

## **Two stage optimization of the PMSM with excitation system composed of different materials**

Lukasz Knypiński, Lech Nowak  
Poznan University of Technology

60 – 965 Poznań, ul. Piotrowo 3a, e-mail: lukasz.knypinski@put.poznan.pl,  
lech.nowak@put.poznan.pl

The paper presents the algorithm and software for the optimization of the rotor structure of the permanent magnet synchronous motor with magnet composed of two materials about different magnetic properties. The software consists of two modules: a numerical model of the PMSM motor and an optimization solver. Numerical implementation is based on finite element method. The optimization module has been elaborated employing the Delphi environment. For the rotor structure optimization the genetic algorithm has been applied. Selected results of the calculation are presented and discussed.

**KEYWORDS:** electric machines, permanent magnet synchronous motor, optimization, genetic algorithm

### **1. Introduction**

The aim of the research is designing series of modern, energy-efficient permanent magnet synchronous motors (PMSM) which in the future will replace commonly used induction motors for pumps and fans drives in the mining industry. There are considered numerous structures of PMSM rotors in literature [1, 2]. Typical structures are shown in Fig. 1.

New structures require developing new methods of the optimum design [9]. In the paper, the comprehensive software, in which the unit using the genetic algorithm has been combined with the module containing the field model of PMSM motor has been elaborated.

Very important problem at formulating the optimization task is the choice of the functional parameters which constitute the objective and constraint functions. In the paper it has been shown that uncritical constructing the objective function could lead to irrational variants of the designed object. Authors pointed out [4] that connecting the useful torque and the cogging torque simultaneously in the one compromise objective function generates ineffective operation of the optimization algorithm and often also leads to the non-optimal result.

Recent years have witnessed rapid development of magnetic powder technology. This concerns both: soft and hard magnetic material. The powder technology enables free formation of the element geometries and control over their magnetic properties –

depending on the admixtures used. The design and construction of excitation systems consisting of two or more different materials is also possible.

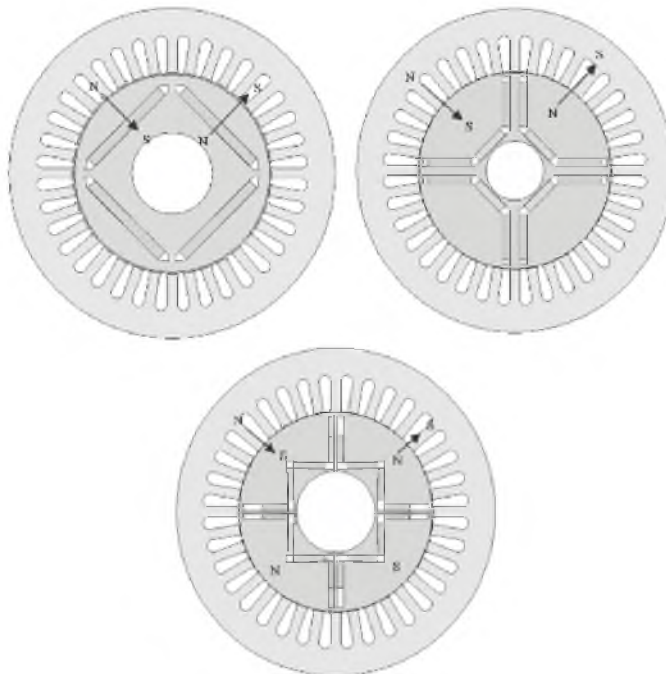


Fig. 1. Structures of the PM synchronous motors

In the paper, the results of the optimization of the motor with magnets built of two materials: (a) sintered NdFeB and (b) powder magnetic material [3] have been presented.

## 2. The structures of the machine

The structures of the considered motors are presented in Figure 2. The PMSM 1 (Fig. 2a) has the excitation system built of two materials with different magnetic properties. The PMSM 2 (Fig. 2b) has homogeneous arc magnets. The design data of the stator are listed in Table 1.

Table 1. Design data of the motor

Number of pole pair	Outer diameter of stator	Inner diameter of stator	Stack length	Air gap length	Number of slots
[-]	[mm]	[mm]	[mm]	[mm]	[-]
2	154	94	125	0.9	36

The rotor core is made of soft powder material (Somaloy 500). The excitation system of first motor consists of two areas with different magnetic properties ( $M1$  and  $M2$ ). Area  $M1$  is made of sintered NdFeB material with properties:  $H_c = 890$  kA/m and  $B_r = 1.23$  T, whereas area  $M2$  is made of powder magnetic material with properties:  $H_c = 404.97$  kA/m and  $B_r = 0.646$  T.

