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Position fixing and its accuracy evaluation

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Abstract
In traditional approach to position fixing navigator exploits mathematical apparatus based on probability theory. Series of assumptions are required in order to use the platform to draw final conclusions. Limited ability is available regarding fix accuracy a posteriori evaluation. In the paper Mathematical Theory of Evidence is exploited in order to introduce new foundations enabling modeling and solving problems with uncertainty. Modified scheme of approach towards making the fix delivers new standpoint for perceiving accuracy of the result.

Introduction

Imprecise and uncertain data dominate in maritime navigation. Imprecision results from wrong calibrated devices their natural limitation, as well as limitation in perceiving ability of an observer. Uncertainty is related to imprecision but also refers to quality of particular measurement. Observed object can be close and clear or far and vague, these two cases contributions to the fix should be differentiated. Positions indicated by various navigational aids are also of different quality. They are randomly distributed around the true place of the ship. Types of distributions of measurements and indications are assumed known although their parameters vary on real scale depending on many factors.

Hierarchy among available data is to be upgraded and included into computation scheme. Unfortunately in traditional approach possibility of doing so is rather limited.

Mathematical Theory of Evidence was proposed by Dempster [1] and Shafer [2], it extends probabilistic approach. Further extensions enabling processing imprecise data [3] create unique platform for modeling uncertain knowledge and ignorance. Evidence combination scheme as mechanism enabling enrichment combined data informative context is exploited in many applications [4, 5]. In nautical applications it can be useful in order to make position fixing and evaluate its accuracy [6, 7]. Scheme of combination is numerically complex; it is exponentially bounded on the number of observations [8]. Therefore, some effort must be done in order to reduce number of required iterations. Some improvement in the matter has already been achieved [9].

Mathematical Theory of Evidence enables upgrading models and solving crucial problems in many disciplines. The matter is rather hampered in traditional, probabilistic approach due to high level of uncertainty. MTE delivers new unique opportunity once possibilistic extension was adopted [10, 11, 12]. Approaches towards theoretical evaluation of tasks including nondeterministic ones and those with imprecise data are to be reconsidered. Despite obvious advantages significant interest in the new opportunity has not been observed so far. Publications devoted to nautical applications are rather scarce, those appeared are delivered by the author. Some of them considered evaluation of navigational situation within confined and congested areas of crossing routes [12]. In order to forecast and evaluate condition within confined region one has to engage possibilitic platform. Statement like: large vessels encounter at the crossing of heavy traffic routes create hazardous situation involves fuzziness. Imprecision refers to classification of ships, sort of traffic and quality of condition. In another publication [13] uncertainty in floating objects detection ability by a group of monitoring stations was considered. Hereto synergetic effort is involved; cumulated ability of detection is of interest. Common ability of discovering floating object by all station covering considered region under certain sea surface conditions is sought, extrapolation, engaging approximate reasoning methods, for various conditions is required [14].

Uncertainty in available detections characteristics and measurements distribution is common feature for all presented problems. Shortcomings of traditional mathematical apparatus caused that this sort of tasks were solved mainly based on the skill and more often on intuition of engaged navigation experts.

Many tasks are realized under uncertainty resulted from variable natural condition of measurements or retrieving data from navigational aids. Variety of data quality can be subjectively classified, introducing this sort of hierarchy hardly matters since, there is not formal apparatus to include them into calculation scheme. Thus various quality data affects final solution in the same manner.

Mathematical Theory of Evidence exploits belief and plausibility measure and operates on belief functions. Belief function is a mapping that consists of pairs: vectors representing fuzzy locations of a set of points within sets related to each measurement – degrees of confidence assigned to these vectors. Degrees of confidence reflect probability that a line of position is being located within given strip area or, during processing, position being located inside two belts intersection region. Appropriate imprecise values are at disposal based upon statistical investigations of measurements distributions. In Mathematical Theory of Evidence belief structures combination is carried out [15, 16]. During combination all pairs of location vectors are associated and product of involved masses is assigned to the result set. Obtained assignment is supposed to increase informative context of the initial structures. Combination of structures embracing measurements data is assumed to result in position fixing. The goal can be achieved provided selection of common points is carried out during association. In navigation points situated within intersection of introduced ranges are to be selected. Selection is done thanks to T-form operations [14] used during association [17]. The simplest T-form results in smaller values being taken from consecutive pairs of associating elements.

