

## Multilevel inverter with optimal THD through the firefly algorithm

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**Abstract:** Reduction of the Total Harmonic Distortion (THD) in multilevel inverters requires resolution of complex nonlinear transcendental equations; in this paper we propose a combination of one of the best existing optimized hardware structures with the recent firefly algorithm, which was used to optimize the THD, through finding the best switching angles and guaranteeing the minimization of harmonics within a user defined bandwidth. The obtained THD through the simulation of the thirteen-level symmetric inverter has been reduced down to 5% (FFT of 60 harmonics). In order to validate the simulation results, a thirteen-level symmetric inverter prototype has been made, and practically experimented and tested with different loads. Consequently, the measured THD with resistive load was 4.7% on a bandwidth of 3 kHz. The main advantage of the achieved work is the reduction of the THD.

**Key words:** firefly algorithm (FFA), genetic algorithm (GA), inverter, multilevel, optimized THD

### 1. Introduction

Through the last few years, there have been extensive researches about substitutes of conventional energies through renewable energies. This new type of energy can be produced by wind turbines and Solar Photovoltaic Energy. Such a solution needs systems to stock and convert the energy. Considered as one of the crucial parts that allow delivering clean Alternating Current (AC), inverters have been one of the important research axes, the main ambition was to increase the quality of the output waveform, in addition to promote and improve friendly environment systems [1, 2].

It is well established that a multilevel inverter (MLI) can produce a high quality signal and low Total Harmonics Distortion (THD) [3, 4, 8, 12]. In fact, during these many years, several applications using multilevel inverters have been proposed and realized.

The most important structures of multilevel inverters can be classified into three types, a diode-clamped, flying capacitors and a cascaded H-bridge [3, 4].

The first diode-clamped inverter and also called neutral-point-clamped was introduced the first time by Nabae in 1981 and was essentially a three-level-clamped inverter, where during the off period of the PWM each output terminal will be clamped to the neutral potential [4, 5], this is shown in Figure 1a.

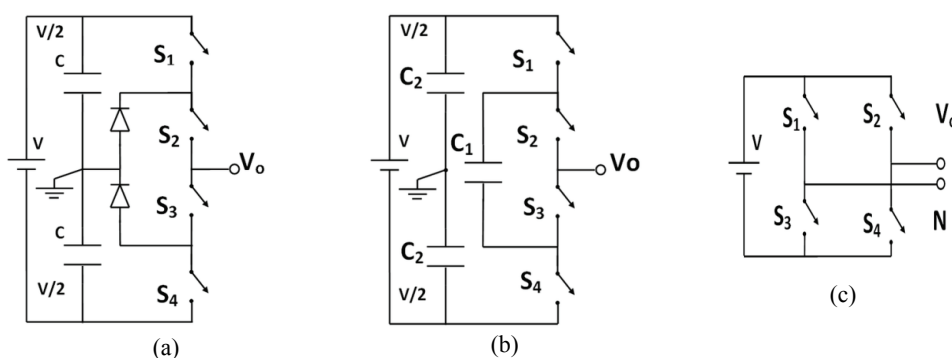


Fig. 1. Three-level-diode-clamped inverter: (a) three-level flying capacitor inverter; (b), three-level H-bridge inverter (c)

The first flying capacitor inverter has been introduced by Meynard in 1992, a 3 level topology single phase of this inverter is illustrated in Figure 1b, it is similar to the first type but the clamping diodes are replaced by a capacitor [4, 6].

The third type is the cascaded H-bridge multilevel inverter, which offers a design with less power devices. This design offers a higher number of levels, but requires an isolated and separated DC source for each H-bridge [3].

This structure has been introduced the first time by Hammond in 1997, Figure 1c [3, 7]. The cascaded multilevel inverter has been previously designed for Static VAR compensator and motor drives, but the topology has been also used to interface with renewable energy sources, because it uses separated dc sources. [3, 4, 7], accordingly, the produced AC voltage is generated by connecting multiple Cells in series. Each cell will generate three levels  $+V_{dc}$ , 0 and  $-V_{dc}$ . Consequently, connecting  $n$  cells will generate  $2n + 1$  levels.

In addition to the three types described above, other Hybrid topologies have been developed, a diode-clamped or flying capacitor with a cascaded H-bridge. [4].