The parameters of the rotor structure have been optimized. The numerical model of the machine is based on the FE method. The genetic algorithm has been applied for solving the optimization process. The object has been described by four design variables (Fig. 2a):  $z_1 = g_{m1}$  – thickness of the NdFeB magnet area,  $z_2 = \xi = g_{m2}/g_{m1}$  – relative thickness of the powder magnet area,  $z_3 = \alpha = (b_{m1} + b_{m2})/\tau$  – angular width of the magnetic material related to pole pitch,  $z_4 = \alpha_1 = b_{m1}/(b_{m1} + b_{m2})$  – relative angular width of area  $M1$ . These variables form design vector  $\mathbf{z} = [g_{m1} \ \xi \ \alpha \ \alpha_1]^T$ . All variables  $z_j$  have been transformed into dimensionless quantities  $s_j$  according to the formula [5]:

$$s_j = (z_j - z_{\min j}) / (z_{\max j} - z_{\min j}) \quad (1)$$

where  $z_{\min j}$  and  $z_{\max j}$  denote the lower and upper limits of each variable  $z_j$ , respectively.

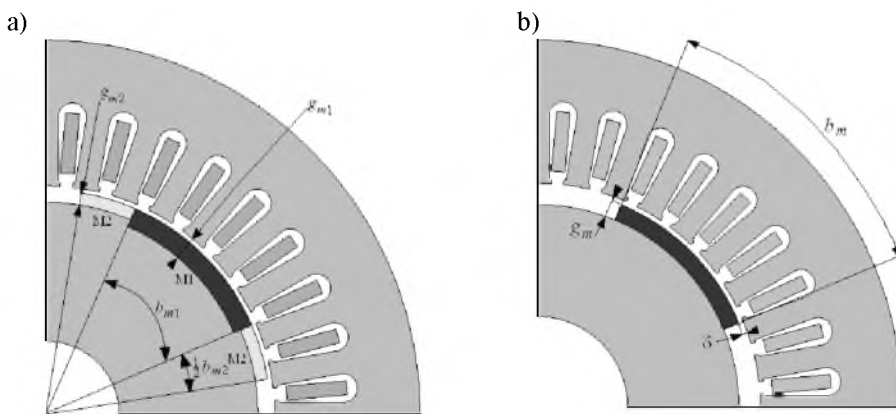


Fig. 2. The PMSM with: a) excitation system, b) homogenous arc magnets

### 3. Optimization strategies

Typical problem of the electromagnetic devices optimization is the constrained optimization. Constrained optimization is the process of optimizing an objective function with constraints on selected parameters or main dimensions of the device. Relations between devices actual parameters  $p_j(\mathbf{s})$  and required value  $p_{jr}$  have been defined as follows:

$$p_j(\mathbf{s}) \geq p_{jr} \quad \text{or} \quad p_j(\mathbf{s}) \leq p_{jr} \quad (2)$$

Here  $\mathbf{s}$  is the vector composed of variables  $s_j$ .

The optimization task is to find such vector  $\hat{\mathbf{s}}$  that:

$$\bigwedge_{\mathbf{s} \in D} f(\mathbf{s}) \leq f(\hat{\mathbf{s}}) \quad (3)$$

Here  $D$  is the permissible search area defined by the constraints  $D = \{\mathbf{s} : g_j(\mathbf{s}) \leq 0; j = 1, 2, \dots, m\}$ .

The constraints can be taken into account in two ways: (a) by the outside penalty term [7] or (b) by attaching them (in the form of additional terms) the compromise objective function [10].

### **Penalty function**

According to this method, the modified objective function  $h_k(\mathbf{s})$  is created. In the modified function, the term representing the penalty for overstepping the constraint is formed. The penalty component  $p_k(\mathbf{s})$  is increasing during optimization process. The  $p_k(\mathbf{s})$  depends on value of constraints and is defined as follows:

$$p_k(\mathbf{s}) = r^k \sum_{j=1}^m \lambda_j g_j(\mathbf{s}) \quad (4)$$

where  $r > 1$  is the penalty coefficient,  $k$  is a number of penalty iteration.