Position fixing

Figure 1 shows traditional way of position fixing with three distances. Three circles intersect at three points in the vicinity of the fixed ship position. Assuming measured distances as mutually independent random variables, the true position is somewhere inside obtained triangle. It is up to navigator's knowledge and experience to estimate the fix. The more accurate the measured distances, the smaller is the triangle and thus the better is the estimation of the fixed position. Obviously an experienced navigator is able to verify acceptable dimensions of such triangle. Intersection area, greater than an average, results in rejection of the observations.

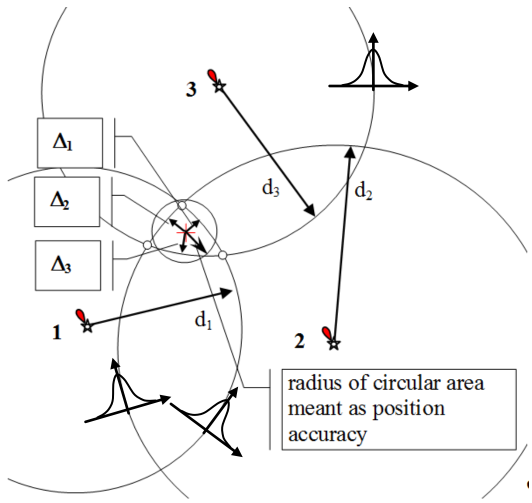

Fig. 1. Example of position fixing based on three imprecise distances

The most common approach to analytical way of position fixing exploits the least square adjustment method. One has to find a point for which expression $\Sigma_k w_k \cdot \Delta^2_k$ reaches its minimum. Sum of weighted squared deflections Δ_k from the measured isolines is calculated. Weights w_k introduce credibility masses attributed to each of the taken distance. Traditional way of position fixing engages:

1) available indications and/or measurements;
2) characteristics of the measured values and type of distribution are not important, although normal distribution is widely assumed and exploited in the least square adjustment method;
3) subjectively evaluated masses of credibility attributed to each of measurements included in analytical approach;
4) measured values as random variable governed by normal distributions, as well as constellation of observed objects are considered in the fix accuracy estimation.

The main disadvantage of traditional approach is the lack of inherited method evaluating quality of the obtained fix. Unfortunately, existing form of accuracy estimation appears to be inadequate in many practical cases.

In the previous papers [5, 18, 19] the author presented concept of engaging MTE extended for fuzzy environment to position fixing computation scheme. Possibilistic extension of the theory appeared to be flexible enough to be used for reasoning on the fix, provided imprecise measurements and/or indications are available. Contrary to the traditional approach it enables embracing knowledge and uncertainty into calculations. Knowledge regarding position fixing includes: characteristics of random distributions of measuring values, as well as ambiguity and imprecision in obtained parameters of such distributions. Moreover, observations can be differentiated by subjectively evaluated masses of confidence attributed to each of them.

The solution proposed and used herein is based on Mathematical Theory of Evidence (MTE), extended to fuzzy environment [12] is more flexible as it enables considering of the following:

1) available indications and/or measurements;
2) various characteristics of the measured values; kind of distribution is important and may affect final solution; empirical and theoretical distribution can be considered;
3) accuracy of measured distances, including ability of engaged aids, their lengths and characteristic of the referenced object;
4) imprecision in accuracy estimation[1];
5) subjectively evaluated masses of credibility attributed to each of measurement;
6) inconsistencies of the computation process;
7) fix adjustment in case of abnormal high inconsistency;
8) evaluation of selected position quality is embedded into computation scheme; plausibility, belief and inconsistency values enable direct assessment of the fix;
9) belief and plausibility measures instead of crisp valued probability are to be used once quality of the fix is evaluated;
10) plausibility of the fix being located within adjacent area is easily available, thus reasoning on the fix accuracy appears to be straightforward.

[1] In books devoted to navigation one can read that mean error attributed to measuring with particular aid is x, but reaching y ($y > x$) value is also possible.

Notes on the fix accuracy estimation

Traditional meaning of the fix accuracy is related to a regular area around the fixed position. Within the area the true position of the ship is located with certain and equal degree of credibility. It is assumed that the area is of circular or elliptical shape within which the fix is located with the same probability. The latest is widely assumed although it is known that condition (1), that contradicts the statement, is to be observed. The formula expresses probability of the fix being located in point (x, y) as a function of probabilities of all isolines embracing given point along with credibility attributed to each of the measurements.

$$p_{f|_{(x,y)}} = f\left(p_{o_i|_{(x,y)}}, \Omega_i\right) \tag{1}$$

where:

- $p_{f|_{(x,y)}}$ – probability that the fix is located in (x, y) point;
- $p_{o_i|_{(x,y)}}$ – probability that the point (x, y) is located at the isoline related to i-th observation;
- Ω_i – credibility attributed to the i-th observation, subjectively evaluated quality of the measurement.

