Regular and Chaotic Dynamics of a 4-DOF Mechanical System with Dry Friction

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Abstract

In this paper the model of four degree-of-freedom mechanical sliding system with dry friction is considered. One of the components of the mentioned system rides on driving belt, which is driven at constant velocity. This model corresponds to a row of carriage laying on a guideway, which moves at constant velocity with respect to the guideway as a foundation. From a mathematical point of view the analyzed problem is governed by four second order differential equations of motion, and numerical analysis is performed in Mathematica software. Some interesting behaviors are detected and reported using Phase Portraits, Poincaré Maps and Lyapunov Exponents. Moreover, Power Spectral Densities obtained by the Fast Fourier Transform technique are reported. The presented results show different behaviors of the system, including periodic, quasi-periodic and chaotic orbits.

Keywords: periodicity, quasi-periodicity, chaos, hyper-chaos, non-regular vibrations

1. Introduction

The comprehension and characterization of dynamical systems belong to a challenging subject in recent years [1], and also nowadays these investigations are still continued. In many real systems (for instance, sliding linear guide systems, brakes, clutches, piston rings in a cylinder, and many other) friction phenomenon and stick-slip effect as a result of relative sliding velocity between surfaces of bodies rubbing themselves have a great impact on the strength of mechanical elements of these systems as well as their dynamics. And although there are numerous papers related to the mentioned problems in the literature, not all effects, associated with the friction phenomenon, have been sufficiently understood so far. In many cases, the presence and the manifestation of some effects depends on the structure of the considered system. In general, friction belongs to the complex processes and depends on various parameters like relative sliding velocity, normal load or surface properties. As an example, a review on different applied in the literature dry friction models can be found in [4], or in the recent paper [5].

The presented in this paper studies are a continuation and extension of research related to the mechanical model presented in [2,3]. In comparison to the mentioned papers, here other new numerical simulations obtained for other system parameters are presented and discussed. In addition, in contrary to the previous numerical investigations, beyond using Phase Portraits and Lyapunov Exponents, also other methods are used and applied like Poincaré Maps and Power Spectral Densities (PSDs).

The rest of the paper is organized as follows. In section 2 mechanical model of the considered system and its equations of motion in the non-dimensional form are introduced. In section 3 assumptions of numerical computations, the applied approximations of non-smooth functions, as well as parameters of the considered system are introduced. Numerical results of our investigations are presented in section 4. Finally, conclusions of our investigations are presented in the last section 5.

2. Mechanical Model and Non-Dimensional Form

The analyzed in this paper four degrees-of-freedom model is shown in Fig. 1.