The modified objective function is formulated:

$$h_k(\mathbf{s}) = \begin{cases} f(\mathbf{s}) & \text{for } \mathbf{s} \in D \\ f(\mathbf{s}) - p_k(\mathbf{s}) & \text{for } \mathbf{s} \notin D \end{cases} \quad (5)$$

In this algorithm, the relative penalty has been usually introduced

$$\kappa(\mathbf{s}) = p_k(\mathbf{s}) / f(\mathbf{s}) \quad (6)$$

After substituting (6) into equation (5), the modified objective function may be expressed in the form:

$$h_k(\mathbf{s}) = \begin{cases} f(\mathbf{s}) & \text{for } \mathbf{s} \in D \\ f(\mathbf{s}) \left( 1 - \frac{p_k(\mathbf{s})}{f(\mathbf{s})} \right) & \text{for } \mathbf{s} \notin D \end{cases} \quad (7)$$

In genetic algorithm, the modified objective function  $h_k(\mathbf{s})$  (individual adaptation) have to be positive. The relative penalty term  $\kappa(\mathbf{s})$  must be less than 1. The sigmoidal transformation has been applied. After conversion  $\hat{\kappa}(\mathbf{s}) = 1 - e^{-\kappa(\mathbf{s})}$ , the modified objective function can be expressed as a primary function multiplied by dimensionless penalty ratio  $\zeta_k(\mathbf{s})$

$$h_k(\mathbf{s}) = f(\mathbf{s}) \zeta_k(\mathbf{s}) \quad (8)$$

in which  $\zeta_k(\mathbf{s}) = e^{-r^k \sum_{j=1}^m \lambda_j g_j(\mathbf{s})}$ .

### **Compromise objective function**

In such strategy the main functional parameters of the design device and the constraints are included into compromise objective function. The function can be represented as an additive or multiplicative compromise objective function.

The additive compromise function has usually the form

$$f(\mathbf{s}) = \sum_k \lambda_k P_k(\mathbf{s}) \tag{9}$$

where  $\lambda_k$  is set of the weighting coefficient.

The multiplicative compromise objective function may be expressed as:

$$f(\mathbf{s}) = \prod_k P_k^{q_k}(\mathbf{s}) \tag{10}$$

where  $q_k$  is set of the weighting coefficient.

## **4. The software structure**

The elaborated software consists of two modules: a numerical model of the PMSM and an optimization solver. The block diagram of the software is presented in Fig. 3.

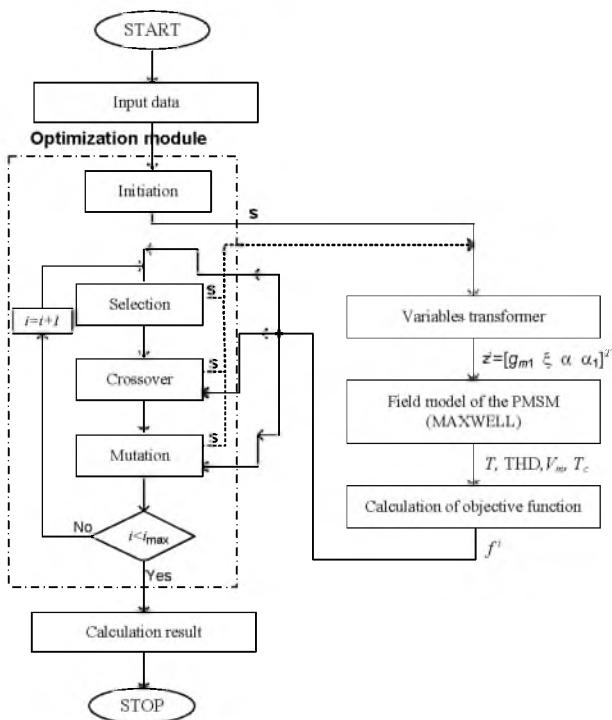


Fig. 3. The block diagram of the software

The optimization module has been developed using Delphi environment. The genetic algorithm has been employed for solving the optimization problem. All genetic operations (reproduction, crossover and mutation) are executed on dimensionless design variables  $s_j \in \langle 0, 1 \rangle$  (see Figure 3). During the crossover and mutation procedures the binary 32-bit representation of design variables is applied. In the studied algorithm, the so called simple elitism procedure has been taken into account [7]. This means that, the best individual is automatically moved to the next generation.

The numerical model of the PMSM has been executed in the Ansys Maxwell. After the calculation of the electromagnetic field distribution with the use of the finite element method, the value of electromagnetic torque  $T$ , the volume of the permanent magnets  $V_m$ , the harmonic distortion coefficient THD of the inducted voltage  $E$  and the maximum value of cogging torque  $T_c$  are determined. The numerical model of the motor require the real value of design variables  $\mathbf{z}$ . Both modules are connected through the variable transformation procedures (variable transformer - see Fig. 3).