In traditional practical approach formulas enabling calculation of the radius or ellipse's parameters are derived for typical schemes of observations followed while a fix is being made [20, 21, 22]. Usually bearings and distances are taken. Two or three bearings combined with distances are often exploited for position fixing. Appropriate formula is to be engaged to evaluate mean error of the fix. Expression (2) (see [20]) is an example to be used when calculating mean error of the fix obtained with three distances. The formula engages mean errors of involved measurements and angles of intersection of lines of position.

$$m_f = \\ = \pm\sqrt{\frac{m_1^2 \cdot m_2^2 + m_2^2 \cdot m_3^2 + m_1^2 \cdot m_3^2}{m_1^2 \cdot \sin^2\Theta_2 + m_2^2 \cdot \sin^2(\Theta_1 + \Theta_2) + m_3^2 \cdot \sin^2\Theta_1}} \tag{2}$$

where:

- m_i – mean error the the i-th observation;
- Θ_1 – angle of intersection of the first and second isoline;
- Θ_2 – angle of intersection of the second and third isoline.

Mean error of the fix is meant as circular area with the centre in the fix. Point representing fixed position is assumed to be located in geometric centre of a figure spanned over selected intersection points of obtained isolines. The formula was derived based on normality of the measurement error distributions. It should be stressed that more observations engage even more complex formulas. For this reason expressions for greater number of measurements are impractical and usually not available in nautical publications.

There is yet another drawback related to traditional way of accuracy estimation. The approach does not correlate quality of observations and accuracy of the obtained fix consequently contradicts expression (1). Figure 2 presents two cases of fixed positions and their accuracy estimations. It should be noted that estimations are the same in both cases. Assuming the same scale and constellation of observed objects, as well as lack of constant errors in case a) quality of observations seems be poorer than in case b). Intersections of three isolines in case a) are spread over much larger area compared to right hand case. Thus accuracy of the fix b) seems be different than in case a). Unfortunately, in traditional approach accuracy estimation does not reflect real quality of the fix, although true the statement seems to be somewhat contradictory and illogic. Obviously supporters of the idea can claim that as long as measurements are random variables it may happen. Under this assumption accuracy estimations remain valid in both cases. Nonetheless allocation of isolines within area close to the fixed position seems important factor when accuracy is being a priori analyzed.

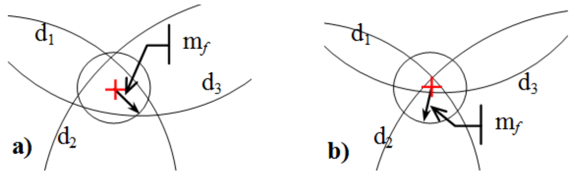

Fig. 2. Two cases of fixed positions and their accuracy estimations

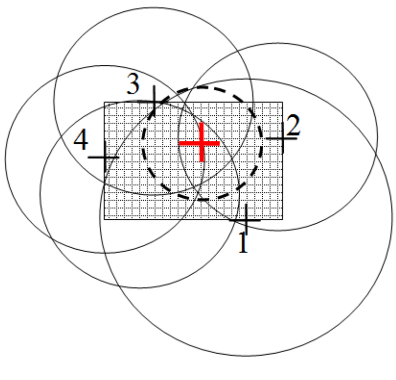

Fig. 3. Fixed position made with four indications delivered by various navigational aids

In monographs devoted to nautical science [21, 22] problem of making a fix based on indications delivered by various navigational aids (example shown in figure 3) is treated superficially, meaningless attention is devoted to accuracy of such fix. Authors suggest using Expression (3) to obtain hints on quality of the fix.

$$\sigma_w = \sqrt{\sigma_1^2 + \sigma_2^2 + \ldots + \sigma_n^2} \qquad (3)$$

Formula (3) estimates mean error provided standard deviations of involved indications are known. Calculated value is a length of the radius defining circle within which the fix is located with probability of 0.68. Particular instance of constellation of indicated positions is not taken into account while estimating accuracy in this way.