Resolving the inverters problem does not depend only on the hardware, it depends also on the sequence and the timing of controlling each switch in the structure, this can become complicated when the number of switches increases, the goal is to get a smooth clean signal at the output of the inverter, choosing the best switching time for each switch will significantly reduce the THD.

Usually solving the switching time problem, or finding the best switching angles requires resolving complicated nonlinear transcendental equations, the number of variables will in-

crease when the number of switches increases, therefore new methods have been introduced such as Particle Swarm Optimization (PSO), Genetic Algorithms (GA) and biogeographical based optimization (BBO) [9, 10, 11]. In our proposed solution, the reduction of the THD is accomplished by using the recent firefly algorithm (FFA).

The rest of the paper is organized as follows: in Section 2, the necessary theory background, signal synthesis, optimization problem definition, in Section 3, introduction of the firefly algorithm and proposition of solving the optimization problematic presented in Section 2, Section 4, contains the used hardware, application of the simulation results to build the 13-level inverters, Section 6, presentation of the experimental results and finally, the last section is a conclusion, reporting the benefits and features of the proposed method.

## 2. Background and problem statement

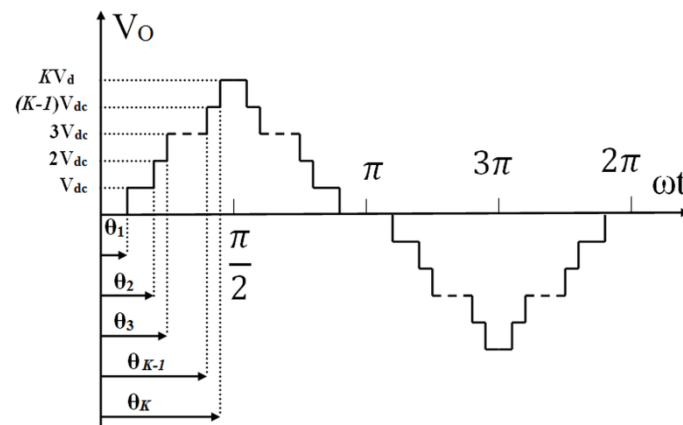


Fig. 2. Typical  $K$  Multilevel inverter output waveform

A typical  $K$  multilevel inverter voltage waveform will look like the waveform in Figure 2, where  $\theta_1, \theta_2, \dots, \theta_K$  are the switching angles,  $\theta_1 < \theta_2 < \dots, < \theta_K < \pi/2$  [10].

Since this function is periodic, as per the Fourier theory the function can be expressed by a sum of sines and cosines.

The output voltage function can be expressed by:

$$f(t) = a_0 + \sum_{n=1}^{\infty} A_n \cos(n\omega t) + B_n \sin(n\omega t), \quad (1)$$

where  $\omega = 2\pi/T$  and  $T$  is the period of the function.

The reference signal is half wave symmetry and odd at the same time, therefore we can write it as:

$$f(t) = \sum_{n=1}^{\infty} B_n \sin(n\omega t), \quad (2)$$

$$B_n = \frac{8}{T} \int_0^{T/4} f(t) \sin(n\omega t) dt. \quad (3)$$

For odd  $ns$ , zero for the others.

$$B_n = \frac{4V_{dc}}{n\pi} \sum_{i=1}^K \cos(n\theta_i). \quad (4)$$

For odd  $ns$ , and zero for even  $ns$  [9, 10],  $K$  is the number of switching angles.

$$f(\omega t) = \sum_{n=1,3,5}^{\infty} \frac{4V_{dc}}{n\pi} [\cos(n\theta_1) + \cos(n\theta_2) + \dots + \cos(n\theta_K)] \sin(n\omega t). \quad (5)$$

Our objective is to have a pure sine wave at the output. Therefore, it is possible to find the angles  $\theta_1 < \theta_2 < \dots < \theta_K < \pi/2$  in order to illuminate the lower order harmonics and have the THD at minimum (ideally zero). The desired output signal is in Equation (6).

$$f(\omega t) = V_1 \sin \omega t, \quad (6)$$

where  $V_1$  is the fundamental.

Actually, this will be achieved by targeting a maximum value of  $V_1$ , and minimizing the amplitude of the other harmonics. In order to solve our problem we have gathered all the parameters in the system of Equations (7) [9, 10].

One solution set is found by zeroing the  $K-1$  equations, in (7).

As an example, for  $K = 6$  the target solution will eliminate all the harmonics up to the 11<sup>th</sup> one.