## 5. Decomposition of the optimization task

At the stage of testing software the PMSM with homogenous arc magnets has been analyzed (Fig. 2b). On the basis of field calculations, the PMSM electromagnetic parameters, e.g.  $T$ ,  $T_c$  and THD have been determined. The magnet volume  $V_m$ , as a an additional economic criterion was considered. The „torque density”  $t_v = T/V_m$  [Nm/m<sup>3</sup>] reflected this criterion.

Two design variables has been assumed: the magnet thickness  $g_m$  and relative arc magnet span  $\alpha = b_m/\tau$  – Fig. 2b. In order to recognize the relations between design variables  $g_m$ ,  $\alpha$  and electromagnetic parameters  $T$ ,  $T_c$ , THD, and  $t_v$  the series of test calculations were carried out. Results are presented in Fig. 4. The output torque  $T$  monotonically increases with both variables, while the cogging torque  $T_c$  is multimodal function of variable  $\alpha = b_m/\tau$ ; it has several extremes. The harmonic distortion coefficient THD has it minimum for  $\alpha \cong 0.7 \div 0.8$ .

The cogging torque as a additional criterion significantly distorts the optimization process. Algorithm is too sensitive to the changes of this torque. The change of the magnet relative span  $\alpha$  of order 0.05 causes only 4 % change of the output torque, while the changes of cogging torque can reach even 400 %. The cogging torque has a significant impact on value  $T$  of compromise objective function. Therefore the decomposition of the task has been proposed. In the first stage the compromise criterion consist of 3 terms: output torque  $T$ , harmonic coefficient THD and magnet volume  $V_m$ . The cogging torque is taken into account in the second stage of optimization.

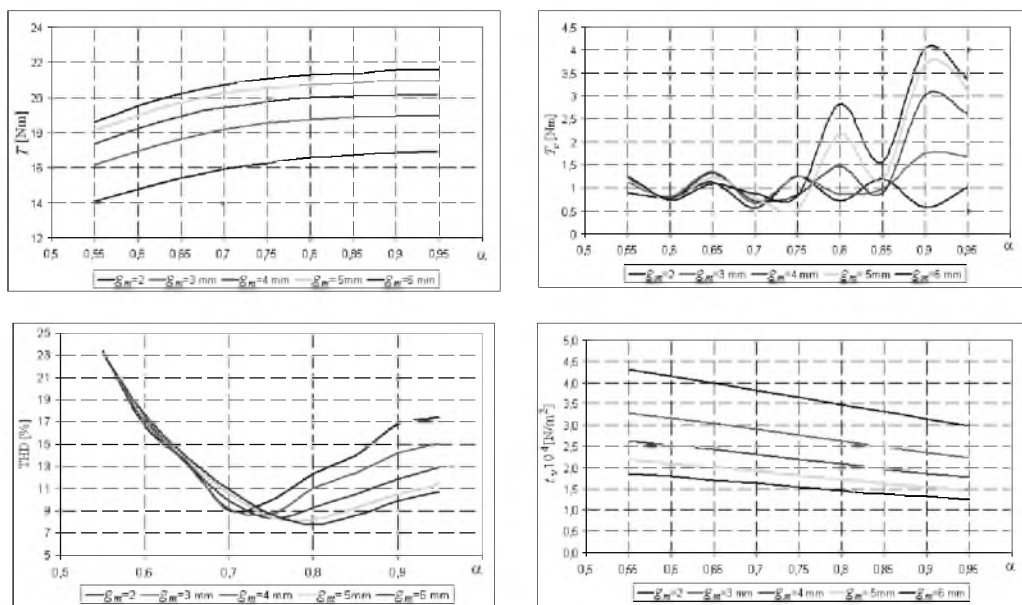


Fig. 4. Effect of variables  $g_m$  and  $\alpha$  on the motor electromagnetic parameters

In the optimization procedure, all parameters have been related to the average values obtained in genetic initiation procedure:  $t = T/T_{0av}$ ,  $t_c = T_c/T_{c0av}$ ,  $h = THD/(THD)_{0av}$ ,  $v_m = V_m/V_{m0av}$ .

## 6. Results of multi-objective optimization

### Stage I. The optimization task taking into account coefficient THD

After many trial calculations, the following compromise additive objective function for  $i$ -th individual has been suggested [4]:

$$f_i = \lambda_1 t + \lambda_2 (2 - h) + \lambda_3 (2 - v_m) \quad (11)$$

Here:  $\lambda_1$ ,  $\lambda_2$ ,  $\lambda_3$  are the weighting factors.