Another view at the fix accuracy

In approach based upon MTE distribution of probabilities of the fix being located within explored area is embedded into methodology. Expression (1) is valid and engaged during calculation. Therefore, accuracy can be perceived as a cohesive area within which probability (plausibility) of the fix location is higher the required threshold value.

Using possibilistic concept that has been explained in previous papers [5, 19] software tools have been implemented. The software was used to make the fix with four distances. Presented in figure 4 illustration include probability distribution for the fix being located in adjacent area. Distributions of figures denote plausibility of the fix within hypothesis frame. Estimated mean errors of each observation [cables], as well as subjective evaluations of measurements are shown in the insertion. It was assumed that mean errors are interval valued. Presented error estimations should be treated as modal values of intervals $[\sigma_i - 0.1 \cdot \sigma_i,\ \sigma_i + 0.1 \cdot \sigma_i]$. Subjective assessments are modal for linguistic terms: “medium” and “very good” fuzzy values.

Iterative procedure was implemented to make the fix [23]. In consecutive iterations decreasing search area was explored. Explored area embraces all maxima points selected in previous iterations. Grid of 10×10 cells was spanned over the area in order to define hipothesis space. Distribution of the fix plausibility measures all over the area examined in the last iteration is shown in the centre part of figure. It should be noted that area within which probabilities reach their maxima is not a circular one. Instead irregular shape of cohesive area with highest plausibility measures represents the fix accuracy.

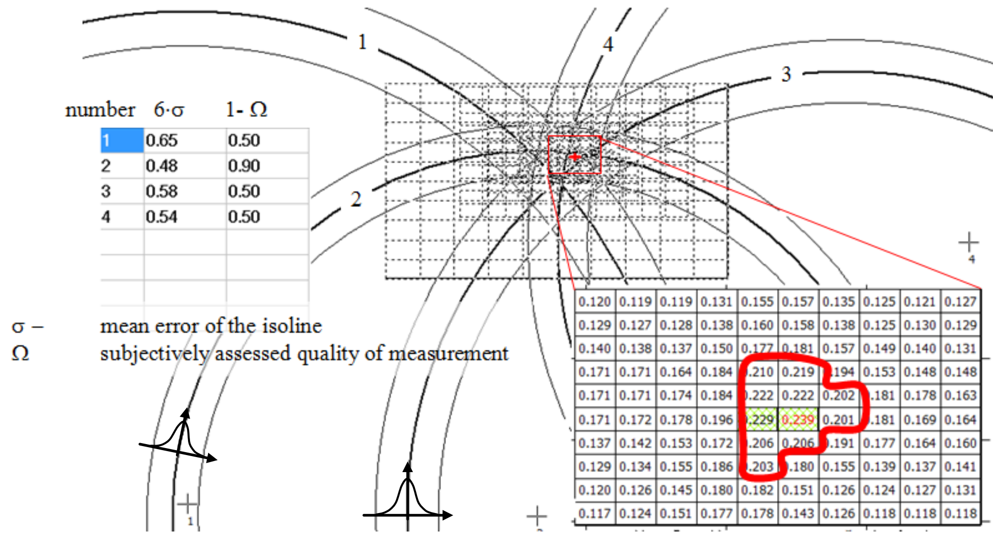

Fig. 4. Making the fix with four distances – output delivered by software implementing possibilistic approach towards position fixing

Representing uncertain evidence in nautical applications

In possibilistic approach uncertain evidence is represented using fuzzy sets. Each set has assigned mass of confidence. Relations between hypothesis and evidence spaces are encoded into evidence representation. Sets (usually fuzzy ones [23]) embrace grades expressing possibilities of belonging of consecutive hypothesis items to the sets related to each piece of evidence. As already mentioned each of the sets has credibility mass assigned. Thus evidence mapping consist of "fuzzy set – probability assigned to the set" pairs. Adequate mapping is expressed by Formula (4).

$$m(e_i) = \{(\mu_{i1}(x_k), f(e_i \to \mu_{i1}(x_k))), \cdots \\ \dots,(\mu_{in}(x_k), f(e_i \to \mu_{in}(x_k)))\} \quad (4)$$

Herein in order to draw useful conclusions simplified evidence representation will be considered. Three distances measured to different objects will be taken into account (see Fig. 5). The drawing also shows example set of points treated as hypothesis frame or a search space. Hypothesis points locations will be encoded in binary terms: for situated within considered area value of 1 is used, for those outside the range 0 is applicable. It should be emphasized that such simplification does not affect generality of the rational in sense of usefulness of drawn conclusions.