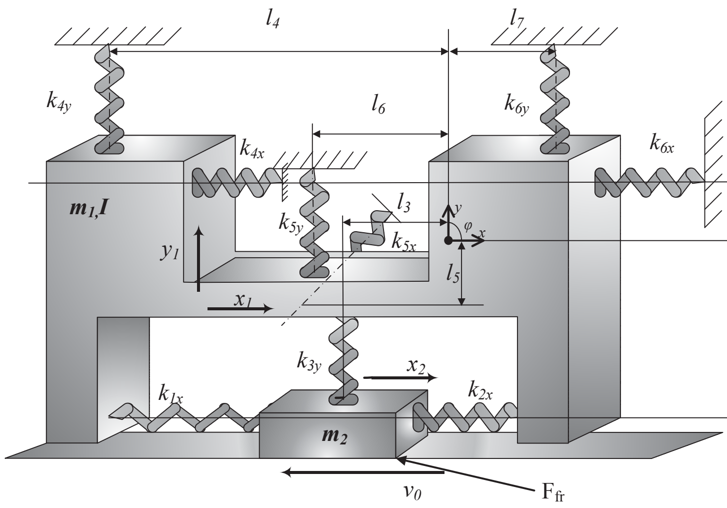

Figure 1. The considered 4-DOF model with dry friction

The state of the considered dynamical system is described by the following variables: x_1, $v_1 = \dot{x}_1$, y_1, $z_1 = \dot{y}_1$, φ, $\omega = \dot{\varphi}$, x_2 and $v_2 = \dot{x}_2$. The body of mass m_1 can rotate about the pivot axis S (moment of inertia about the pivot axis S of this mass is equal to I). The whole system is characterized by lengths l_i (i = 1,2,...,6) and springs with stiffness coefficients k_{ix}, k_{jy} (i =1,2,4,5,6; j =3,4,5,6). Moreover, additional body of mass m_2 is placed on the belt as a foundation, which moves with a constant velocity v_0. Between the mentioned mass m_2 and the belt dry friction force occurs, which is a function of the relative sliding velocity $v_0 - \dot{x}_2$. Equations of motion of the system are obtained using the second kind of Lagrange equations (presented in detail in [2]) and have the following non-dimensional form

$$\begin{cases} \ddot{x}_1 + a_1 x_1 + a_2 \varphi - a_3 x_2 = 0, \\ \ddot{y}_1 + b_1 y_1 - b_2 \varphi + f_g = 0, \\ \ddot{\varphi} + c_1 x_1 - c_2 y_1 + c_3 \varphi - c_4 x_2 = 0, \\ \ddot{x}_2 - x_1 - \varphi + x_2 = f_k(v_0 - \dot{x}_2) \cdot [f_g - (e_1 y_1 - e_2 \varphi)] \cdot \mathbf{1}(f_g - (e_1 y_1 - e_2 \varphi)), \end{cases} \tag{1}$$

where x_1, $\dot{x}_1$, y_1, $\dot{y}_1$, φ, $\dot{\varphi}$, x_2, $\dot{x}_2$ denote now non-dimensional state variables. Other non-dimensional parameters and functions of Eqn. (1) are introduced in section 3.

3. The Applied Approximations and Parameters

Numerical simulations are obtained in Mathematica software via the fourth order Runge-Kutta method, and the trajectories are started from zeros initial conditions. The values of non-dimensional system parameters are as follows:

$$a_1 = 0.08,\ a_2 = 0.03,\ a_3 = 0.04,\ b_1 = 0.09,\ b_2 = 0.03,\ c_1 = 0.03,\ c_2 = 0.03,$$
$$c_3 = 0.06,\ c_4 = 0.03,\ f_g = 0.01,\ e_1 = 1.38,\ e_2 = 0.47,\ v_0 = \text{var.},$$

and their estimation is explained in [2]. Kinetic friction function $f_k(v_0 - v_2)$ in our model is described by the Stribeck function. Since the classical signum function is discontinuous, it has been approximated by the hyperbolic tangent function with control parameter ε in the following way

$$f_k(v_0 - v_2) = \mu_0 \tanh\left(\frac{v_0 - v_2}{\varepsilon}\right) - \alpha(v_0 - v_2) + \beta(v_0 - v_2)^3 \tag{2}$$

with fixed parameters $\mu_0 = 0.8$, $\alpha = 15{,}59$, $\beta = 4252{,}12$ and $\varepsilon = 10^{-4}$. Because the unit step function $\mathbf{1}(f_g - (e_1 y_1 - e_2 \varphi))$ is also discontinuous, the following approximation is also applied in our computations

$$f_n(f_g - (e_1 y_1 - e_2 \varphi)) = \left[\tanh\left(\frac{f_g - (e_1 y_1 - e_2 \varphi)}{\varepsilon}\right)\right]^3 \cdot \mathbf{1}(f_g - (e_1 y_1 - e_2 \varphi)). \tag{3}$$

4. Results

Figs. 2-4 present numerical simulations for different parameter v_0. The presented results vary from each other, depending on the used value of v_0 parameter.