The methods used to find solution include but are not limited to iterative approaches and optimization methods [9].

$$\left. \begin{aligned} B_1 &= \frac{4V_{dc}}{\pi} \sum_{i=1}^k \cos \theta_i = V_1, \\ B_3 &= \frac{4V_{dc}}{3\pi} \sum_{i=1}^k \cos 3\theta_i = V_3, \\ B_5 &= \frac{4V_{dc}}{5\pi} \sum_{i=1}^k \cos 5\theta_i = V_5, \\ &\dots \\ &\dots \\ B_n &= \frac{4V_{dc}}{n\pi} \sum_{i=1}^k \cos n\theta_i = V_n. \end{aligned} \right\} \quad (7)$$

For  $n = 1, 3, 5, \dots, \infty$  (for odd  $ns$ ) zero for the others. Where in this equations:  $\theta_1, \theta_2, \dots, \theta_K$  are the switching angles.

In order to process the optimization and find the best angles, we need to define an objective function, this one will need to maintain the fundamental harmonic at maximum, and

minimize the effects of the other harmonics up to a predefined range, under the constraint  $\theta_1 < \theta_2 < \dots < \theta_K < \pi/2$ .

One possible objective function is described in (8):

$$F(\theta_1, \theta_2, \dots, \theta_K) = (\sum_{i=1}^k \cos \theta_i - m)^2 + (\sum_{i=1}^k \cos 3\theta_i)^2 + (\sum_{i=1}^k \cos 5\theta_i)^2 \dots + (\sum_{i=1}^k \cos n\theta_i)^2, \quad (8)$$

where:

$$m = V_1 / (4V_{dc} / \pi) \quad (9)$$

and  $n$  is the highest rank of harmonics we want to reduce and is an odd number.

The best solution for  $(\theta_1, \theta_2, \dots, \theta_K)$  will be found by reducing  $F(\theta_1, \theta_2, \dots, \theta_K)$  under the constraint  $\theta_1 < \theta_2 < \dots < \theta_K < \pi/2$  [9, 10].

Another frequently used equivalent cost function is the Total Harmonic Distortion (THD). By reducing the THD we ensure that our signal strongest contributor is coming from the first harmonic.

$$F(\theta_1, \theta_2, \dots, \theta_K) = THD(\theta_1, \theta_2, \dots, \theta_K) = \sqrt{\frac{V_3^2 + V_5^2 + V_7^2 + \dots + V_n^2}{V_1^2}} \times 100, \quad (10)$$

where  $n$  is the order of the highest harmonic and needs to be predefined.

Now we have to choose an algorithm for optimization, this later needs to be able to reduce the function THD  $(\theta_1, \theta_2, \dots, \theta_K)$ , below a preferred value, once this value is reached, the optimization conditions are satisfied.

We have chosen to use the recent firefly algorithm to find the best solution and reduce the THD  $(\theta_1, \theta_2, \dots, \theta_K)$ .

### 3. Firefly algorithm

Today's world development and growth are calling for more and more resources, with the fact that most of the resources are limited, this reality has called for a new ways to manage the available resources, and optimization is one of these preferred methods. The evolution of computers has influenced the optimization of a wide spectrum of interests, as many new optimization methods are being developed, such as artificial intelligence and nature – inspired metaheuristic algorithms [13].

In the late of 2007 and early 2008, Xin-She Yang has developed a new firefly algorithm (FA), which is one of the metaheuristic algorithms, and is based on social behavior, interaction and communication between fireflies. Fireflies generate bioluminescence flashes to communicate with mates or to attract prey.

Three rules below are defining the firefly algorithm [13]:

- a) Fireflies are unisex so that one firefly will be attracted to other fireflies regardless of their sex.

- b) The attractiveness is proportional to the brightness, and they both decrease as the distance between fireflies increases. Thus for any two flashing fireflies, the less bright one will move towards the brighter one. If there is no brighter one than a particular firefly, they will move randomly.
- c) The brightness of a firefly is determined by the landscape of the objective function.

Fireflies' attractiveness is proportional to the light intensity seen by adjacent fireflies, we can now define the variation of fireflies attractiveness  $\beta$  with the distance  $r$  by:

$$\beta = \beta_0 e^{-\gamma r^2}, \quad (11)$$

where  $\beta_0$  is the attractiveness at  $r = 0$ .