Number of individuals in each generation was equal to  $L=60=\text{const}$ . The following parameters of the genetic algorithm has been assumed: the probability of mutation  $p_m = 0.005$ , the maximum number of genetic algorithm populations  $(N_p)_{\max} = 35$ . The calculations were carried out for  $\lambda_1 = 1$ ,  $\lambda_2 = 0.75$ ,  $\lambda_3 = 0.25$ . The comparison of the optimization results for the best individual in successive populations is shown in Table 2.

The comparison of the values of components of compromise objective function (11) in selected generations is presented in Figure 5.

Table 2. The first stage of the optimization process

$N_p$	$g_{ml}$	$\xi$	$\alpha$	$\alpha_1$	$T$	THD	$V_m$	$T_c$	$f$
	[mm]	[-]	[-]	[-]	[Nm]	[%]	[cm <sup>3</sup> ]	[Nm]	[-]
1	4.022	1.019	0.799	0.545	14.400	8.982	24.066	0.583	1.246841
3	3.197	1.195	0.809	0.629	16.190	6.825	23.903	1.039	1.425541
5	3.144	1.072	0.782	0.704	16.308	7.736	21.701	0.655	1.470121
10	3.144	0.900	0.791	0.704	16.296	7.499	20.908	0.247	1.473461
16	3.144	0.824	0.821	0.704	16.561	7.189	21.206	0.925	1.474563
25	3.019	0.806	0.812	0.704	16.172	7.356	19.605	0.571	1.495876
30	3.019	0.749	0.817	0.704	16.236	7.064	19.838	0.751	1.496811
35	3.019	0.749	0.817	0.704	16.236	7.064	19.838	0.751	1.496812

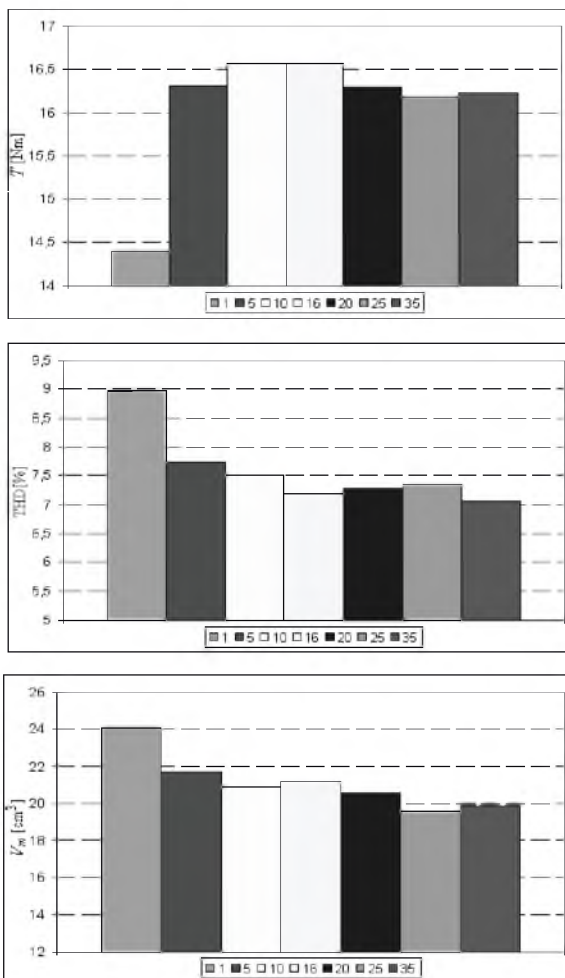


Fig. 5. Comparison of  $T$ , THD and  $V_m$  in selected generations of the optimization process



**Stage II. The optimization task taking into account cogging torque  $T_c$**

In the second stage, instead of magnet volume  $V_m$ , the cogging torque  $T_c$  has been taken into account. The cogging torque strongly depends on the span  $\alpha$  of the magnetic material.

