Kołowrocki Krzysztof
ORCID ID: 0000-0002-4836-4976

Kuligowska Ewa
ORCID ID: 0000-0002-6704-9541

Soszyńska-Budny Joanna
ORCID ID: 0000-0003-1525-9392

Torbicki Mateusz
ORCID ID: 000-0003-4182-1932
Gdynia Maritime University, Gdynia, Poland

Integrated impact model on critical infrastructure safety related to its operation process including operating environment threat and climate-weather change process including extreme weather hazards

Keywords

Integrated model, critical infrastructure, operation process, environment threat, weather hazard, safety.

Abstract

The paper presents the general safety analytical model of a complex technical system under the influence of the operation process including its operating environment threats related to climate-weather change process, The system operation process including environment threats under influence of climate-weather variable conditions is defined. Moreover, the safety model of multistate systems at variable conditions related to operating environment threats and extreme weather hazards is proposed. The conditional safety functions at the operation process including operating environment threats and related to climate-weather change process particular states, the unconditional safety function and the risk function of the complex system at changing in time operation and climate-weather conditions and other, practically significant, critical infrastructure safety indices are defined. Furthermore, the same safety indicators are defined for exponential systems.

1. Introduction

The time dependent interactions between the operation process including operating environment threats related to the climate-weather change process including extreme weather hazards states varying at the system operating area and the system safety structure and its components safety states changing are evident features of most real technical systems, including critical infrastructures as well. The common critical infrastructure safety and the operation process related to climate-weather change process at its operating area analysis is of great value in the industrial practice because of negative impacts of operating environment threats and extreme weather hazards on the critical infrastructure safety. The convenient tools for analysing this problem are the multistate critical infrastructures safety modelling [Kołowrocki, Soszyńska-Budny, 2011; Xue, 1985; Xue, Yang, 1995a-b] commonly used with the semi-Markov modelling [Ferreira, Pacheco, 2007; Glynn, Hass, 2006; Grabski, 2014; Kołowrocki 2005; Limnios, Oprisan, 2005; Mercier 2008] of the operation processes related to operating environment threats and extreme weather hazards at their operating areas, leading to the construction the joint general safety models of the critical infrastructures under the influence of their operation processes including operating environment threats and climate-weather change processes including extreme weather hazards at their operating areas.

Thus, in the paper the operating environment threats and climate-weather hazards influence on the safety of a critical infrastructure are presented. In the result of research, a general safety analytical model of a complex technical system under the influence of the operation process including its operating environment threats related to climate-weather change process is proposed. It is the integrated model of complex technical system safety, linking its multistate safety model and the model of its operation process including operating environment threats and related to climate-weather change process at its operating area, considering variable at the different operation and climate-weather states impacted by them the system safety structures and its components safety parameters. The conditional safety functions at the operation process including operating environment threats and related to climate-weather change process particular states, the unconditional safety function and the risk function of the complex system at changing in time operation and climate-weather conditions are defined. Other, practically significant, critical infrastructure safety indices introduced in the report are its mean lifetime up to the exceeding a critical safety state, the moment when its risk function value exceeds the acceptable safety level, the intensities of ageing of the critical infrastructure and its components and the coefficients of the operating environment threats and climate-weather hazards impact on the critical infrastructure and its components intensities of ageing are defined.

In the case of critical infrastructure safety analysis, the determination of its safety function and its risk function which graph corresponds to the fragility curve are crucial indices for its operators and users. These safety indices are defined in general for any critical infrastructure.

2. System operation process including operating environment threats related to climate-weather variable conditions

2.1. System operation process including operating environment threats

We assume that the critical infrastructure during its operation process including operating environment threats is taking $v', v' \in N$, different operation states $z'_1, z'_2, \ldots, z'_{v'}$. Further, we define the critical infrastructure operation process including operating environment threats $Z'(t)$, $t \in <0,+\infty)$, with discrete operation states from the set $\{z'_1, z'_2, \ldots, z'_v\}$.

Moreover, we assume that the critical infrastructure operation process including operating environment threats $Z'(t)$ is a semi-Markov process [Grabski, 2002], [Limnios, 2005], [Mercier, 2008], [Soszyńska, 2007], [Kołowrocki, Soszyńska-Budny, 2011] with the conditional sojourn times θ'_{bl} at the operation states z'_b when its next operation state is z'_l, $b, l = 1,2,...,v'$, $b \neq l$. Under these assumptions, the critical infrastructure operation process including operating environment threats may be described by:

– the vector $[p'_b(0)]_{1 \times v'}$ of the initial probabilities $p'_b(0) = P(Z'(0) = z'_b)$, $b = 1,2,...,v'$, of the critical infrastructure operation process including operating environment threats $Z'(t)$ staying at particular operation states at the moment $t = 0$;
– the matrix $[p'_{bl}]_{v' \times v'}$ of probabilities p'_{bl}, $b, l = 1,2,...,v'$, of the critical infrastructure operation process including operating environment threats $Z'(t)$ transitions between the operation states z'_b and z'_l;
– the matrix $[H'_{bl}(t)]_{v' \times v'}$ of conditional distribution functions $H'_{bl}(t) = P(\theta'_{bl} < t)$, $b, l = 1,2,...,v'$, of the critical infrastructure operation process including operating environment threats $Z'(t)$ conditional sojourn times θ'_{bl} at the operation states.

As the mean values of the conditional sojourn times θ'_{bl} are given by [Kołowrocki, Soszyńska-Budny, 2011]

$$M'_{bl} = E[\theta'_{bl}] = \int_0^\infty t dH'_{bl}(t) = \int_0^\infty t h'_{bl}(t) dt,$$
$$b, l = 1,2,...,v', \ b \neq l, \tag{1}$$

From the formula for total probability, it follows that the unconditional distribution functions of the sojourn times θ'_b, $b = 1,2,...,v'$, of the system operation process including operating environment threats $Z'(t)$ at the operation states z'_b, $b = 1,2,...,v'$, are given by [Kołowrocki, Soszyńska-Budny, 2011]

$$H'_b(t) = \sum_{l=1}^{v} p'_{bl} H_{bl}(t), \ b = 1,2,...,v'. \tag{2}$$

Hence, the mean values $E[\theta'_b]$ of the system operation process $Z'(t)$ unconditional sojourn times θ'_b, $b = 1,2,...,v'$, at the operation states are given by

$$M'_b = E[\theta'_b] = \sum_{l=1}^{v'} p'_{bl}\, M_{bl}\,,\; b = 1,2,...,v', \qquad (3)$$

where M'_{bl} are defined by the formula (1) in a case of any distribution of sojourn times θ'_{bl} and by the formulae (2)-(8) in the cases of particular defined respectively by (2.5)-(2.11) [EU-CIRCLE Report D2.1-GMU2, 2016], distributions of these sojourn times.

The limit values of the system operation process $Z'(t)$ transient probabilities at the particular operation states

$$p'_b(t) = P(Z'(t) = z_b)\,,\; t \in <0,+\infty),\; b = 1,2,...,v',$$

are given by [Kołowrocki, Soszyńska-Budny, 2011]

$$p'_b = \lim_{t\to\infty} p'_b(t) = \frac{\pi_b M'_b}{\sum_{l=1}^{v'} \pi_l M'_l},\; b = 1,2,...,v', \qquad (4)$$

where M'_b, $b = 1,2,...,v'$, are given by (10), while the steady probabilities π_b of the vector $[\pi_b]_{1xv'}$ satisfy the system of equations

$$\begin{cases} [\pi_b] = [\pi_b][p'_{bl}] \\ \sum_{l=1}^{v'} \pi_l = 1. \end{cases} \qquad (5)$$

In the case of a periodic system operation process including operating environment threats, the limit transient probabilities p'_b, $b = 1,2,...,v'$, at the operation states defined by (4), are the long term proportions of the system operation process $Z'(t)$ sojourn times at the particular operation states z'_b, $b = 1,2,...,v'$.

Other interesting characteristics of the system operation process including operating environment threats $Z'(t)$ possible to obtain are its total sojourn times $\hat{\theta}'_b$ at the particular operation states z'_b, $b = 1,2,...,v$, during the fixed system opetation time. It is well known [Kołowrocki, Soszyńska-Budny, 2011] that the system operation process total sojourn times $\hat{\theta}'_b$ at the particular operation states z'_b, for sufficiently large operation time θ, have approximately normal distributions with the expected value given by

$$\hat{M}'_b = E[\hat{\theta}'_b] = p'_b\,\theta,\; b = 1,2,...,v', \qquad (6)$$

where p'_b are given by (4).

2.2. Climate-weather change process including extreme weather hazards

To model the climate-weather change process for the critical infrastructure operating area we assume that the climate-weather in this area is taking w, $w \in N$, different climate-weather states $c_1, c_2, \ldots, c_w$. Further, we define the climate-weather change process $C(t)$, $t \in <0,+\infty)$, with discrete operation states from the set $\{c_1, c_2, \ldots, c_w\}$. Assuming that the climate-weather change process $C(t)$ is a semi-Markov process it can be described by [EU-CIRCLE Report D, 2016]:

- the vector $[q_b(0)]_{1Xw}$ of the initial probabilities $q_b(0) = P(C(0) = c_b)$, $b = 1,2,\ldots, w$, of the climate-weather change process $C(t)$ staying at particular climate-weather states c_b at the moment $t = 0$;
- the matrix $[q_{bl}]_{w\mathrm{x}w}$ of the probabilities of transitions q_{bl}, b, l = 1,2,…, w, $b \neq l$, of the climate-weather change process $C(t)$ from the climate-weather states c_b to c_l;
- the matrix $[C_{bl}(t)]_{w\mathrm{x}w}$ of the conditional distribution functions $C_{bl}(t) = P(C_{bl} < t)$, b, l = 1,2,…, w, of the conditional sojourn times C_{bl} at the climate-weather states c_b when its next climate-weather state is c_l, $b, l = 1,2,\ldots, w$, $b \neq l$.

Assuming that we have identified the above parameters of the climate-weather change process semi-Markov model, we can predict this process basic characteristics.