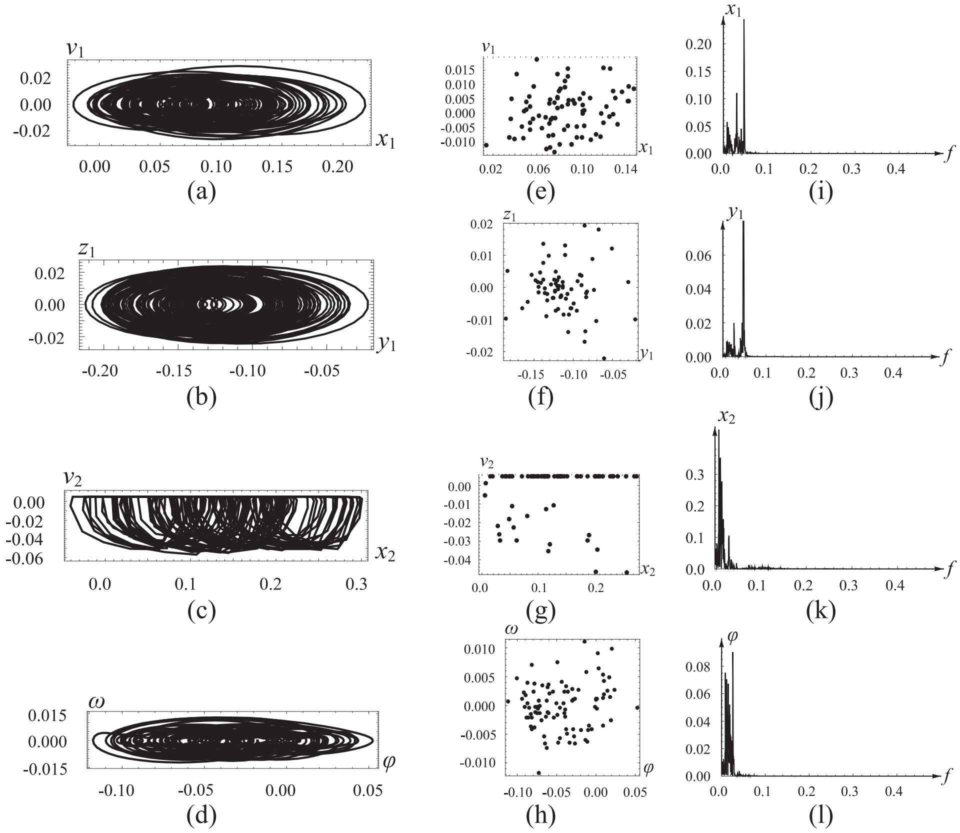

Figure 2. Phase portraits (a,b,c,d), Poincaré sections (e,f,g,h) and PSDs (i,j,k,l) for $v_0 = 0.005$ in time interval $\tau \in (20000,22000)$

As can be seen, for $v_0 = 0.005$ the character of motion is chaotic. Presented in Fig. 2 phase portraits, Poincaré sections and PSDs confirm its irregular dynamics. The chaotic attractor has different forms on different Poincaré maps. Moreover, it should be emphasized that the characters of motion differ is very sensitive to the changes of the belt velocity v_0. In particular stick-slip chaotic dynamics is clearly exhibited by the phase portrait shown in Fig. 2c and the Poincaré map reported in Fig. 2g.

