TOWARDS THE CONSTRUCTION OF ARTISTIC VISUAL IMAGES BY MEANS OF ANALYTICAL GEOMETRY AND COMPUTER GRAPHICS

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Abstract. In the new software application "ArtMathGraph" (AMG), the construction of artistic visual images of realistic objects is based on the analytical approximation of complicated geometrical forms by the parametric equations of the mathematical transformed surfaces and also on the interactive visual mathematical modeling of predetermined geometrical forms as visual models of realistic objects. Consecutive mathematical transformations of the analytical representations of initial well-known classical surfaces (such as plane, cone, cylinder, sphere, ellipsoid, etc.) allowed obtaining visual images of various realistic objects, including the objects of Nature such as vegetables, fruits, flowers, leafs, etc. The AMG application provides the possibility to combine the individual images into artistic compositions. By experience, consecutive use of all means of the AMG application allows obtaining various artistic compositions by methods of analytical geometry and computer graphics only.

Key Words: Analytical Approximation**,** Computer Visualization**,** Artistic Images.

1. Introduction

Search for computer solutions of geometry problems on the way to the construction of artistic visual images of realistic objects, including the objects of nature, by means of analytical geometry and computer graphics is the main purpose of this research.

The following operations of analytical geometry and computer graphics must be implemented for obtaining good results:

- to construct a visual representation of individual geometrical forms, as models of realistic objects;
- to combine individual images into a composition of derivable visual images;
- to arrange each individual image and a composition of individual images in the space with its 3D-representation on the screen;
- to choose the color for the individual images;
- to choose the direction of light and to construct the corresponding distribution of light and shade;
- to consider the perspective as a system of representation of visual images on the screen.

Some of these mathematical and computer graphics problems are solved in the new software application 'ArtMathGraph' (AMG).

Constructing the visual images of complicated geometrical forms as visual models of realistic objects in the AMG application is based on:

- analytical approximation of complicated geometrical forms by parametric equations of mathematical transformed surfaces;
- computer visualization of mathematical transformed surfaces;
- interactive visual mathematical modeling of predetermined complicated geometrical forms.

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The mathematically transformed surfaces are a result of applying the mathematical transformations to the analytical representations of initial well-known classical surfaces such as plane, cone, cylinder, sphere, ellipsoid, etc. The analytical statement and software application for constructing and visualizing a large number of new Mathematical Transformed Surfaces was developed previously, based on conical transformations of torses as examples of Initial Surfaces [1, 2]. Those Mathematical Transformed Surfaces were analyzed as Kinematic Surfaces [1, 2].

The ability of analytical approximation of complicated geometrical forms as visual models of realistic objects is based on the constructing multiple different sets of new transformed surfaces by combining available values of variable parameters in the equations of initial surfaces and/or of mathematical transformations.

For methodical example, the graphic result of some mathematical transformations of Cone or Cylinder Surfaces as initial surfaces is shown in the Figures 1–12 [3]. The analytical representations of these initial and mathematical transformed surfaces are cited by the parametric equations (1) – (8).

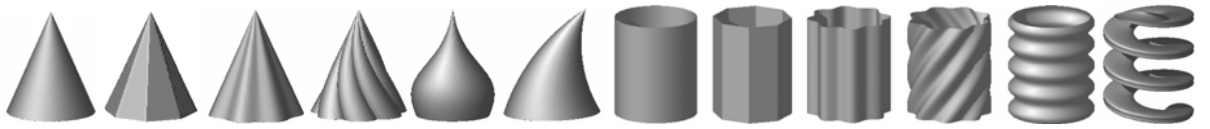

Figure.1 Figure.2 Figure3 Figure 4 Figure 5 Figure 6 Figure 7 Figure 8 Figure 9 Figure 10 Figure 11 Figure 12

Initial Cone Surface (Figure 1):

$$x = av\cos u;\ \ y = av\sin u;\quad z = v; \tag{1}$$

$$0 < u \le 2\pi,\ \ 0 \le v \le h.$$

Initial Cylinder Surface (Figure 7):

$$x = a\cos u;\quad y = a\sin u;\quad z = v; \tag{2}$$

$$0 \le u \le 2\pi,\ 0 \le v \le h.$$

Transformed Cone Surfaces (Figures 2-6):

1. Cutting (Fig. 2)

$$x = a\frac{\cos\frac{\pi}{k}\cdot\cos u}{\cos(\frac{\pi}{k}-\varphi)}v;\quad y = a\frac{\cos\frac{\pi}{k}\cdot\sin u}{\cos(\frac{\pi}{k}-\varphi)}v;\quad z = v; \tag{3}$$

$$\varphi = u - [\frac{k}{2\pi}\cdot u]\frac{2\pi}{k},\ \ 0 \le u \le 2\pi,\quad 0 \le v \le h\ .$$

where k – a quantity of sides.

2. Sinus-waviness (Figure 3)

$$x = av(1 + d\sin ku)\cos u;\ \ y = av(1 + d\sin ku)\sin u;\ \ z = v; \tag{4}$$

$$0 < u \le 2\pi,\ \ 0 \le v \le h,$$

where k – a quantity of waves, d – a height of a wave.