The mean values of the conditional sojourn times C_{bl}, are given by [EU-CIRCLE Report D2.1-GMU3, 2016]

$$N_{bl} = E[C_{bl}] = \int_0^\infty t\,dC_{bl}(t) = \int_0^\infty t\,c_{bl}(t)\,dt,$$
$$b, l = 1,2,...,w. \qquad (7)$$

From the formula for total probability, it follows that the unconditional distribution functions of the sojourn times C_b, $b = 1,2,...,w$, of the climate-weather change process $C(t)$ at the climate-weather states c_b $b = 1,2,...,w$, are given by [EU-CIRCLE Report D2.1-GMU3, 2016]

$$C_b(t) = \sum_{l=1}^{v} q_{bl} C_{bl}(t),\; b = 1,2,...,w, \qquad (8)$$

Hence, the mean values $E[C_b]$ of the climate-weather change process $C(t)$ unconditional sojourn times C_b, $b = 1,2,...,w$, at the climate-weather states are given by

$$N_b = E[C_b] = \sum_{l=1}^{v} q_{bl} N_{bl}\,,\; b = 1,2,...,w, \qquad (9)$$

where N_{bl} are defined by the formula (7) in a case of any distribution of sojourn times C_{bl} and by the formulae (3.2)-(3.8) given in [EU-CIRCLE Report D2.1-GMU3, 2016] in the cases of particular defined respectively by (2.5)-(2.11) in [EU-CIRCLE Report D2.1-GMU2, 2016], distributions of these sojourn times.

The limit values of the climate-weather change process $C(t)$ transient probabilities at the particular operation states

$$q_b(t) = P(C(t) = c_b), \ t \in <0,+\infty), \quad b = 1,2,...,w, \tag{10}$$

are given by [EU-CIRCLE Report D2.1-GMU3, 2016],

$$q_b = \lim_{t\to\infty} q_b(t) = \frac{\pi_b N_b}{\sum_{l=1}^{v} \pi_l N_l}, \ b = 1,2,...,w, \tag{11}$$

where N_b, $b = 1,2,...,w$, are given by (9), while the steady probabilities π_b of the vector $[\pi_b]_{1\times w}$ satisfy the system of equations

$$\begin{cases} [\pi_b] = [\pi_b][q_{bl}] \\ \sum_{l=1}^{v} \pi_l = 1. \end{cases} \tag{12}$$

In the case of a periodic climate-weather change process, the limit transient probabilities q_b, $b = 1,2,...,w$, at the climate-weather states defined by (11), are the long term proportions of the climate-weather change process $C(t)$ sojourn times at the particular climate-weather states C_b, $b = 1,2,...,w$.

Other interesting characteristics of the system climate-weather change process $C(t)$ possible to obtain are its total sojourn times $\hat{C}_b$ at the particular climate-weather states c_b , $b = 1,2,...,w$, during the fixed time. It is well known, [Kołowrocki, Soszyńska-Budny, 2011], [EU-CIRCLE Report D2.1-GMU3, 2016], that the climate-weather change process total sojourn times $\hat{C}_b$ at the particular climate-weather states c_b for sufficiently large time C have approximately normal distributions with the expected value given by

$$\hat{N}[\hat{C}_b] = q_b C, \ b = 1,2,...,w, \tag{13}$$

where q_b are given by (11).

2.3. Critical infrastructure operation process related to operating environment threats and extreme weather hazards

Assuming that we have identified the unknown parameters of the critical infrastructure operation process related to operating environment threats and extreme weather hazards $Z'C(t)$, $t \in <0,+\infty)$, that can take $v'w, v', w \in N$, different operation states $z'c_{11}, z'c_{12}, \ldots, z'c_{v'w}$, described by :

- the vector $[p'q_{ij}(0)]_{1xv'w}$ of initial probabilities of the critical infrastructure operation process related to operating environment threats and extreme weather hazards $Z'C(t)$ staying at the initial moment $t = 0$ at the operation and climate-weather states $z'c_{ij}$, $i = 1,2,...,v' \in N$, $j = 1,2,..., w$;
- the matrix $[pq_{ij\,kl}]_{v'wxv'w}$ of the probabilities of transitions of the critical infrastructure operation process related to operating environment threats and extreme weather hazards $Z'C(t)$ between the operation states $z'c_{ij}$ and $z'c_{kl}$, $i = 1,2,...,v'$, $j = 1,2,..., w$, $k = 1,2,...,v'$, $l = 1,2,..., w$;
- the matrix $[H'C_{ij\,kl}(t)]_{v'wxv'w}$ of the matrix of conditional distribution functions of the critical infrastructure operation process related to operating environment threats and extreme weather hazards $Z'C(t)$ conditional sojourn times $\theta'C_{ij\,kl}$, $i = 1,2,...,v'$, $j = 1,2,..., w$, $k = 1,2,...,v'$, $l = 1,2,..., w$, at the operation state $z'c_{ij}$, $i = 1,2,...,v'$, $j = 1,2,..., w$, when the next operation state is $z'c_{kl}$, $k = 1,2,...,v'$, $l = 1,2,..., w$,

we can predict this process basic characteristics.

2.4. Critical infrastructure operation process related to operating environment threats and extreme weather hazards characteristics - independent critical infrastructure operation process and climate-weather change process

The mean values of the conditional sojourn times $\theta'C_{ij\,kl}$, $i = 1,2,...,v'$, $j = 1,2,..., w$, $k = 1,2,...,v'$, $l = 1,2,..., w$, at the operation state $z'c_{ij}$, $i = 1,2,...,v'$, $k = 1,2,..., w$, when the next operation state is $zc_{\substack{j\\kl}}$, $k = 1,2,...,v'$, $l = 1,2,..., w$, are defined by [Kołowrocki, Soszyńska-Budny, 2011]

$$M'N_{ij\,kl} = E[\theta' C_{ij\,kl}] = \int_0^\infty t dH' C_{ij\,kl}(t)dt$$
$$= \int_0^\infty t h' c_{ij\,kl}(t)dt,\ i = 1,2,\dots,\nu',\ \ j = 1,2,\dots,w,$$
$$k = 1,2,\dots,\nu',\ \ l = 1,2,\dots,w. \quad (14)$$

In the case when the processess $Z'(t)$ and $C(t)$ are independent, according to (23) the expessions (14) tasks the form

$$M'N_{ij\,kl} = E[\theta' C_{ij\,kl}]$$
$$= \int_0^\infty t[h'_{ik}(t)C_{jl}(t) + H'_{ik}(t)c_{jl}(t)]dt,\ i = 1,2,\dots,\nu',$$
$$j = 1,2,\dots,w,\ \ k = 1,2,\dots,\nu',\ \ l = 1,2,\dots,w. \quad (15)$$

Since from the formula for total probability, it follows that the unconditional distribution functions of the conditional sojourn times $\theta' C_{ij}$, of the critical infrastructure operation process related to operating environment threats and extreme weather hazards $Z'C(t)$ at the operation states state $z'c_{ij}$, $i = 1,2,\dots,\nu'$, $j = 1,2,\dots,w$, are given by

$$H'C_{ij}(t) = \sum_{k=1}^{\nu'}\sum_{l=1}^{w} p'_{ij\,kl} H'C_{ij\,kl}(t),\ t \in <0,+\infty),$$
$$i = 1,2,\dots,\nu',\ \ j = 1,2,\dots,w, \quad (16)$$

In the case when the processess $Z'(t)$ and $C(t)$ are independent, according to (4) and (11) the expessions (16) tasks the form

$$H'C_{ij}(t) = \sum_{k=1}^{\nu'}\sum_{l=1}^{w} p'_{ik}\, q_{jl} H'_{ik}(t)C_{jl}(t),\ t \in <0,+\infty),$$
$$i = 1,2,\dots,\nu',\ \ j = 1,2,\dots,w, \quad (17)$$

From (16) it follows that the mean values $E[\theta' C_{ij}]$ of the unconditional distribution functions of the conditional sojourn times $\theta' C_{ij}$, of the critical infrastructure operation process related to operating environment threats and extreme weather hazards $Z'C(t)$ at the operation states $z'c_{ij}$, $i = 1,2,\dots,\nu'$, $j = 1,2,\dots,w$, are given by

$$M'N_{ij} = E[\theta' C_{ij}] = \sum_{k=1}^{\nu'}\sum_{l=1}^{w} p'_{ij\,kl} M'N_{ij\,kl},$$
$$i = 1,2,\dots,\nu',\ \ j = 1,2,\dots,w, \quad (18)$$

where $M'N_{ij\,kl}$ are given by the formula (14).

In the case when the processess $Z'(t)$ and $C(t)$ are independent, considering (17) and (18) [EU-CIRCLE Report D2.1-GMU3, 2016] the expession (18) takes the form

$$M'N_{ij} = E[\theta' C_{ij}] = \sum_{k=1}^{\nu'}\sum_{l=1}^{w} p'_{ik}\, q_{jl} M'N_{ij\,kl},$$
$$i = 1,2,\dots,\nu',\ \ j = 1,2,\dots,w, \quad (19)$$

where $M'N_{ij\,kl}$ are given by the formula (15).

The transient probabilities of the critical infrastructure operation process related to operating environment threats and extreme weather hazards $Z'C(t)$ at the operation states $z'c_{ij}$, $i = 1,2,\dots,\nu'$, $j = 1,2,\dots,w$, can be defined by

$$p'q_{ij}(t) = P(Z'C(t) = z'c_{ij}),\ t \in <0,+\infty),$$
$$i = 1,2,\dots,\nu',\ \ j = 1,2,\dots,w. \quad (20)$$

In the case when the processess $Z'(t)$ and $C(t)$ are independent the expession (20) for the transient probabilities can be expressed in the following way

$$p'q_{ij}(t) = P(Z'C(t) = z'c_{ij})$$
$$= P(Z'(t) = z'_i \cap C(t) = c_j)$$
$$= P(Z'(t) = z'_i) \cdot P(C(t) = c_j) = p'_i(t) \cdot q_j(t),$$
$$t \in <0,+\infty),\ i = 1,2,\dots,\nu',\ \ j = 1,2,\dots,w, \quad (21)$$

where

$$p'_i(t) = P(Z'(t) = z'_i),\ t \in <0,+\infty),\ i = 1,2,\dots,\nu', \quad (22)$$

are the transient probabilities of the operation process $Z'(t)$ defined in Chapter 2 and

$$q_j(t) = P(C(t) = c_j),\ t \in <0,+\infty),$$
$$j = 1,2,\dots,w, \quad (23)$$

are the transient probabilities of the climate-weather change process $C(t)$ defined in Chapter 4.