Reducing scope of interest to measured distances sets related to each piece of evidence can be limited to the following items: $e_1 \to \{d_1\}$, $e_2 \to \{d_2\}$ and $e_3 \to \{d_3\}$. Thus membership function grades take the form of expression: $\mu_i(\{x_k\}) = g(\{x_k\} \to \{d_1, d_2, d_3\})$. The expression means that membership grades are degrees of inclusion of hypothesis points within evidence frames (in the example they refer to circles confined by appropriate distance). Grades identify whether respective point is located closer to observed objects than measured distance. Considering single grade $\mu_i\{x_k\}$ one can use formula (5) to obtain its binary value:

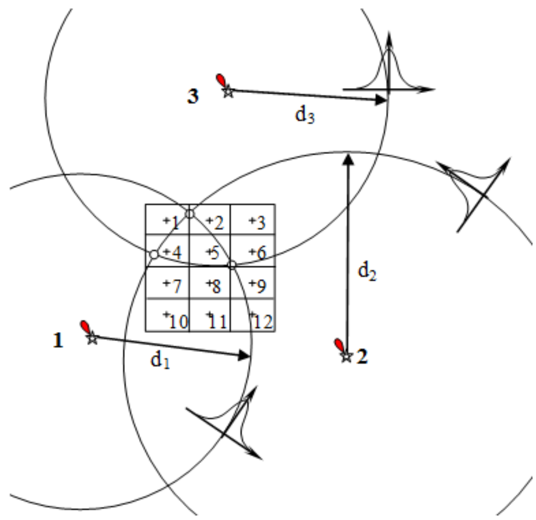

Fig. 5. Example of three distances and a set of hypothesis points

$$\mu_i(x_k) = \begin{cases} 1 & \text{if } d(x_k) \le d_i \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

where: $d(x_k)$ is the distance between k-th point and i-th observed landmark.

Figure 4 presents example of distances taken to three different objects and a set of hypothesis points. Using formula (5) grades of sets related to taken distances were obtained and presented in table 1. Row headers named as: μ_1, μ_2, μ_3 show locations hypothesis points within sets related to measured isolines. Vectors together with assigned, example masses presented in the last column are constituents of the evidence representation as specified by formula (4).

Table 1. Location vectors and results of their combinations

	1	2	3	4	5	6	7	8	9	10	11	12	$m(..)$
μ_1	{1	0	0	1	1	0	1	1	0	1	1	0}	0.6
μ_2	{0	1	1	1	1	1	1	1	1	1	1	1}	0.5
$\mu_1 \wedge \mu_2$	{0	0	0	1	1	0	0	1	0	1	1	0}	0.3
μ_3	{1	1	1	1	1	1	0	0	0	0	0	0}	0.7
$\mu_1 \wedge \mu_2 \wedge \mu_3$	{0	0	0	1	1	0	0	0	0	0	0	0}	0.21

Two evidence representations can be combined. Result grades of membership functions are selected using T-norm operation; for calculation details see previous publications [8, 23]. In the first step of combination data in row $\mu_1 \wedge \mu_2$ were obtained. Next the same procedure was used to associate row $\mu_1 \wedge \mu_2$ and row μ_3. Two steps combination yields data presented in row $\mu_1 \wedge \mu_2 \wedge \mu_3$. It should be noted that result set embraces two points situated within common area for three circles related to taken distances. It was achieved thanks to T-norm operation used during association.

Summary and conclusions

In the paper comparison of traditional way of position fixing and approach based on theory of evidence was presented. Main advantage of the proposed scheme of reasoning is that it engages possibilistic approach [24]. The approach is justified whenever insufficient data samples are available. It is quite often when dealing with estimations of measurements distributions. Possibilistic mechanisms engage belief and plausibility measures. Adequate formulas were derived based on exploration of knowledge base obtained as a result of evidence combination.

In proposed approach knowledge included into computational scheme is something what creates new opportunity. New standpoint for perceiving accuracy of the fix is possible when using reasoning mechanism. Traditional understanding and estimating of accuracy is inadequate in most cases. Appropriate formulas are intended for particular observations schemes that include at most three measurements. Although basic set of data (mean errors and constellation of observed objects) are included in accuracy estimation, applying the same mean error measure for different distributions of isolines seems unjustified. In the new approach accuracy estimation is embedded into reasoning scheme. Obtained results emphasize obvious shortcomings of the traditional approach.

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