Considering fireflies  $X_i$  and  $X_j$ ,  $r_{ij}$  is the distance between firefly  $X_i$  and  $X_j$ .

$$r_{ij} = \|X_i - X_j\|. \quad (12)$$

The attractiveness will be expressed as per Equation (13):

$$\beta = \beta_0 e^{-\gamma r_{ij}^2}. \quad (13)$$

The firefly  $i$  is attracted to another more attractive (brighter) firefly  $j$  according to movement equation:

$$X_i^{t+1} = X_i^t + \beta_0 e^{-\gamma r_{ij}^2} (X_j^t - X_i^t) + \alpha_t \varepsilon_i^t. \quad (14)$$

The second term is due to the attraction. The third term is randomization with  $\alpha_t$  the randomization parameter, and  $\varepsilon_i^t$  is a vector of random numbers drawn from a Gaussian distribution or uniform distribution at time  $t$  [13].

The FFA has been developed and executed through Matlab, the pseudo code of the FFA is given in Figure 5.

The work done in [15] has used more complex hardware structure, cascaded H-bridges, and has used the FFA not to optimize directly the THD, but to resolve the nonlinear transcendental equations. A similar approach has been also used in [16, 17], in our case we have used the THD  $(\theta_1, \theta_2, \dots, \theta_K)$  as the light intensity  $I$ , the best solution will check  $F(X_{\text{best}}) = \text{THD}(\theta_1^*, \theta_2^*, \dots, \theta_K^*)$  which is the lowest THD.

The optimization of the THD will guarantee the minimization of the harmonics within the selected bandwidth (in our case, 3 kHz bandwidth where the fundamental is 50 Hz, 60 harmonics were involved in the THD minimization), while in [15], the author is targeting the minimization of the 5<sup>th</sup>, 7<sup>th</sup>, 11<sup>th</sup> and 13<sup>th</sup> harmonics. This will not be efficient for the higher harmonics (from the 15th harmonic and above).

The FA pseudo code presented in Figure 3 has been translated to a Matlab program. In all performed simulations we have used the parameters below:

$$\begin{aligned} IT = 40 & \quad \text{maximum generation,} \\ P = 50 & \quad \text{population size,} \end{aligned}$$

$K = 6$       number of variables,  
 $\beta_0 = 100$     attractiveness at  $r = 0$ ,  
 $\gamma = 1$       light absorption coefficient,  
 $\alpha = 100$     randomization parameter.

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*Firefly Algorithm*

*Objective (Cost) function*

$f(\mathbf{x}), \mathbf{x} = (\mathbf{x}_1, \dots, \mathbf{x}_d)^T$

Generate initial population of fireflies  $\mathbf{x}_i$  ( $i = 1, 2, \dots, n$ )

Light intensity  $I_i$  at  $\mathbf{x}_i$  is evaluated as  $f(\mathbf{x}_i)$

Define light absorption coefficient  $\gamma$

while ( $t < \text{MaxGeneration}$ )

  for  $i = 1 : n$  all  $n$  fireflies

    for  $j = 1 : i$  all  $n$  fireflies

      if ( $I_i > I_j$ ) /\*in the case of maximization  $I_i < I_j$  \*/

        Move firefly  $i$  towards  $j$  in  $d$ -dimension;

      end if

    Attractiveness varies according to the distance  $r$  via  $\exp[-\gamma r]$

    Evaluate new solutions and update light intensity

  end for  $j$

end for  $i$

Rank the fireflies and find the current best

end while

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*Postprocess results and visualization*

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Fig. 3. Pseudo code of the firefly algorithm (FA) [14]

The best THD has been found to be 5% for  $N = 60$ ,  $N$  is the order of the highest harmonic of the spectrum, corresponds to 3 kHz.

The fitness function we have used is the THD ( $\theta_1, \theta_2, \dots, \theta_K$ ) of the produced signal based on the first  $N$  harmonics of the FFT. The order of the highest harmonic  $N$  has been chosen as 60 to cover for all the low frequencies, practically we are working up to 3 kHz.

We have built in a program to reduce the THD for a  $K$  level inverter. The program entries are the number of switching angles.

It is obvious that more switching angles will generate a better signal and therefore less THD. Below the simulation results for 3 and 6 angles corresponding to 7 and 13-level symmetric inverter.

