetic and ionospheric conditions, and the GNSS positioning performance, respectively. This often results with description models of modest quality, and inability to forecast correctly the GNSS positioning performance response to space weather, geomagnetic and ionospheric disturbances.

The change-point detection methodology presented here points clearly and accurately to the instant of time when the GNSS positioning performance starts to deteriorate, using the analysis of statistical nature of time series dynamics. The accuracy of the time onset estimation appears to be related to the number of expected change-points (*k*, Section 2), when Binary Segmentation method with Schwarz Information Criterion Penalty is considered. This problem is to be addressed by our team in continuation of this research, with the aim of development and deployment of a tailored optimisation algorithm for the *k* parameter selection.

The opportunity for the change-point detection procedure application on a smoothed time series of GPS northing, easting, and height positioning errors time series was also discussed. While determination of change-points would go more smoothly in such a case, information on the process under observation may be lost, especially in the case of short-term local disruptions. We concluded that application of the Binary Segmentation change-point detection method with Schwarz Information Criterion Penalty (Killick, 2016) with reduced number of expected change-points but applied on original (non-smoothed) data set will yield more accurate results, without missing or incorrectly identified change-points. The methodology applied split the original data set into sub-sets with related variance, pointing out to periods and data of anomalous behaviour of GNSS positioning performance. Sub-set formation in consideration of local variance may be extended further to analyses of the cause and result. Overlapping intervals of the same-level variance in different descriptors time series will allow determination of the cause-effect assessment more, such as in studies (Filić and Filjar, 2019) and (Filić and Filjar, 2018) conducted earlier, more accurately and efficiently.

Finally, the demonstration of the change-point detection methodology application on GNSS positioning error time series improves numerous GNSS application scenarios involving post-processing of GNSS observations, including, but not limited to: science, GNSS error modelling and mitigation, GNSS forensics (event reconstruction), and actuarial science. We identified a potential for a significant improvement of GNSS positioning performance understanding, modelling and forecasting using bespoke tuned change-point detection methods applied on time series of GNSS positioning error components.

In summary, this manuscript presented the change-point detection methodology as a valuable addition to the process of modelling GNSS positioning performance, thus improving GNSS positioning performance models, and widening space for the post-processing GNSS applications in disciplines and scenarios that require knowledge on, and understanding of the GNSS positioning performance degradations.

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