The limit values of the critical infrastructure operation process related to operating environment threats and extreme weather hazards $Z'C(t)$ at the operation states $z'c_{ij}$, $i = 1,2,\dots,\nu'$, $j = 1,2,\dots,w$, can be found from [Kołowrocki, Soszyńska-Budny, 2011]

$$p'q_{ij} = \lim_{t\to\infty} \frac{\pi_{ij} M'N_{ij}}{\sum_{i=1}^{\nu'}\sum_{j=1}^{w} \pi_{ij} M'N_{ij}},\ i = 1,2,\dots,\nu',$$
$$j = 1,2,\dots,w, \quad (24)$$

where $M'N_{ij}$, $i=1,2,...,v'$, $j=1,2,...,w$, are given by (19), while the steady probabilities π_{ij}, $i=1,2,...,v'$, $j=1,2,...,w$, of the vector $[\pi_{ij}]_{1xv'w}$ satisfy the system of equations

$$\begin{cases} [\pi_{ij}][p'q_{ij\,kl}] = [\pi_{ij}] \\ \sum_{i-=1}^{v'}\sum_{j=1}^{w}\pi_{ij} = 1, \end{cases} \quad (25)$$

where $p'q_{ij\,kl}$, $i=1,2,...,v'$, $j=1,2,...,w$, $k=1,2,...,v'$, $l=1,2,...,w$, are given by (24).

In the case of a periodic system operation process related to operating environment threats and extreme weather hazards, the limit transient probabilities $p'q_{ij}$, $i=1,2,...,v'$, $j=1,2,...,w$, at the operation states given by (24), are the long term proportions of the critical infrastructure operation process $Z'C_{ij}(t)$ sojourn times at the particular operation states $z'c_{ij}$, $i=1,2,...,v'$, $j=1,2,...,w$.

Other interesting characteristics of the critical infrastructure operation process related to operating environment threats and extreme weather hazards $Z'C_{ij}(t)$ possible to obtain are its total sojourn times $\hat{\theta}'\hat{C}_{ij}$, $i=1,2,...,v'$, $j=1,2,...,w$, at the particular operation states zc_{ij}, $i=1,2,...,v$, $j=1,2,...,w$, during the fixed system opetation time. It is well known [Kołowrocki, Soszyńska-Budny, 2011] that the system operation process total sojourn times $\hat{\theta}'\hat{C}_{ij}$, at the particular operation states $z'c_{ij}$, for sufficiently large operation time θ, have approximately normal distributions with the expected value given by

$$\hat{M}'\hat{N}_{ij} = E[\hat{\theta}'\hat{C}_{ij}] = p'q_{ij}\theta, \; i=1,2,...,v', \; j=1,2,...,w, \quad (26)$$

where $p'q_{ij}$, $i=1,2,...,v'$, $j=1,2,...,w$, are given by (24).

2.5. Critical infrastructure operation process related to operating environment threats and extreme weather hazards characteristics - dependent critical infrastructure operation process and climate-weather change process

The mean values of the conditional sojourn times $\theta'C_{ij\,kl}$, $i=1,2,...,v'$, $j=1,2,...,w$, $k=1,2,...,v'$, $l=1,2,...,w$, at the operation state $z'c_{ij}$, $i=1,2,...,v'$, $j=1,2,...,w$, when the next operation state is $z'c_{kl}$, $k=1,2,...,v'$, $l=1,2,...,w$, are defined by [Kołowrocki, Soszyńska-Budny, 2011]

$$M'N_{ij\,kl} = E[\theta'C_{ij\,kl}] = \int_0^\infty t dH'C_{ij\,kl}(t)dt = \int_0^\infty t h'c_{ij\,kl}(t)dt, \; i=1,2,...,v', \; j=1,2,...,w, \; k=1,2,...,v', \; l=1,2,...,w. \quad (27)$$

Since from the formula for total probability, it follows that the unconditional distribution functions of the conditional sojourn times $\theta'C_{ij}$, of the critical infrastructure operation process related to operating environment threats and extreme weather hazards $Z'C(t)$ at the operation states state $z'c_{ij}$, $i=1,2,...,v'$, $j=1,2,...,w$, are given by

$$H'C_{ij}(t) = \sum_{k=1}^{v'}\sum_{l=1}^{w} p'_{ij\,kl} H'C_{ij\,kl}(t), \; t \in <0,+\infty), \; i=1,2,...,v', \; j=1,2,...,w, \quad (28)$$

Hence, the mean values $E[\theta'C_{ij}]$ of the unconditional distribution functions of the conditional sojourn times $\theta'C_{ij}$, of the critical infrastructure operation process related to operating environment threats and extreme weather hazards $Z'C(t)$ at the operation states $z'c_{ij}$, $i=1,2,...,v'$, $j=1,2,...,w$, are given by

$$M'N_{ij} = E[\theta'C_{ij}] = \sum_{k=1}^{v'}\sum_{l=1}^{w} p'_{ij\,kl} M'N_{ij\,kl}, \; i=1,2,...,v', \; j=1,2,...,w, \quad (29)$$

where $M'N_{ij\,kl}$ are defined by the formula (27).

The transient probabilities of the critical infrastructure operation process related to operating environment threats and extreme weather hazards $Z'C(t)$ at the operation states $z'c_{ij}$, $i=1,2,...,v'$, $j=1,2,...,w$, can be defined by

$$p'q_{ij}(t) = P(Z'C(t) = z'c_{ij}), \; t \in <0,+\infty), \; i=1,2,...,v', \; j=1,2,...,w. \quad (30)$$

In the case when the processess $Z'(t)$ and $C(t)$ are dependent the transient probabilities can be expresed either by

$$p'q_{ij}(t) = P(Z'C(t) = z'c_{ij})$$
$$= P(Z'(t) = z'_i \cap C(t) = c_j)$$
$$= P(Z'(t) = z'_i) \cdot P(C(t) = c_j \mid Z'(t) = z'_i)$$
$$= p'_i(t) \cdot q_{j/i}(t),\ t \in <0,+\infty),\ i = 1,2,...,\nu',$$
$$j = 1,2,...,w, \quad (31)$$

where

$$p'_i(t) = P(Z'(t) = z'_i),\ t \in <0,+\infty),$$
$$i = 1,2,...,\nu', \quad (32)$$

are transient probabilities of the operation process *Z'*(*t*) defined in Chapter 2 and

$$q_{j/i}(t) = P(C(t) = c_j \mid Z'(t) = z'_i),\ t \in <0,+\infty),$$
$$i = 1,2,...,\nu',\ \ j = 1,2,...,w, \quad (33)$$

are conditional transient probabilities of the climate-weather change process *C*(*t*) defined in Chapter 4 in case they are not conditional or by

$$p'q_{ij}(t) = P(Z'C(t) = z'c_{ij})$$
$$= P(Z'(t) = z'_i \cap C(t) = c_j)$$
$$= P(C(t) = C_j) \cdot P(Z'(t) = z'_i \mid C(t) = c_j)$$
$$= q_j(t) \cdot p'_{i/j}(t),\ t \in <0,+\infty),\ i = 1,2,...,\nu',$$
$$j = 1,2,...,w, \quad (34)$$

where

$$q_j(t) = P(C(t) = c_j),\ t \in <0,+\infty),$$
$$j = 1,2,...,w, \quad (35)$$

are transient probabilities of the operation process *C*(*t*) defined in Chapter 4 and

$$p'_{i/j}(t) = P(Z'(t) = z'_i \mid C(t) = c_j),$$
$$t \in <0,+\infty),\ i = 1,2,...,\nu',\ \ j = 1,2,...,w, \quad (36)$$

are conditional transient probabilities of the climate-weather change process *Z'*(*t*) defined in Chapter 2 in case they are not conditional.

The limit values of the critical infrastructure operation process related to operating environment threats and extreme weather hazards $Z'C(t)$ at the operation states $z'c_{ij}$, $i = 1,2,...,\nu'$, $j = 1,2,...,w$, can be found from [Kołowrocki, Soszyńska-Budny, 2011]

$$p'q_{ij} = \lim_{t\to\infty} \frac{\pi_{ij} M'N_{ij}}{\sum_{i=1}^{\nu'}\sum_{j=1}^{w} \pi_{ij} M'N_{ij}},$$
$$i = 1,2,...,\nu',\ \ j = 1,2,...,w, \quad (37)$$

where $M'N_{ij}$, $i = 1,2,...,\nu'$, $j = 1,2,...,w$, are given by (29), while the steady probabilities π_{ij}, $i = 1,2,...,\nu'$, $j = 1,2,...,w$, of the vector $[\pi_{ij}]_{1x\nu'w}$ satisfy the system of equations

$$\begin{cases} [\pi_{ij}][p'q_{ij\,kl}] = [\pi_{ij}] \\ \sum_{i-=1}^{\nu'}\sum_{j=1}^{w} \pi_{ij} = 1. \end{cases} \quad (38)$$

In the case of a periodic system operation process related to operating environment threats and extreme weather hazards, the limit transient probabilities $p'q_{ij}$, $i = 1,2,...,\nu'$, $j = 1,2,...,w$, at the operation states given by (37), are the long term proportions of the critical infrastructure operation process related to operating environment threats and extreme weather hazards $Z'C_{ij}(t)$ sojourn times at the particular operation states $z'c_{ij}$, $i = 1,2,...,\nu'$, $j = 1,2,...,w$.

Other interesting characteristics of the critical infrastructure operation process related to operating environment threats and extreme weather hazards possible to obtain are its total sojourn times $\hat{\theta}'\hat{C}_{ij}$, $i = 1,2,...,\nu'$, $j = 1,2,...,w$, at the particular operation states $z'c_{ij}$, $i = 1,2,...,\nu'$, $j = 1,2,...,w$, during the fixed system opetation time. It is well known [Kołowrocki, Soszyńska-Budny, 2011] that the system operation process related to operating environment threats and extreme weather hazards total sojourn times $\hat{\theta}'\hat{C}_{ij}$, at the particular operation states $z'c_{ij}$, for sufficiently large operation time θ, have approximately normal distributions with the expected value given by

$$\hat{M}'\hat{N}_{ij} = E[\hat{\theta}'\hat{C}_{ij}] = p'q_{ij}\theta,$$
$$i = 1,2,...,\nu',\ j = 1,2,...,w, \quad (39)$$

where $p'q_{ij}$, $i = 1,2,...,\nu'$, $j = 1,2,...,w$, are given by (37).