3. Quasi-epicycloidal lobularity with twisting along axis *oz* (Figure 4)

$$X = x\cos\omega z + y\sin\omega z;\ \ Y = -x\sin\omega z + y\cos\omega z;\ \ Z = z, \tag{5}$$

where ω – determines direction and degree of twisting,

x, y, z are calculated according to the equations of quasi-epicycloidal lobularity:

$$x = av((1+k)\cos u - \cos((1+k)u));\ \ y = av((1+k)\sin u - \sin((1+k)u));\ \ z = v; \tag{6}$$

$$0 < u \le 2\pi,\ \ 0 \le v \le h,$$

where k – a quantity of lobes.

4. Thickening (Figure 5)

$$X = \frac{x}{1 + p(v - v_c)^2}; \quad Y = \frac{y}{1 + p(v - v_c)^2}; \quad Z = z, \tag{7}$$

where *x, y, z* are calculated according to the equations (1),
p – determines a degree of thickening,
v_c – determines a position of thickening along axis *oz*.

5. Bending along axis *oy* (Figure 6)

$$X = q - (q - x)\cos\frac{z}{q}; \qquad Y = y; \qquad Z = (q - x)\sin\frac{z}{q}, \tag{8}$$

where *x, y, z* are calculated according to the equations (1),
q - determines direction and degree of bending.
(The parametric equations of Transformed Cylinder Surfaces (Figures 7-12) and Transformed Cone Surfaces (Figures 2-6) are analogous.)

In addition to the basic set of Initial Surfaces and some groups of mathematical transformations, the AMG application also includes the means of interactive visual mathematical modeling. Each user's action, such as selection of initial surfaces and transformations or adjusting their parameters, is visualized on a screen. The AMG application allows the user, guided by a screen view, creating visual images, adjusting parameters, as well as deriving realistic three-dimensional images.

As some illustrations of the AMG application possibilities, several "Mathematical Pictures" are shown in the Figures 13 – 17.

Figure 13

Figure 14

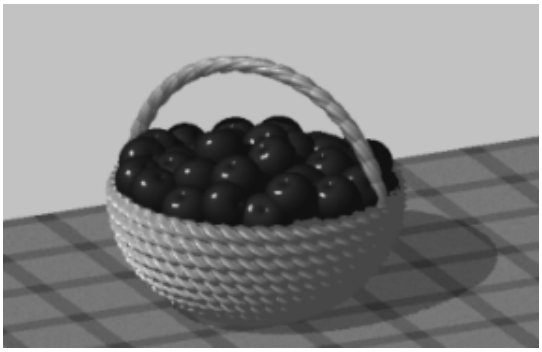

Figure 15

Figure 16

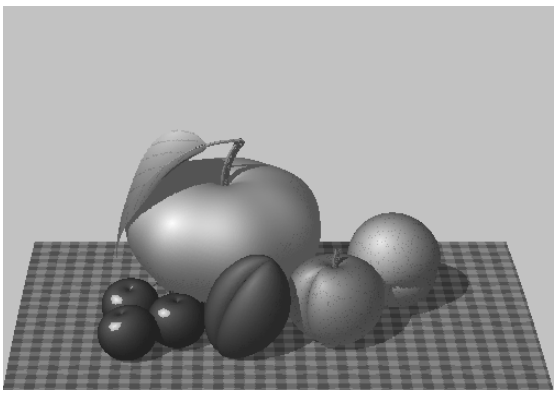

Figure 17

By experience, consecutive use of all means of the AMG application allows obtaining various artistic compositions of visual images of realistic objects, including the objects of Nature, by methods of Analytical Geometry and Computer Graphics only.

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MOŻLIWOŚCI KONSTRUKCJI OBRAZÓW ARTYSTYCZNYCH ZA POMOCĄ GEOMETRII ANALITYCZNEJ I GRAFIKI KOMPUTEROWEJ

Nowy program „ArtMathGraph” (AMG) służy do tworzenia artystycznych obrazów z realistycznych obiektów poprzez aproksymację skomplikowanych form geometrycznych, opisanych równaniami parametrycznymi, przekształconymi matematycznie powierzchniami. Wykorzystuje on modelowanie matematyczne predefiniowanych obiektów geometrycznych do uzyskania interaktywnego obrazu. W efekcie przekształceń matematycznych dobrze znanych klasycznych powierzchni (takich jak płaszczyzna, stożek, walec, sfera, elipsoida itp.) zadanych analitycznymi reprezentacjami, uzyskuje się wizualne obrazy realistycznych obiektów, nie wyłączając obiektów natury takich jak: warzywa, owoce, kwiaty czy liście. Aplikacja AMG daje możliwości zestawiania pojedynczych obrazów w nawet bardzo skomplikowane kompozycje artystyczne wykorzystując jedynie geometrię analityczną i grafikę komputerową.

W artykule przedstawiono kilka graficznych przykładów matematycznie przekształconych podstawowych powierzchni geometrycznych.