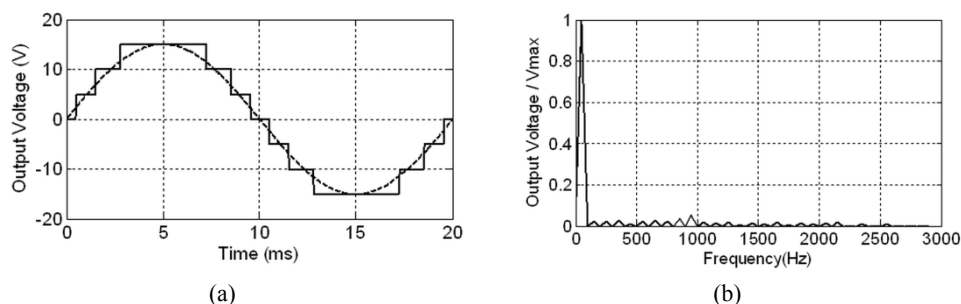


Fig. 4. Seven-level inverter output: time domain  $\theta_1 = 9.1^\circ$ ,  $\theta_2 = 27.5^\circ$ ,  $\theta_3 = 50.4^\circ$  (a); FFT, THD = 11% (b)

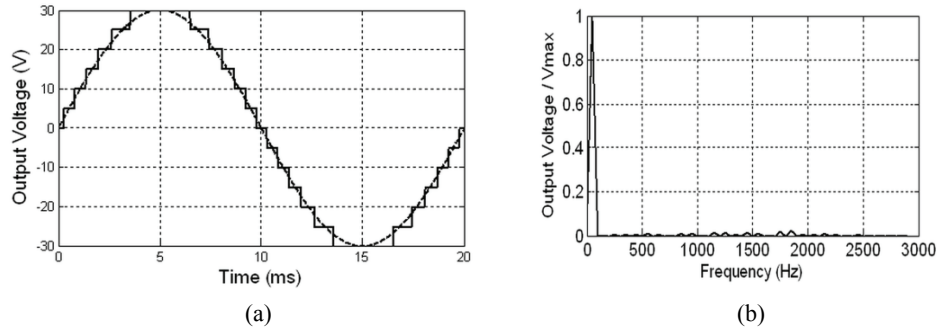


Fig. 5. Thirteen-level inverter output: time domain  $\theta_1 = 5.0^\circ$ ,  $\theta_2 = 14.3^\circ$ ,  $\theta_3 = 24.5^\circ$ ,  $\theta_4 = 35.3^\circ$ ,  $\theta_5 = 46.2^\circ$ ,  $\theta_6 = 63.7^\circ$  (a); FFT, THD = 5% (b)

Figure 4 shows the simulation results and the output of the 7-level inverter, Figure 4a shows the waveform in the time domain, with the perfect waveform, a pure sine wave, Figure 4b shows the FFT of the output signal.

Compared to the work done in [11], where the obtained THD through the simulation was 12.39%, the optimized angles were found through the BBO algorithm, the waveform is similar to the one generated by a symmetrical multilevel inverter, with a step of 100. While in our proposed method we have reached 11% THD, the result obtained in running 10 iterations of the built program which was based on the recent FFA.

In order to reach the 5% THD we had to go up to a 13-level inverter, simulation results are shown in Figure 5.

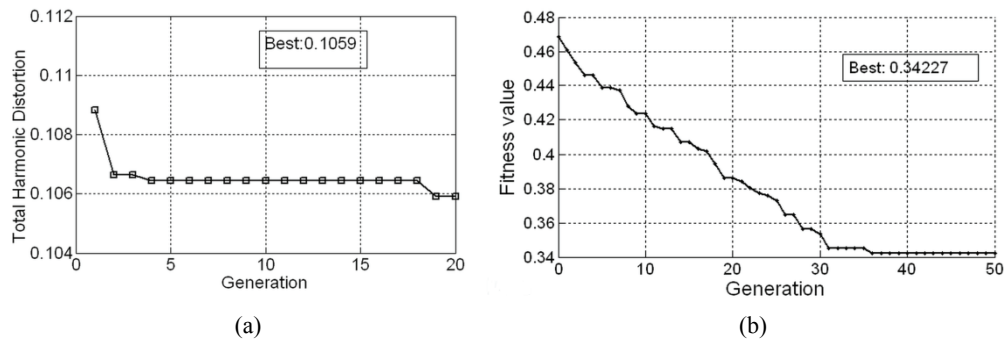


Fig. 6. Firefly algorithm THD vs. generation: (a) genetic algorithm THD vs. generation (b)

We have also compared the proposed FFA to the GA tool on Matlab, see Figure 6, the proposed FFA with the proposed parameters converges faster, it is 90% close to the optimum solution after the fourth iteration and usually the optimum solution is obtained before the 10<sup>th</sup> iteration. When using the GA with the same parameters,  $IT = 20$ , population size = 50, the best obtained fitness value with the GA is 3.9 times the value obtained by the FFA. To obtain similar result with the GA we need 150 generations.