3. Safety of multistate systems at variable conditions related to operating environment threats and extreme weather hazards

We assume that the changes of the operation process related to operating environment threats and extreme weather hazards $Z'C(t)$, $t \in <0,+\infty)$, states $z'c_{11}, z'c_{12}, \ldots, z'c_{\nu' w}$, have an influence on the system multistate components E_i, $i=1,2,...,n$, safety. Consequently, we denote the system multistate component E_i, $i=1,2,...,n$, conditional lifetime in the safety state subset $\{u,u+1,...,z\}$ while the operation process related to operating environment threats and extreme weather hazards $Z'C(t)$ is at the state zc_{bl}, $b=1,2,...,\nu'$, $l=1,2,...,w$, by $T''''^{(bl)}_i(u)$ and its conditional safety function by the vector

$$[S''''_i(t,\cdot)]^{(bl)}$$
$$= [1, [S''''_i(t,1)]^{(bl)}, ..., [S''''_i(t,z)]^{(bl)}], \ t \in <0,\infty),$$
$$b=1,2,...,\nu', \ l=1,2,...,w, \ i=1,2,...,n, \quad (40)$$

with the coordinates defined by

$$[S''''_i(t,u)]^{(bl)} = P(T''''^{(bl)}_i(u) > t \,|\, Z'C(t) = z'c_{bl}) \quad (41)$$

for $t \in <0,\infty)$, $u=1,2,...,z$, $b=1,2,...,\nu'$, $l=1,2,...,w$.

The safety function $[S''''_i(t,u)]^{(bl)}$ is the conditional probability that the component E_i lifetime $T''^{(b)}_i(u)$ in the safety state subset $\{u,u+1,...,z\}$ is greater than t, while the operation process related to operating environment threats and extreme weather hazards $Z'C(t)$ at the system operating area is at the state $z'c_{bl}$, $b=1,2,...,\nu'$, $l=1,2,...,w$.

In the case, the system components E_i, $i=1,2,...,n$, at the operation process related to operating environment threats and extreme weather hazards $Z'C(t)$ states $z'c_{bl}$, $b=1,2,...,\nu$, $l=1,2,...,w$, have exponential safety functions, the coordinates of the vector (40) are given by

$$[S''''_i(t,u)]^{(bl)} = P(T''''^{(bl)}_i(u) > t \,|\, Z'C(t) = z'c_{bl})$$
$$= \exp[-[\lambda''''_i(u)]^{(bl)} t], \ t \in <0,\infty), b=1,2,...,\nu',$$
$$l=1,2,...,w, \ i=1,2,...,n. \quad (42)$$

Existing in (42) the intensities of ageing of the system components E_i, $i=1,2,...,n$, (the intensities of the system components E_i, $i=1,2,...,n$, departure from the safety state subset $\{u,u+1,...,z\}$) at the operation process related to operating environment threats and extreme weather hazards $Z'C(t)$ states $z'c_{bl}$, $b=1,2,...,\nu'$, $l=1,2,...,w$, i.e. the coordinates of the vector

$$[\lambda''''_i(\cdot)]^{(bl)} = [0, [\lambda''''_i(1)]^{(bl)}, \ldots, [\lambda''''_i(z)]^{(bl)}],$$
$$t \in <0,+\infty), b=1,2,...,\nu',$$
$$l=1,2,...,w, \ i=1,2,...,n, \quad (43)$$

are given by

$$[\lambda''''_i(u)]^{(bl)} = \rho''''^{(bl)}_i(u) \cdot \lambda_i(u), \ u=1,2,...,z,$$
$$b=1,2,...,\nu', \ l=1,2,...,w, \ i=1,2,...,n, \quad (44)$$

where $\lambda_i(u)$ are the intensities of ageing of the system components E_i, $i=1,2,...,n$, (the intensities of the system components E_i, $i=1,2,...,n$, departure from the safety state subset $\{u,u+1,...,z\}$) without operation threats and climate-weather hazards impact, i.e. the coordinate of the vector

$$\lambda_i(\cdot) = [0, \lambda_i(1), \ldots, \lambda_i(z)], \ i=1,2,...,n, \quad (45)$$

and

$$[\rho''''_i(u)]^{(bl)}, \ u=1,2,...,z, \ b=1,2,...,\nu',$$
$$l=1,2,...,w, \ i=1,2,...,n, \quad (46)$$

are the coefficients of operation threats and climate-weather hazards impact on the system components E_i, $i = 1,2,...,n$, intensities of ageing (the coefficients of operation threats and climate-weather hazards impact on critical infrastructure component E, $i = 1,2,...,n$, intensities of departure from the safety state subset $\{u,u+1,...,z\}$) at the operation process related to operating environment threats and extreme weather hazards states $z'c_{bl}$, $b=1,2,...,\nu'$, $l=1,2,...,w$, i.e. the coordinate of the vector

$$[\rho''''_i(\cdot)]^{(bl)} = [0, [\rho''''_i(1)]^{(bl)}, \ldots, [\rho''''_i(z)]^{(bl)}],$$
$$b=1,2,...,\nu', \ l=1,2,...,w, \ i=1,2,...,n. \quad (47)$$

The system component safety function (1), the system components intensities of ageing (4) and the coefficients of the operation threats and climate-weather hazards impact on the system components intensities of ageing (47) are main system component safety indices.

Similarly, we denote the system conditional lifetime in the safety state subset $\{u, u+1, ..., z\}$ while the operation process related to operating environment threats and extreme weather hazards *Z'C*(*t*) at the state $z'c_{bl}$, $b = 1,2,...,\nu'$, $l = 1,2,..., w$, by $T''''^{(bl)}(u)$ and the conditional safety function of the system by the vector

$$[\boldsymbol{S''''}(t,\cdot)]^{(bl)} = [1, [\boldsymbol{S''''}(t,1)]^{(bl)}, ..., [\boldsymbol{S''''}(t,z)]^{(bl)}], \quad (48)$$

with the coordinates defined by

$$[\boldsymbol{S''''}(t,u)]^{(bl)} = P(T''''^{(bl)}(u) > t | Z'C(t) = z'c_{bl}) \quad (49)$$

for $t \in < 0, \infty)$, $u = 1,2,...,z$, $b = 1,2,...,\nu'$, $l = 1,2,..., w$.

Further, we denote the system unconditional lifetime in the safety state subset $\{u, u+1, ..., z\}$ by $T''''(u)$ and the unconditional safety function of the system by the vector

$$\boldsymbol{S''''}(t,\cdot) = [1, \boldsymbol{S''''}(t,1), ..., \boldsymbol{S''''}(t,z)], \quad (50)$$

with the coordinates defined by

$$\boldsymbol{S''''}(t,u) = P(T''''(u) > t) \quad (51)$$

for $t \in < 0, \infty)$, $u = 1,2,...,z$.
In the case when the system operation time θ is large enough, the coordinates (12) of the unconditional safety function of the system defined by (11) are given by

$$\boldsymbol{S''''}(t,u) \cong \sum_{b=1}^{\nu'} \sum_{l=1}^{w} p'q_{bl} [\boldsymbol{S''''}(t,u)]^{(bl)} \text{ for } t \geq 0, \quad u = 1,2,...,z, \quad (52)$$

where $[\boldsymbol{S''''}(t,u)]^{(bl)}$, $u = 1,2,...,z$, $b = 1,2,...,\nu'$, $l = 1,2,..., w$, are the coordinates of the system conditional safety functions defined by (48)-(49) and $p'q_{bl}$, $b = 1,2,...,\nu'$, $l = 1,2,..., w$, are the operation process related to operating environment threats and extreme weather hazards *Z'C*(*t*) at the system operating area limit transient probabilities at the state $z'c_{bl}$, $b = 1,2,...,\nu'$, $l = 1,2,..., w$, given by (37).
The exemplary graph of a five-state (z = 4) critical infrastructure safety function

$$\boldsymbol{S''''}(t,\cdot) = [1, \boldsymbol{S''''}(t,1), \boldsymbol{S''''}(t,2), \boldsymbol{S''''}(t,3), \boldsymbol{S''''}(t,4)], \quad t \in < 0, \infty),$$

is shown in *Figure 1*.

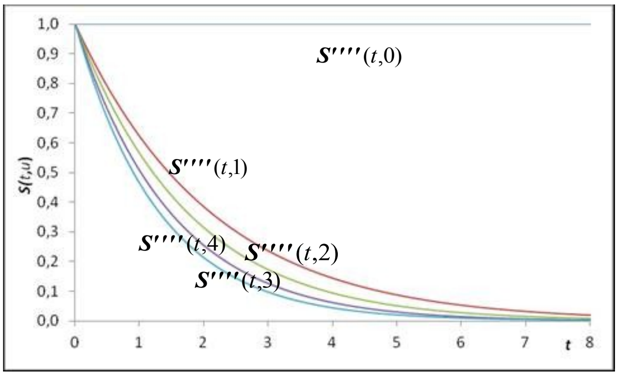

Figure 1. The graphs of a five-state critical infrastructure safety function $\boldsymbol{S''''}(t,\cdot)$ coordinates

The mean value of the system unconditional lifetime $T''''(u)$ in the safety state subset $\{u, u+1, ..., z\}$ is given by [Soszyńska, 2010; Kołowrocki, Soszyńska-Budny, 2011], [EU-CIRCLE Report D2.1-GMU3, 2016]

$$\mu''''(u) \cong \sum_{b=1}^{\nu'} \sum_{l=1}^{w} p'q_{bl} \mu''''_{bl}(u), \quad u = 1,2,...,z, \quad (53)$$

where $\mu''''_{bl}(u)$ are the mean values of the system conditional lifetimes $T''''^{(bl)}(u)$ in the safety state subset $\{u, u+1, ..., z\}$ at the operation process related to operating environment threats and extreme weather hazards *Z'C*(*t*) state $z'c_{bl}$, $b = 1,2,...,\nu'$, $l = 1,2,..., w$, given by

$$\mu''''_{bl}(u) = \int_0^\infty [\boldsymbol{S''''}(t,u)]^{(bl)} dt, \quad u = 1,2,...,z, \quad (54)$$

$[\boldsymbol{S''''}(t,u)]^{(bl)}$, $u = 1,2,...,z$, $b = 1,2,...,\nu'$, $l = 1,2,..., w$, are defined by (48)-(49) and $p'q_{bl}$ are given by (37). Whereas, the variance of the system unconditional lifetime $T'''(u)$ is given by

$$\sigma''''^2(u) = 2\int_0^\infty t \, \boldsymbol{S''''}(t,u) dt - [\mu''''(u)]^2, \quad u = 1,2,...,z, \quad (55)$$

where $\boldsymbol{S''''}(t,u)$, $u = 1,2,...,z$, are given by (51)-(52) and $\mu''''(u)$, $u = 1,2,...,z$, are given by (53)-(54).