Table 3. Results of the second stage of the optimization process

$N_p$	$\alpha$	$\alpha_1$	$T$	THD	$V_m$	$T_c$	$f$
	[-]	[-]	[Nm]	[%]	[cm <sup>3</sup> ]	[Nm]	[-]
1	0.7741	0.8683	16.875	8.575	19.590	0.1850	2.63135471
2	0.8876	0.7697	17.323	12.015	21.899	0.1005	2.70846627
5	0.7750	0.8683	16.884	8.615	19.611	0.1827	2.71009390
7	0.7498	0.8683	16.643	7.689	18.976	0.3015	2.71009390
10	0.7499	0.7695	15.963	7.964	18.500	0.1648	2.71059382
12	0.7499	0.7695	15.963	7.964	18.500	0.1648	2.71059382

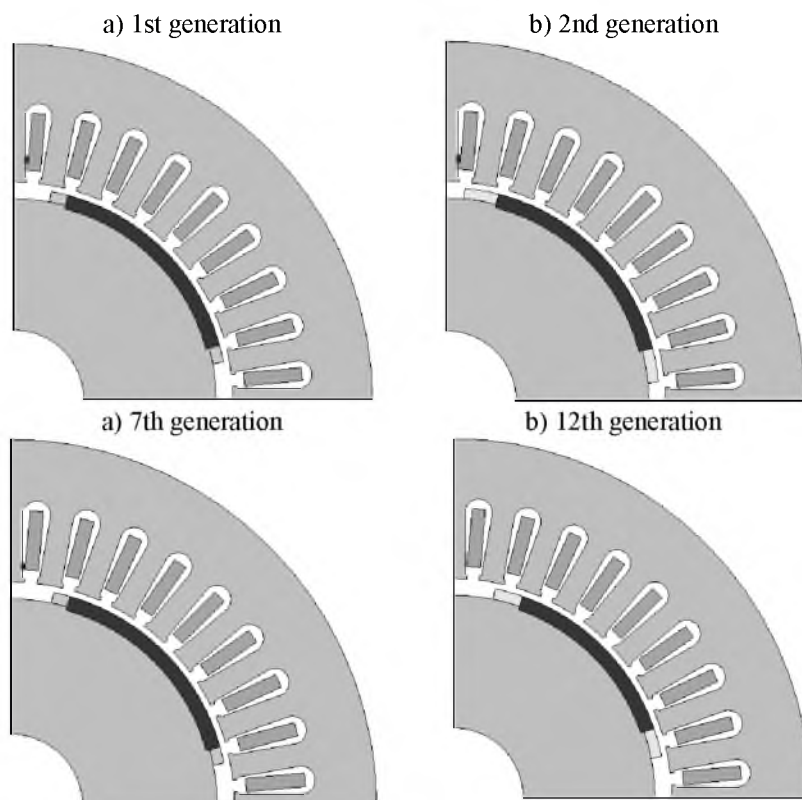


Fig. 6. The comparison of the best individual in the selected generation

In the area of one pole pitch  $\tau$ , it has several maxima and minima. The period of variation is related to the slot pitch  $\tau_{s1}$ . In the second stage, the calculation has been performed for the following range of  $\alpha$ :

$$\left\langle \hat{\alpha} - 0.52 \frac{\tau_{s1}}{\tau}, \hat{\alpha} + 0.52 \frac{\tau_{s1}}{\tau} \right\rangle \quad (12)$$

where  $\hat{\alpha} = 0.817$  is the optimal value of  $\alpha$  obtained from the first stage of the optimization process.

The following multiplicative compromise objective function has been studied [4]:

$$f_i = t^{q_1} h^{q_2} t_c^{q_3} \quad (13)$$

he optimal values:  $\hat{g}_{m1} = 3.019$  and  $\hat{\xi} = 0.749$  from the first stage have been taken into account. Basing on many test calculations, the values of the weighting coefficients  $q_1 = 2$ ,  $q_2 = -1/2$ ,  $q_3 = -1/2$  have been supposed. The optimization results are presented in Table 3.

Figure 6 illustrates a cross section of the best individual in the selected generations.

As a result of the second stage of optimization more than a 300 % reduction of the cogging torque has been achieved, while the output torque decreased only by 1.5%. An economic effect has also been achieved, namely the magnet volume decreased by 10%.

## 7. Conclusion

The paper presents an application of the genetic algorithm for rotor structure optimization of the synchronous motor with excitation system composed of different materials. Research on the optimization of PMSM motors showed that uncritical formation of the objective function leads to incorrect results of the optimization algorithm. As a result of the optimization process, irrational structures were received. In order to correct solve the optimization problem, the two-stage optimization has been applied.

The application of magnets composed of two areas with different magnetic properties has a better influence on the distribution of the magnetic field in the air gap than the use of homogenous magnets. The use of excitation by a system of magnets significantly reduces the cogging torque and the THD coefficient.

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