#### 4. Hardware implementation

For the experiment we have chosen an already optimized structure, presented by Ebrahimi et al., in [18].

Figure 7a shows the sub-multilevel inverter topology proposed in [18], the typical output voltage of the sub-multilevel inverter is given in Figure 7b.

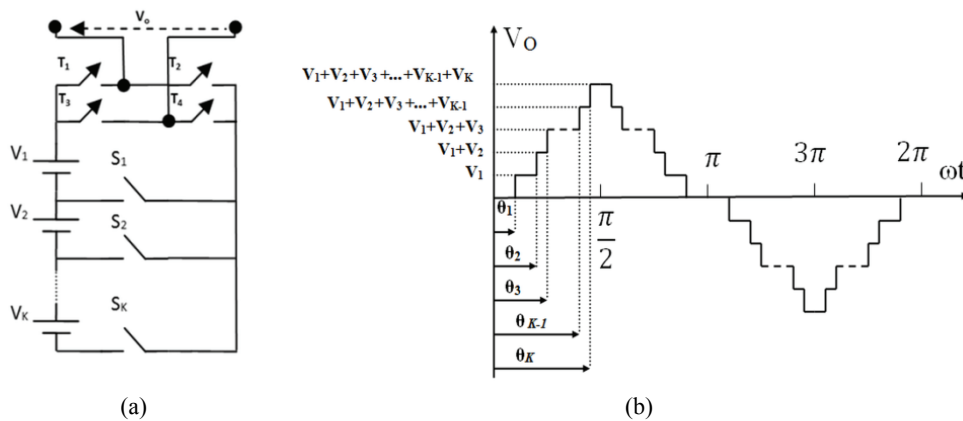


Fig. 7. Sub-multilevel topology [18] (a); typical output voltage of MLI in [18] (b)

The block diagram of the hardware implementation is shown in Figure 8. The thirteen-level inverter was built around a microchip microcontroller type PIC16F628A.

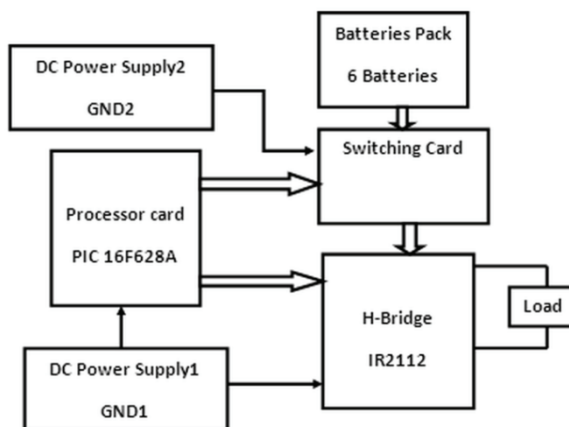


Fig. 8. Block diagram of the multilevel inverter

The multilevel inverter consists of 4 major parts:

- 1) Processor card: this is the brain of the inverter; it generates all the switching angles to activate the switches, to generate the exact angles, to minimize the THD. The best

values found through the simulation have been programmed on the PIC 16F628A, targeting the 5% THD.

- 2) Switching card: this card contains the switching devices and isolates the control from the power (photo couplers), it takes controls from the processor card and switches power from the batteries pack to the H-bridge.
- 3) Batteries pack: directly connected to the switching card and both make the level generator.
- 4) H-bridge card: the input of the H-bridge is a half wave, the H-bridge will make the full wave at its output, and feeds the load. The H-bridge is also controlled by the micro-processor.

Three prototype cards have been made: a processor card, level generator card and the H-bridge card.