Hence, according to (1.19) [Kołowrocki, Soszyńska-Budny, 2011], we get the following formulae for the mean values of the unconditional lifetimes of the system in particular safety states

$$\bar{\mu}''''(u) = \mu''''(u) - \mu''''(u+1), \ u = 1,2,...,z-1,$$
$$\bar{\mu}''''(z) = \mu''''(z), \quad (56)$$

where $\mu''''(u), \ u = 1,2,...,z,$ are given by (53).

Moreover, according (1.20)-(1.21) [Kołowrocki, Soszyńska-Budny, 2011], if r is the system critical safety state, then the system risk function

$$\boldsymbol{r}''''(t) = P(S''''(t) < r \mid S''''(0) = z)$$
$$= P(T''''(r) \le t), \ t \in <0,\infty), \quad (57)$$

defined as a probability that the system is in the subset of safety states worse than the critical safety state r, $r \in \{1,...,z\}$ while it was in the safety state z at the moment $t = 0$ [Kołowrocki, 2014], [Kołowrocki, Soszyńska-Budny, 2011] is given by

$$\boldsymbol{r}''''(t) = 1 - \boldsymbol{S}''''(t,r), \ t \in <0,\infty), \quad (58)$$

where $\boldsymbol{S}'''(t,r)$ is the coordinate of the system unconditional safety function given by (52) for $u = r$.

The graph of the system risk function presented in *Figure 2* is called the fragility curve of the system.

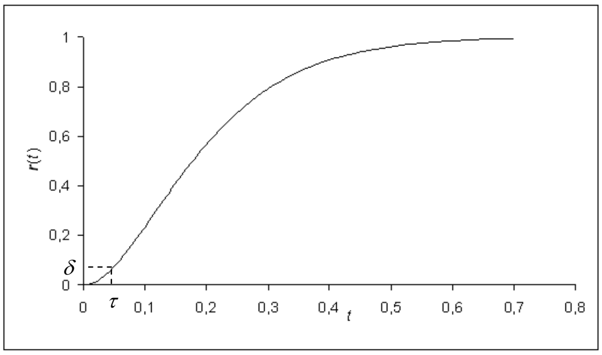

Figure 2. The graph (The fragility curve) of a system risk function $\boldsymbol{r}''''(t)$

The system safety function, the system risk function and the system fragility curve are main system safety indices. Other practically useful system safety indices are:

- the mean value of the unconditional system lifetime $T'''(r)$ up to the exceeding the critical safety state r given by

$$\mu''''(r) \cong \sum_{b=1}^{v'} \sum_{l=1}^{w} p'q_{bl}\, \mu''''_{bl}(r), \quad (59)$$

where $\mu''''_{bl}(r)$ are the mean values of the system conditional lifetimes $T''''^{(b)}(r)$ in the safety state subset $\{r, r+1,...,z\}$ at the operation process related to operating environment threats and extreme weather hazards $Z'C(t)$ state $z'c_{bl}$, $b = 1,2,...,v'$, $l = 1,2,...,w,$ given by

$$\mu''''_{bl}(r) = \int_0^\infty [\boldsymbol{S}''''(t,r)]^{(bl)} dt, \ b = 1,2,...,v',$$
$$l = 1,2,...,w, \quad (60)$$

$[\boldsymbol{S}''''(t,r)]^{(bl)}$, $u = 1,2,...,z,$ $b = 1,2,...,v'$, $l = 1,2,...,w,$ are defined by (48)-(49) and $p'q_{bl}$ are given by (37);

- the standard deviation of the system lifetime $T''''(r)$ up to the exceeding the critical safety state r given by

$$\sigma''''(r) = \sqrt{n''''(r) - [\mu''''(r)]^2}, \quad (61)$$

where

$$n''''(r) = 2\int_0^\infty t\, \boldsymbol{S}''''(t,r) dt, \quad (62)$$

where $\boldsymbol{S}''''(t,r)$ is given by (49) and $\mu''''(r)$ is given by (59) for $u = r$.

- the moment τ the system risk function exceeds a permitted level δ given by

$$\tau = \boldsymbol{r}''''^{-1}(\delta), \quad (63)$$

and illustrated in Figure 2, where $\boldsymbol{r}^{-1}(t)$, if it exists, is the inverse function of the risk function $\boldsymbol{r}''''(t)$ given by (58).

Other critical infrastructure safety indices are:

- the intensities of ageing of the critical infrastructure (the intensities of critical infrastructure departure from the safety state subset $\{u, u+1,...,z\}$) related to the operation threats and climate- weather hazards impact, i.e. the coordinates of the vector

$$\boldsymbol{\lambda}''''(t,\cdot) = [0, \boldsymbol{\lambda}''''(t,1), ..., \boldsymbol{\lambda}''''(t,z)],$$
$$t \in <0,+\infty), \quad (64)$$

where

$$\boldsymbol{\lambda}''''(t,u) = \frac{-\frac{d\boldsymbol{S}''''(t,u)}{dt}}{\boldsymbol{S}''''(t,u)},$$
$$t \in <0,+\infty),\ u = 1,2,...,z; \quad (65)$$

- the coefficients of the operation and operation threats and climate-weather hazards impact on the critical infrastructure intensities of ageing (the coefficients of the operation threats and climate-weather hazards impact on critical infrastructure intensities of departure from the safety state subset $\{u,u+1,...,z\}$), i.e. the coordinates of the vector

$$\boldsymbol{\rho}''''(t,\cdot) = [0,\ \boldsymbol{\rho}''''(t,1),\ \ldots,\boldsymbol{\rho}''''(t,z)\,],$$
$$t \in <0,+\infty), \quad (66)$$

where

$$\boldsymbol{\lambda}''''(t,u) = \boldsymbol{\rho}''''(t,u)\cdot\lambda(t,u),$$
$$t \in <0,+\infty),\ u = 1,2,...,z. \quad (67)$$

and $\lambda(t,u)$ are the intensities of ageing of the critical infrastructure (the intensities of the critical infrastructure departure from the safety state subset $\{u,u+1,...,z\}$) without of operation threats and climate-weather hazards impact, i.e. the coordinate of the vector

$$\lambda(t,\cdot) = [0,\ \lambda(t,1),\ \ldots,\lambda(t,z)\,],$$
$$t \in <0,+\infty). \quad (68)$$

In the case, the critical infrastructure have the exponential safety functions, i.e.

$$\boldsymbol{S}''''(t,\cdot) = [0,\ \boldsymbol{S}''''(t,1),\ \ldots,\boldsymbol{S}''''(t,z)\,],$$
$$t \in <0,+\infty), \quad (69)$$

where

$$\boldsymbol{S}''''(t,r) = \exp[-\boldsymbol{\lambda}''''(u)t],\ t \in <0,+\infty),$$
$$\boldsymbol{\lambda}''''(u) \geq 0,\ u = 1,2,\ldots,z, \quad (70)$$

the critical infrastructure safety indices defined by (64)-(68) take forms:
- the intensities of ageing of the critical infrastructure (the intensities of critical infrastructure departure from the safety state subset $\{u,u+1,...,z\}$) related to operation threats and climate-weather hazards impact, i.e. the coordinates of the vector

$$\boldsymbol{\lambda}''''(\cdot) = [0,\ \boldsymbol{\lambda}''''(1),\ \ldots,\boldsymbol{\lambda}''''(z)\,], \quad (71)$$

- the coefficients of the operation threats and climate-weather hazards impact on the scritical infrastructure intensities of ageing (the coefficients of the operation threats and climate-weatcher hazards impact on critical infrastructure intensities of departure from the safety state subset $\{u,u+1,...,z\}$), i.e. the coordinate of the vector

$$\boldsymbol{\rho}''''(\cdot) = [0,\ \boldsymbol{\rho}''''(1),\ \ldots,\boldsymbol{\rho}''''(z)\,], \quad (72)$$

where

$$\boldsymbol{\lambda}''''(u) = \boldsymbol{\rho}''''(u)\cdot\lambda(u),\ u = 1,2,...,z. \quad (73)$$

and $\lambda(u)$ are the intensities of ageing of the critical infrastructure (the intensities of the critical infrastructure departure from the safety state subset $\{u,u+1,...,z\}$) without of operation threats and climate-weather hazards impact, i.e. the coordinate of the vector

$$\lambda(\cdot) = [0,\ \lambda(1),\ \ldots,\lambda(z)\ ]. \quad (74)$$

4. Safety of multistate exponential systems related to operating environment threats and extreme weather hazards

We assume that the system components at the operation process related to operating environment threats and extreme weather hazards $Z'C(t)$ at the system operating area states have the exponential safety functions. This assumption and the results given in Chapter 1 [Kołowrocki, Soszyńska-Budny, 2011] yield the following results formulated in the form of the following proposition.

Proposition 1
If components of the multi-state system at the operation process related to operating environment threats and extreme weather hazards $Z'C(t)$ at the system operating area states $z'c_{bl},\ b = 1,2,...,v',$ $l = 1,2,...,w,$ have the exponential safety functions given by

$$[S''''_i(t,\cdot)]^{(bl)}$$
$$= [1,[S''''_i(t,1)]^{(bl)},\ldots,[S''''_i(t,z)]^{(bl)}],\ t \in <0,\infty),$$
$$b = 1,2,...,v',\ l = 1,2,...,w,\ i = 1,2,...,n, \quad (75)$$

with the coordinates

$$[S''''_i(t,u)]^{(bl)} = P(T''''^{(bl)}_i(u) > t \mid Z'C(t) = z'c_{bl})$$
$$= \exp[-[\lambda''''_i(u)]^{(bl)}t],\ t \in <0,\infty),\ b = 1,2,...,v',$$
$$l = 1,2,...,w,\ i = 1,2,...,n, \quad (76)$$

and the intensities of ageing of the system components E_i, $i = 1,2,...,n$, (the intensities of the system components E_i, $i = 1,2,...,n$, departure from the safety state subset $\{u, u+1,...,z\}$) related to operation threats and climate-weather hazards impact, existing in (76), are given by

$$[\lambda''''_i(u)]^{(bl)} = \rho''''^{(bl)}_i(u) \cdot \lambda_i(u),\ u = 1,2,...,z,$$
$$b = 1,2,...,\nu',\ l = 1,2,..., w,\ i = 1,2,...,n, \quad (77)$$

where $\lambda_i(u)$ are the intensities of ageing of the system components E_i, $i = 1,2,...,n$, (the intensities of the system components E_i, $i = 1,2,...,n$, departure from the safety state subset $\{u, u+1,...,z\}$) without operation threats and climate-weather hazards impact and

$$[\rho'''_i(u)]^{(bl)},\ u = 1,2,...,z,\ b = 1,2,...,\nu',$$
$$l = 1,2,..., w,\ i = 1,2,...,n, \quad (78)$$

are the coefficients of the operation and climate-weather impact on the system components E_i, $i = 1,2,...,n$, intensities E_i, $i = 1,2,...,n$, of ageing (the coefficients of operation and climate-weather impact on critical infrastructure component E_i, $i = 1,2,...,n$, intensities of departure from the safety state subset $\{u, u+1,...,z\}$) without operation threats and climate-weather change hazards impact, in the case of series, parallel, "*m* out of *n*", consecutive "*m* out of *n*: F" systems
and respectively by

$$[S''''_{ij}(t,\cdot)]^{(bl)}$$
$$= [1, [S''''_{ij}(t,1)]^{(bl)}, \ldots, [S''''_{ij}(t,z)]^{(bl)}],$$
$$t \in <0,\infty),\ b = 1,2,...,\nu',\ l = 1,2,..., w,$$
$$i = 1,2,...,k,\ j = 1,2,...,l_i, \quad (79)$$

with the coordinates

$$[S''''_{ij}(t,u)]^{(bl)} = P(T''''^{(bl)}_{ij}(u) > t | Z'C(t) = z'c_b)$$
$$= \exp[-[\lambda''''_{ij}(u)]^{(bl)} t],\ t \in <0,\infty),\ b = 1,2,...,\nu',$$
$$l = 1,2,..., w,\ i = 1,2,...,k,\ j = 1,2,...,l_i, \quad (80)$$

and the intensities of ageing of the system components E_{ij}, i = 1,2,...,k, j = 1,2,...,l_i, (the intensities of the system components E_{ij}, i = 1,2,...,k, j = 1,2,...,l_i, departure from the safety state subset $\{u, u+1,...,z\}$) related to operation threats and climate-weather hazards impact, existing in (80), are given by

$$[\lambda''''_{ij}(u)]^{(bl)} = \rho''''^{(bl)}_{ij}(u) \cdot \lambda_{ij}(u),\ u = 1,2,...,z,$$
$$b = 1,2,...,\nu',\ l = 1,2,..., w,\ i = 1,2,...,k,$$
$$j = 1,2,...,l_i, \quad (81)$$

where $\lambda_{ij}(u)$ are the intensities of ageing of the system components E_{ij}, i = 1,2,...,k, j = 1,2,...,l_i, (the intensities of the system components E_{ij}, i = 1,2,...,k, j = 1,2,...,l_i, departure from the safety state subset $\{u, u+1,...,z\}$) without operation threats and climate-weather hazards impact and

$$[\rho'''_{ij}(u)]^{(bl)},\ u = 1,2,...,z,\ b = 1,2,...,\nu',$$
$$l = 1,2,..., w,\ i = 1,2,...,k, j = 1,2,...,l_i, \quad (82)$$

are the coefficients of the operation threats and climate-weather hazards impact on the system components E_{ij}, i = 1,2,...,k, j = 1,2,...,l_i, intensities E_{ij}, i = 1,2,...,k, j = 1,2,...,l_i, of ageing (the coefficients of operation threats and climate-weather hazards impact on critical infrastructure component E, i = 1,2,...,n, intensities of departure from the safety state subset $\{u, u+1,...,z\}$) without operation threats and climate-weather hazards impact, in the case of series-parallel, parallel-series, series-"*m* out of *k*", "m_i out of l_i"-series, series-consecutive "*m* out of *k*: F" and consecutive "m_i out of l_i: F"-series systems and the system operation time θ is large enough, then its multistate unconditional safety function is given by the vector:

i) for a series system

$$\boldsymbol{S''''}(t,\cdot) = [1,\ \boldsymbol{S''''}(t,1), ...,\ \boldsymbol{S''''}(t,z)]$$
$$\text{for } t \geq 0, \quad (83)$$

where

$$\boldsymbol{S''''}(t,u) \cong \sum_{b=1}^{\nu'} \sum_{l=1}^{w} p' q_b \exp[-\sum_{i=1}^{n} [\lambda''''_i(u)]^{(bl)} t]$$
$$\text{for } t \geq 0,\ u = 1,2,...,z; \quad (84)$$

ii) for a parallel system

$$\boldsymbol{S''''}(t,\cdot) = [1,\ \boldsymbol{S''''}(t,1), ...,\ \boldsymbol{S''''}(t,z)]$$
$$\text{for } t \geq 0, \quad (85)$$

where

$$\boldsymbol{S''''}(t,u) \cong 1-\sum_{b=1}^{v'}\sum_{l=1}^{w} p'q_b \prod_{i=1}^{n}[1-\exp[-[\lambda''''_i(u)]^{(bl)}t]]$$
for $t \ge 0$, $u = 1,2,\ldots,z$; (86)

iii) for a "*m* out of *n*" system

$$\boldsymbol{S''''}(t,\cdot) = [1, \boldsymbol{S''''}(t,1), \ldots, \boldsymbol{S''''}(t,z)]$$
for $t \ge 0$, (87)

where

$$\boldsymbol{S''''}(t,u) \cong 1-\sum_{b=1}^{v'}\sum_{l=1}^{w} p'q_b \sum_{\substack{r_1,r_2,\ldots,r_n=0\\ r_1+r_2+\ldots+r_n\le m-1}}^{1} \prod_{i=1}^{n}\exp[-r_i[\lambda''''_i(u)]^{(bl)}t]$$
$$[1-\exp[-[\lambda''''_i(u)]^{(bl)}t]]^{1-n}$$ for $t \ge 0$,
$u = 1,2,\ldots,z$, (88)

or

$$\boldsymbol{S''''}(t,\cdot) = [1, \boldsymbol{S''''}(t,1), \ldots, \boldsymbol{S''''}(t,z)]$$
for $t \ge 0$, (89)

where

$$\boldsymbol{S''''}(t,u) \cong \sum_{b=1}^{v'}\sum_{l=1}^{w} p'q_b \sum_{\substack{r_1,r_2,\ldots,r_n=0\\ r_1+r_2+\ldots+r_n\le \bar{m}}}^{1} \prod_{i=1}^{n}[1-\exp[-[\lambda''''_i(u)]^{(bl)}t]]^{r_i}$$
$$\exp[-(1-r_i)[\lambda''''_i(u)]^{(bl)}t]$$ for $t \ge 0$,
$u = 1,2,\ldots,z$, (90)

and $\bar{m} = n-m$;

iv) for a consecutive "*m* out of *n*: F" system

$$\boldsymbol{CS''''}(t,\cdot) = [1, \boldsymbol{CS''''}(t,1), \ldots, \boldsymbol{CS''''}(t,z)]$$
for $t \ge 0$, (91)

where

$$\boldsymbol{CS''''}(t,u) \cong \sum_{b=1}^{v'}\sum_{l=1}^{w} p'q_b \prod_{i=1}^{k}[\boldsymbol{CS''''}(t,u)]^{(bl)}$$
for $t \ge 0$, $u = 1,2,\ldots,z$, (92)

and $[\boldsymbol{CS''''}(t,u)]^{(bl)}$, $t \ge 0$, $i = 1, 2,\ldots, k$, $b = 1,2,\ldots,v'$, $l = 1,2,\ldots,w$, are given by

$$[\boldsymbol{CS''''}(t,u)]^{(bl)} \cong \begin{cases} 1 & \text{for } n < m, \\ 1-\sum_{b=1}^{v'}\sum_{l=1}^{w} p'q_b \prod_{i=1}^{n}[1-\exp[-[\lambda''''_i(u)]^{(bl)}t]] & \text{for } n = m, \\ \sum_{b=1}^{v'}\sum_{l=1}^{w} p'q_b[\exp[-[\lambda''''_n(u)]^{(bl)}t][\boldsymbol{CS''''}_{n-1}(t,u)]^{(bl)} & \\ +\sum_{i=1}^{m-1}\exp[-[\lambda''''_{n-i}(u)]^{(bl)}t][\boldsymbol{CS''''}_{n-i-1}(t,u)]^{(bl)} & \\ \prod_{j=n-i+1}^{n}[1-\exp[-[\lambda''''_j(u)]^{(bl)}t]] & \text{for } n > m, \end{cases}$$

for $t \ge 0$, $u = 1,2,\ldots,z$; (93)

v) for a series-parallel system

$$\boldsymbol{S''''}(t,\cdot) = [1, \boldsymbol{S''''}(t,1), \ldots, \boldsymbol{S''''}(t,z)]$$
for $t \ge 0$, (94)

where

$$\boldsymbol{S''''}(t,u) \cong 1-\sum_{b=1}^{v'}\sum_{l=1}^{w} p'q_b \prod_{i=1}^{k}[1-\exp[-\sum_{j=1}^{l_i}[\lambda''''_{ij}(u)]^{(bl)}t]]$$
for $t \ge 0$, $u = 1,2,\ldots,z$; (95)

vi) for a parallel-series system

$$\boldsymbol{S''''}(t,\cdot) = [1, \boldsymbol{S''''}(t,1), \ldots, \boldsymbol{S''''}(t,z)]$$
for $t \ge 0$, (96)

where

$$\boldsymbol{S''''}(t,u) \cong \sum_{b=1}^{v'}\sum_{l=1}^{w} p'q_b \prod_{i=1}^{k}[1-\prod_{j=1}^{l_i}[1-\exp[-[\lambda''''_{ij}(u)]^{(bl)}t]]]$$
for $t \ge 0$, $u = 1,2,\ldots,z$; (97)

vii) for a series-"*m* out of *k*" system

$$\boldsymbol{S''''}(t,\cdot) = [1, \boldsymbol{S''''}(t,1), \ldots, \boldsymbol{S''''}(t,z)]$$
for $t \ge 0$, (98)

where

$$\boldsymbol{S''''}(t,u) \cong 1-\sum_{b=1}^{v'}\sum_{l=1}^{w} p'q_b \sum_{\substack{r_1,r_2,\ldots,r_k=0\\ r_1+r_2+\ldots+r_k\le m-1}}^{1} \prod_{i=1}^{k}[\prod_{j=1}^{l_i}\exp[-[\lambda''''_{ij}(u)]^{(bl)}t]]^{r_i}$$
$$\cdot[1-\prod_{j=1}^{l_i}\exp[-[\lambda''''_{ij}(u)]^{(bl)}t]]^{1-r_i}$$

for $t \geq 0,\ u = 1,2,...,z,$ (99)