## 5. Experimental tests and discussion

Tests have been conducted with different loads to check the performance of the MLI. The first set of experiments has been done with a resistive load. Figure 9 shows experimental hardware, the 13-level inverter running with a resistive load, and an ITT OX 710c oscilloscope. Please note that we have also used a USB oscilloscope, type PicoScope 3204, this one was used to view the signal, the spectrum and to measure the frequency and the THD.

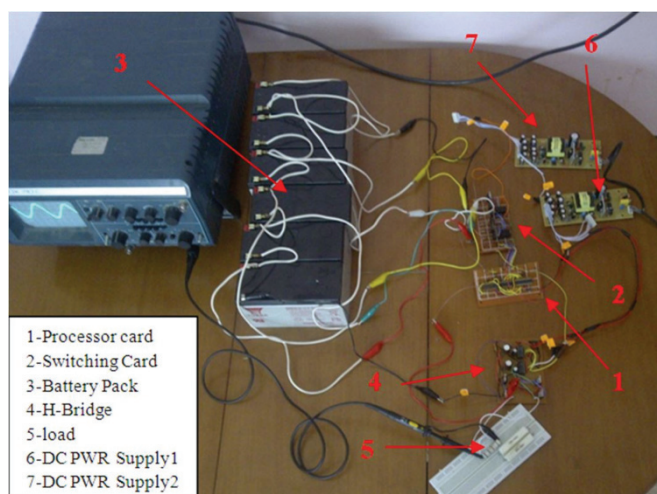


Fig. 9. The 13-level inverter running with resistive load

The experimental results have confirmed the theory, with an R load the THD was 4.6% on a spectrum of 3 kHz. Figure 10 shows the current with a resistive load, on a digital scope, the signal frequency is 50 Hz, while Figure 11 shows the spectrum of the output signal on a 3 kHz bandwidth. The measured value of the THD was 4.6%. This is a satisfactory result as we were targeting 5% THD.

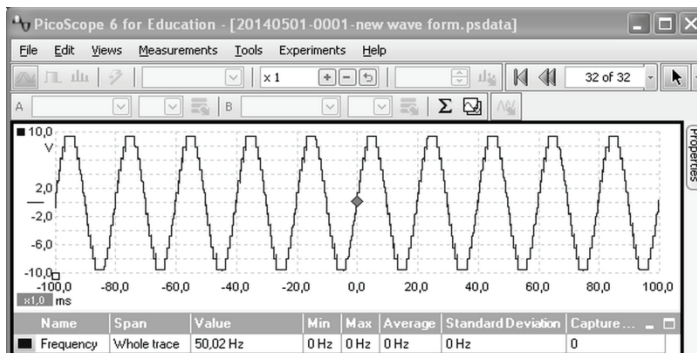


Fig. 10. current waveform of the 13-level inverter with a resistive load

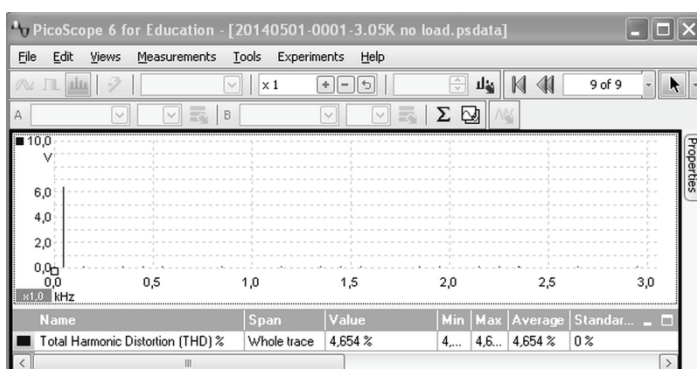


Fig. 11. Current FFT on 3 kHz bandwidth of the 13-level inverter with a resistive load THD = 4.65%

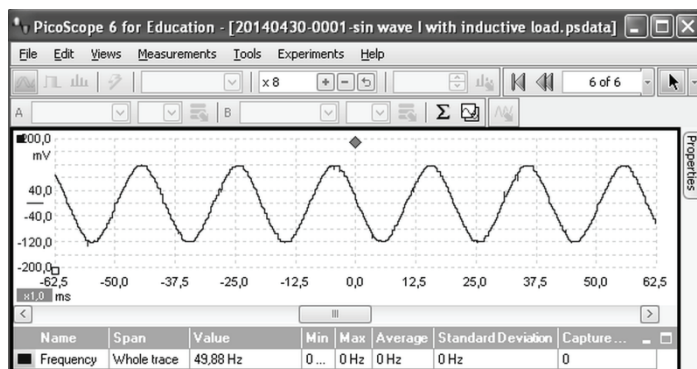


Fig. 12. Current waveform of the 13-level inverter with an inductive load  $L = 100$  mH

The second set of experiments was done with a 100 mH induction load. Figure 12 shows the current waveform through the inductive load. More harmonics have been filtered and we see that the resulting THD is 3.2%, this is shown in Figure 13.