or

$$\boldsymbol{S}''''(t,\cdot) = [1,\ \boldsymbol{S}''''(t,1),\ ...,\ \boldsymbol{S}''''(t,z)\,]$$
for $t \geq 0,$ (100)

where

$$\boldsymbol{S}''''(t,u)$$

$$\cong \sum_{b=1}^{v'}\sum_{l=1}^{w} p' q_b \sum_{\substack{r_1,r_2,...,r_k=0 \\ r_1+r_2+...+r_k \leq \bar{m}}}^{1} \prod_{i=1}^{k} [1 - [\prod_{i=1}^{l_i} \exp[-[\lambda''''_{ij}(u)]^{(bl)} t]]^{r_i}$$

$$\cdot [\prod_{j=1}^{l_i} \exp[-[\lambda''''_{ij}(u)]^{(bl)} t]]^{1-r_i} \text{ for } t \geq 0,\ \bar{m} = k - m,$$
$u = 1,2,...,z;$ (101)

viii) for a "m_i out of l_i"-series system

$$\boldsymbol{S}''''(t,\cdot) = [1,\ \boldsymbol{S}''''(t,1),\ ...,\ \boldsymbol{S}''''(t,z)\,]$$
for $t \geq 0,$ (102)

where

$$\boldsymbol{S}''''(t,u) \cong \sum_{b=1}^{v'}\sum_{l=1}^{w} p' q_b \prod_{i=1}^{k} [1 - \sum_{\substack{r_1,r_2,...,r_{l_i}=0 \\ r_1+r_2+...+r_{l_i} \leq m_i - 1}}^{1} \prod_{j=1}^{l_i} \exp[-r_j [\lambda''''_{ij}(u)]^{(bl)} t]$$
$$\cdot [1 - \exp[-[\lambda''''_{ij}(u)]^{(bl)} t]]^{1-r_j}\,]$$
for $t \geq 0,\ u = 1,2,...,z,$ (103)

or

$$\boldsymbol{S}''''(t,\cdot) = [1,\ \boldsymbol{S}''''(t,1),...,\boldsymbol{S}''''(t,z)\,]$$
for $t \geq 0,$ (104)

where

$$\boldsymbol{S}''''(t,u)$$
$$\cong \sum_{b=1}^{v'}\sum_{l=1}^{w} p' q_b \prod_{i=1}^{k} [\sum_{\substack{r_1,r_2,...,r_{l_i}=0 \\ r_1+r_2+...+r_{l_i} \leq \bar{m}_i}}^{1} \prod_{j=1}^{l_i} [1 - \exp[-[\lambda''''_{ij}(u)]^{(bl)} t]]^{r_j}$$
$$\cdot \exp[-(1-r_j)[\lambda''''_{ij}(u)]^{(bl)} t]] \text{ for } t \geq 0,$$
$\bar{m}_i = l_i - m_i,\ i = 1,2,...,k,\ u = 1,2,...,z;$ (105)

ix) for a series-consecutive "*m* out of *k*: F" system

$$\boldsymbol{CS}''''(t,\cdot) = [1,\ \boldsymbol{CS}''''(t,1),\ ...,\ \boldsymbol{CS}''''(t,z)\,]$$
for $t \geq 0,$ (106)

where

$$\boldsymbol{CS}''''^{m}_{k;l_1,l_2,...,l_k}(t,u) \cong \sum_{b=1}^{v'}\sum_{l=1}^{w} p' q_b [\ \boldsymbol{CS}''''^{m}_{k;l_1,l_2,...,l_k}(t,u)]^{(bl)}$$
for $t \geq 0,\ u = 1,2,...,z,$ (107)

and $[\boldsymbol{CS}''''(t,u)]^{(bl)},\ t \geq 0,\ b = 1,2,...,v',$ $l = 1,2,...,w,$ are given by

$$[\boldsymbol{CS}''''(t,u)]^{(bl)}$$
$$= \begin{cases} 1 & \text{for } k < m, \\ 1 - \prod_{i=1}^{k}[1 - \exp[-\sum_{j=1}^{l_i}[\lambda''''_{ij}(u)]^{(bl)} t]] & \text{for } k = m, \\ \exp[-\sum_{j=1}^{l_k}[\lambda''''_{kj}(u)]^{(bl)} t][\boldsymbol{CS}''''_{k-1;l_1,l_2,...l_k}(t,u)]^{(bl)} & \\ + \sum_{j=1}^{m-1}[\exp[-\sum_{v=1}^{l_{k-j}}[\lambda''''_{k-jv}(u)]^{(bl)} t]] & \\ \cdot[\boldsymbol{CS}''''_{k-j-1;l_1,l_2,...l_k}(t,u)]^{(bl)} & \\ \cdot \prod_{i=k-j+1}^{k}[1 - \exp[-\sum_{v=1}^{l_i}[\lambda''''_{iv}(u)]^{(bl)} t]] & \text{for } k > m, \end{cases}$$
for $t \geq 0,\ u = 1,2,...,z;$ (108)

x) for a consecutive "m_i out of l_i: F"-series system

$$\boldsymbol{CS}''''(t,\cdot) = [1,\ \boldsymbol{CS}''''(t,1),\ ...,\ \boldsymbol{CS}''''(t,z)\,]$$
for $t \geq 0,$ (109)

where

$$\boldsymbol{CS}''''(t,u) \cong \sum_{b=1}^{v'}\sum_{l=1}^{w} p' q_b \prod_{i=1}^{k} [\boldsymbol{CS}''''_{i,l_i}(t,u)]^{(bl)}$$
for $t \geq 0,\ u = 1,2,...,z,$ (110)

and $[\ \boldsymbol{CS}''''_{i,l_i}(t,u)]^{(bl)},\ t \geq 0,\ i = 1, 2, \dots, k,$ $b = 1,2,...,v',\ l = 1,2,...,w,$ are given by

$$[\boldsymbol{CS}''''^{m_i}_{i,l_i}(t,u)]^{(bl)}$$
$$= \begin{cases} 1 & \text{for } l_i < m_i, \\ 1 - \prod_{j=1}^{l_i}[1 - \exp[-[\lambda''''_{ij}(u)]^{(bl)} t]] & \text{for } l_i = m_i, \\ \exp[-[\lambda''''_{il_i}(u)]^{(bl)} t][\boldsymbol{CS}''''_{i,l_i-1}(t,u)]^{(bl)} & \\ + \sum_{j=1}^{m_i-1}\exp[-[\lambda''''_{il_i-j}(u)]^{(bl)} t] & \\ \cdot[\boldsymbol{CS}''''_{i,l_i-j-1}(t,u)]^{(bl)} & \\ \cdot \prod_{v=l_i-j+1}^{l_i}[1 - \exp[-[\lambda''''_{iv}(u)]^{(bl)} t]] & \text{for } l_i > m_i, \end{cases}$$
for $t \geq 0,\ u = 1,2,...,z.$ (111)

Remark 1

The formulae for the safety functions stated in *Proposition 1* are valid for the considered systems under the assumption that they do not change their structures at different operation process related to $\bar{\boldsymbol{S}}''''_n(t,u)$ $Z'C(t)$ at the system operating area states $z'c_{bl}$, $b=1,2,...,\nu'$, $l=1,2,...,w$. This limitation can be simply omitted by the replacement in these formulae the system's structure shape constant parameters n, m, k, m_i, l_i, respectively by their changing at different operation states $z'c_{bl}$, $b=1,2,...,\nu'$, $l=1,2,...,w$, equivalent structure shape parameters $n^{(bl)}$, $m^{(bl)}$, $k^{(bl)}$, $m_i^{(bl)}$, $l_i^{(bl)}$, $b=1,2,...,\nu'$, $l=1,2,...,w$.

For the exponential complex technical systems, considered in *Proposition 1*, we determine the mean values $\mu''''(u)$ and the standard deviations $\sigma''''(u)$ of the unconditional lifetimes of the system in the safety state subsets $\{u,u+1,...,z\}$, $u=1,2,...,z$, the mean values $\bar{\mu}''''(u)$ of the unconditional lifetimes of the system in the particular safety states u, $u=1,2,...,z$, the system risk function $\boldsymbol{r}''''(t)$ and the moment τ'''' when the system risk function exceeds a permitted level δ respectively defined by (53)-(63), after substituting for $\boldsymbol{S}''''(t,u)$, $u=1,2,...,z$, the coordinates of the unconditional safety functions given respectively by (83)-(111).

5. Conclusions

The integrated general model of complex systems' safety, linking their safety models and their operation processes models and considering variable at different operation states their safety structures and their components safety parameters is constructed. The material given in this report delivers the procedures and algorithms that allow to find the main an practically important safety characteristics of the complex technical systems at the variable operation conditions. Next the results are applied to the safety evaluation of the port oil piping transportation system and the maritime ferry technical system. The predicted safety characteristics of these exemplary critical infrastructures operating at the variable conditions are different from those determined for this system operating at constant conditions [Kołowrocki, Soszyńska-Budny, 2011]. This fact justifies the sensibility of considering real systems at the variable operation conditions that is appearing out in a natural way from practice. This approach, upon the sufficient accuracy of the critical infrastructures' operation processes and the critical infrastructures' components safety parameters identification, makes their safety prediction much more precise.