Figure 14a shows the shape of the output voltage of the 13-level inverter on an analogue scope ITT OX 710 C. Figure 14b shows the current through a motor load, current pick is 2.5 A.

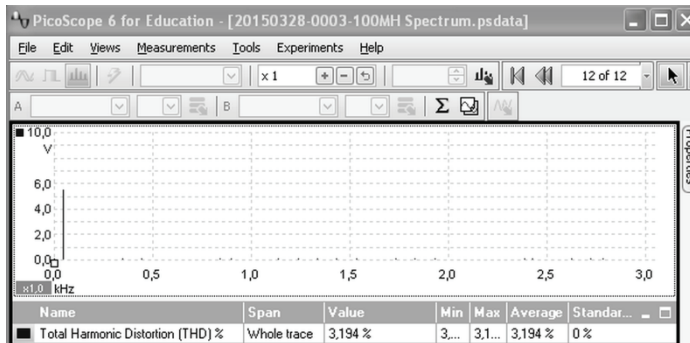


Fig. 13. Current FFT on 3 kHz bandwidth THD = 3.2%. The 13-level inverter with an inductive load,  $L = 100$  mH

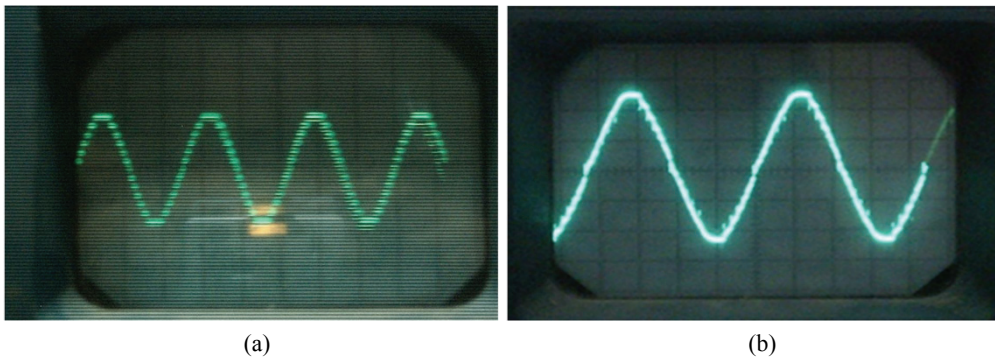


Fig. 14. 13-level inverter: output voltage on an analogue scope ITT OX 710 C (a), current through motor load on an analogue scope ITT OX 710 C (b)

## 8. Conclusion

In this paper, the recent firefly algorithm has been used to find the best switching angles in order to minimize the THD on a user defined bandwidth. The method focuses on optimizing directly the THD and not to resolve the equations or minimizes a selected number of harmonics. The THD over the predefined range was minimized. The best THD found through the simulation is 5% on a bandwidth of 3 kHz.

One of the best optimized hardware structures of MLI has been used [18] to verify and validate the simulation results.

The signal generated by 13-level symmetric inverter has a low THD, and the measured THD is 4.8% on a bandwidth of 3 kHz, which is a satisfactory result. Simulation results as well as experimental results have been shared to prove the performance of software solution and the used hardware structure.

Under the same conditions (3 kHz bandwidth and 50 Hz fundamental), we have compared the proposed FFA to the BBO algorithm developed in [11] and the GA implemented in Matlab. Obtained THD by the BBO algorithm [11] was 12.39% while the obtained THD by using the suggested FFA was 11%.

In this specific problem, the FFA, compared to the GA, converged faster.

As for future development, we need to mention that the work performed in [19, 20] is an excellent work, and can be considered as one of the opportunities of expansion and improvement of the actual work, instead of choosing the staircase basic waveform as a target output voltage, we can choose to combine it with the PWM, this will reduce the number of levels, therefore the required hardware, the FFA will allow us to find the optimized switching angles to reduce the THD even further.

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