Acknowledgements

The paper presents the results developed in the scope of the EU-CIRCLE project titled "A pan – European framework for strengthening Critical Infrastructure resilience to climate change" that has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 653824. http://www.eu-circle.eu/

References

Barbu V., Limnios N., Empirical estimation for discrete-time semi-Markov processes with applications in reliability. Journal of Nonparametric Statistics, Vol. 18, No. 7-8, 483-498, 2006

Collet J., Some remarks on rare-event approximation. IEEE Transactions on Reliability 45, 106-108, 1996

EU-CIRCLE Report D1.1, EU-CIRCLE Taxonomy, 2015

EU-CIRCLE Report D2.1-GMU3, Modelling Climate-Weather Change Process Including Extreme Weather Hazards, 2016

EU-CIRCLE Report D2.1-GMU2, Modelling outside dependences influence on Critical Infrastructure Safety (CIS) – Modelling Critical Infrastructure Operation Process (CIOP) including Operating Environment Threats (OET), 2016

EU-CIRCLE Report D2.1-GMU3, Modelling outside dependences influence on Critical Infrastructure Safety (CIS) – Modelling Climate-Weather Change Process (C-WCP) including Extreme Weather Hazards (EWH), 2016

EU-CIRCLE Report D2.1-GMU4, Modelling outside dependences influence on Critical Infrastructure Safety (CIS) - Designing Critical Infrastructure Operation Process General Model (CIOPGM) related to Operating Environment Threats (OET) and Extreme Weather Hazards (EWH) by linking CIOP and C-WCP models, 2016

EU-CIRCLE Report D2.2-GMU1, Modelling port piping transportation system operation process at the southern Baltic Sea area using the Critical Infrastructure Operation Process General Model (CIOPGM) related to Operating Environment Threats (OET) and Extreme Weather Hazards (EWH) in this region, 2016

EU-CIRCLE Report D2.2-GMU2, Modelling

maritime ferry transportation system operation process at the Baltic Sea area using the Critical Infrastructure Operation Process General Model (CIOPGM) related to Operating Environment Threats (OET) and Extreme Weather Hazards (EWH) in this region, 2016

EU-CIRCLE Report D2.2-GMU3, Modelling port, shipping and ship traffic and port operation information critical infrastructures network operation process at the Baltic Sea area using the Critical Infrastructure Operation Process General Model (CIOPGM) related to Operating Environment Threats (OET) and Extreme Weather Hazards (EWH) in this region, 2016

EU-CIRCLE Report D2.2-GMU4, Modelling the operation process of the Baltic Sea critical infrastructures global network of interconnected and interdependent critical infrastructures located within the Baltic Sea and ashore around that function collaboratively using the Critical Infrastructure Operation Process General Model (CIOPGM) related to Operating Environment Threats (OET) and Extreme Weather Hazards (EWE) in its operating environment ("network of networks" approach), 2016

EU-CIRCLE Report D2.3-GMU1, Identification methods and procedures of Critical Infrastructure Operation Process (CIOP) including Operating Environment Threats (OET), 2016

EU-CIRCLE Report D2.3-GMU2, Identification methods and procedures of Climate-Weather Change Process (C-WCP) including Extreme Weather Hazards (EWH), 2016

EU-CIRCLE Report D2.3-GMU3, Identification methods and procedures of unknown parameters of Critical Infrastructure Operation Process General Model (CIOPGM) related to Operating Environment Threats (OET) and Extreme Weather Hazards (EWH), 2016

EU-CIRCLE Report D2.3-GMU4, Evaluation of unknown parameters of a port oil piping transportation system operation process related to Operating Environment Threats (OET) and Extreme Weather Hazards (EWH) at the southern Baltic Sea area, 2016

EU-CIRCLE Report D2.3-GMU5. Evaluation of unknown parameters of a maritime ferry transportation system operation process related to Operating Environment Threats (OET) and Extreme Weather Hazards (EWH) at the Baltic Sea area, 2016

EU-CIRCLE Report D3.3-GMU0, How to model and analyse climate-weather change process influence on critical infrastructure safety – General approach, 2016

EU-CIRCLE Report D3.3-GMU1, Modelling inside dependences influence on safety of multistate ageing systems – Modelling safety of multistate ageing systems, 2016

EU-CIRCLE Report D3.3-GMU2, Modelling inside dependences influence on safety of multistate ageing systems – Modelling safety of multistate ageing systems with dependent components and subsystems, 2016

EU-CIRCLE Report EU-CIRCLE Report D3.3-GMU3, Modelling inside and outside dependences influence on safety of complex multistate ageing systems (critical infrastructures) – Integrated Model of Critical Infrastructure Safety (IMCIS) related to its operation process including operating environment threats (with other critical infrastructures influence, without climate-weather change influence), 2016

EU-CIRCLE Report D3.3-GMU12, Integration of the Integrated Model of Critical Infrastructure Safety (IMCIS) and the Critical Infrastructure Operation Process General Model (CIOPGM) into the General Integrated Model of Critical Infrastructure Safety (GIMCIS) related to operating environment threads (OET) and climate-weather extreme hazards (EWH), 2017

EU-CIRCLE Report D3.3-GMU14, Application of the General Integrated Model of Critical Infrastructure Safety (GIMCIS) to port oil piping transportation system safety modelling, identification and prediction (with climate-weather change influence), 2017

EU-CIRCLE Report D3.3-GMU15, Application of the General Integrated Model of Critical Infrastructure Safety (GIMCIS) to maritime ferry safety modelling, identification and prediction (with climate-weather change influence), 2017

Ferreira F., Pacheco A., Comparison of level-crossing times for Markov and semi-Markov processes. Statistics and Probability Letters, Vol. 7, No 2, 151-157, 2007

Gamiz M. L., Roman Y., Non-parametric estimation of the availability in a general repairable. Reliability Engineering & System Safety, Vol. 93, No. 8, 1188-1196, 2008

Giudici P., Figini S., Applied data mining for business and industry. John Wiley & Sons Ltd, 2009.

Onisawa T. and Kacprzyk J., Eds., Physica Verlag, Heidelberg, 169-182, 1995

Glynn P.W., Haas P.J., Laws of large numbers and functional central limit theorems for generalized semi-Markov processes. Stochastic Models,Vol. 22, No 2, 201-231, 2006

Grabski F., Semi-Markov Processes: Application in System Reliability and Maintenance, Amsterdam, Boston, Heidelberd, London, New York, Oxford, Paris, San Diego, San Francisco, Singapore, Sidney, Tokyo, Elsevier, 2014

Guze S., Kołowrocki K., Soszyńska J., Modeling environment and infrastructure influence on reliability and operation processes of port transportation systems. Journal of Polish Safety and Reliability Association, Summer Safety and Reliability Seminars,Vol. 2, No 1, 179-188, 2008

Habibullah M. S., Lumanpauw E., Kolowrocki K., Soszynska J., Ming N. G., A computational tool for general model of operation processes in industrial systems operation processes. Electronic Journal Reliability & Risk Analysis: Theory & Applications, Vol. 2, No 4, 181-191, 2009

Helvacioglu S, Insel M, expert system applications in marine technologies. Ocean Engineering, Vol. 35, No. 11-12, 1067-1074, 2008

Hryniewicz O., Lifetime tests for imprecise data and fuzzy reliability requirements. Reliability and Safety Analyses under Fuzziness

Kołowrocki K., Reliability of Large and Complex Systems, Amsterdam, Boston, Heidelberd, London, New York, Oxford, Paris, San Diego, San Francisco, Singapore, Sidney, Tokyo, Elsevier, 2014

Kołowrocki K., Soszyńska J., A general model of industrial systems operation processes related to their environment and infrastructure. Summer Safety & Reliability Seminars. Journal of Polish Safety and Reliability Association, Issue 2, Vol. 2, 223-226, 2008

Kolowrocki K., Soszynska J. Modeling environment and infrastructure influence on reliability and operation process of port oil transportation system. Electronic Journal Reliability & Risk Analisys: Theory & Applications, Vol. 2, No 3, 131-142, 2009a

Kolowrocki K., Soszynska J., Safety and risk evaluation of Stena Baltica ferry in variable operation conditions.ElectronicJournal Reliability & Risk Analisys: Theory & Applications, Vol. 2, No 4, 168-180, 2009b

Kołowrocki K., Soszyńska J., Statistical identification and prediction of the port oil pipeline system's operation process and its reliability and risk evaluation. Summer Safety & Reliability Seminars. Journal of Polish Safety and Reliability Association, Issue 3, Vol. 2, 241-250, 2009c

Kołowrocki K., Soszyńska J., Methods and algorithms for evaluating unknown parameters of operation processes of complex technical systems. Summer Safety & Reliability Seminars. Journal of Polish Safety and Reliability Association, Issue 3, Vol. 1, 2, 211-222, 2009d

Kołowrocki K., Soszyńska J., Methods and algorithms for evaluating unknown parameters of components reliability of complex technical systems. Summer Safety & Reliability Seminars. Journal of Polish Safety and Reliability Association, Issue 4, Vol. 2, 223-230, 2009e

Kołowrocki K., Soszyńska J., Statistical identification and prediction of the port oil pipeline system's operation process and its reliability and risk evaluation. Summer Safety & Reliability Seminars. Journal of Polish Safety and Reliability Association, Issue 4, Vol. 2, 241-250, 2009f

Kolowrocki K., Soszynska J., Reliability modeling of a port oil transportation system's operation processes. International Journal of Performability Engineering, Vol. 6, No 1, 77-87, 2010a

Kolowrocki K., Soszynska J., Reliability, availability and safety of complex technical systems: modelling –identification – prediction – optimization.Journal of Polish Safety and Reliability Association, Summer Safetyand Reliability Seminars,Vol. 4, No 1, 133-158, 2010b

Kołowrocki K., Soszyńska-Budny J., Reliability and Safety of Complex Technical Systems and Processes: Modeling - Identification - Prediction - Optimization, London, Dordrecht, Heildeberg, New York, Springer, 2011

[Kołowrocki K., Soszyńska-Budny J., Torbicki M., Critical Infrastructure Operation Area Climate-Weather Change Process (C-WCP) Including Extreme Weather Hazards (EWH), C-WCP Model, 2016]

Limnios N., Oprisan G., Semi-Markov Processes and Reliability. Birkhauser, Boston, 2005

Mercier S., Numerical bounds for semi-Markovian quantities and application to reliability. Methodologyand Computing in Applied Probability, Vol. 10, No 2, 179-198, 2008

Rice J. A., Mathematical statistics and data analysis. Duxbury. Thomson Brooks/Cole. University of California. Berkeley, 2007

Soszyńska J., Systems reliability analysis in variable operation conditions. PhD Thesis, Gdynia MaritimeUniversity-System Research Institute Warsaw, 2007 (in Polish)

Soszyńska J., Reliability and risk evaluation of a port oil pipeline transportation system in variable operation conditions Int J of Press Vessel and Pip 87(2-3):81-87, 2010

Soszyńska J., Kołowrocki K., Blokus-Roszkowska A., Guze S.,Prediction of complex technical systems operation processes.Journal of Polish Safety and Reliability Association, Summer Safety and Reliability Seminars, Vol. 4, No 2, 379-510, 2010

Vercellis S., Data mining and optimization for decision making. John Wiley & Sons Ltd., 2009

Wilson A. G., Graves T. L., Hamada M. S., et al, Advances in data combination, analysis and collection for system reliability assessment. Statistical Science, Vol. 21, No. 4, 514-531, 2006

Xue J., On multi-state system analysis, *IEEE Trans on Reliab.* 34, 329-337, 1985

Xue J., Yang K., Dynamic reliability analysis of coherent multi-state systems, *IEEE Trans on Reliab.* 4(44), 683-688, 1995a

Xue J., Yang K., Symmetric relations in multi-state systems, *IEEE Trans on Reliab* 4(44), 689-693, 1995