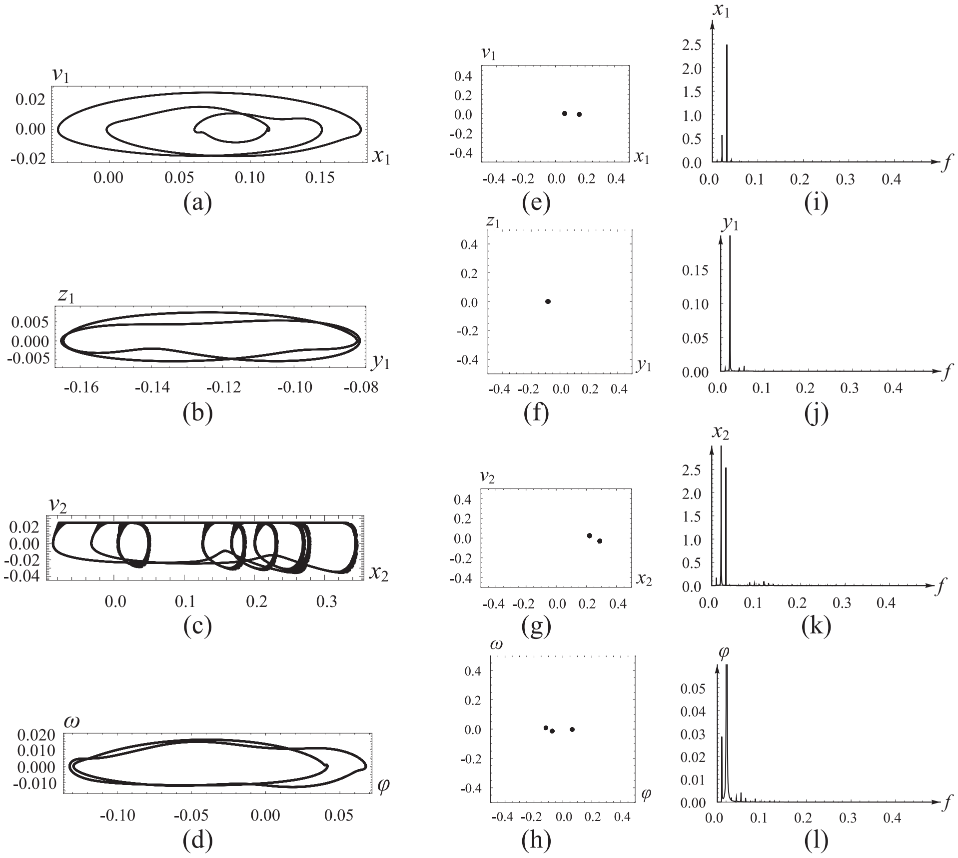

Figure 3. Phase portraits (a,b,c,d), Poincaré sections (e,f,g,h) and PSDs (i,j,k,l) for $v_0 = 0.025$ in time interval $\tau \in (20000, 22000)$

When $v_0 = 0.025$, for variable x_1 there is a periodic-two cycle orbit, which is represented by two points in the Poincaré section (Fig. 3e) and is depicted as the trajectory crosses itself in phase portrait (Fig. 3a). The same situation occurs for state variable x_2. While for y_1 a period-one harmonic appears (Fig. 3f), it is worth noting that this is a closed curve in the phase plane (Fig. 3b). A three cycle period behavior is presented for ω (Fig. 3d,h).

Frequencies, at which the energies are strong and at which variations energies are weak, are shown in the Fig. 3 (i,j,k,l) for $v_0 = 0.025$. For the following state variables: x_1, y_1, x_2 and ω the energy is the strongest at two, single, two and three frequencies, respectively.

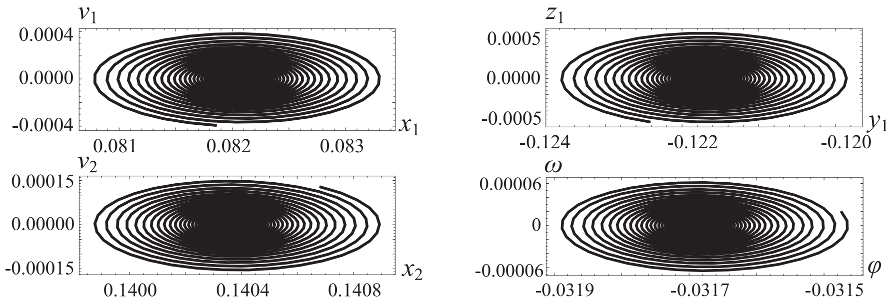

Figure 4. Trajectories of the system for $v_0 = 0.05$ in time interval $\tau \in (10000{,}12000)$

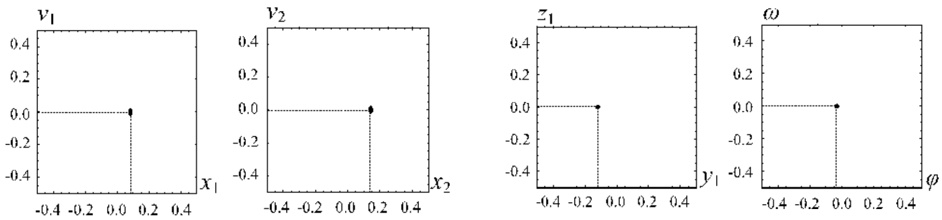

Figure 5. Poincaré sections for $v_0 = 0.05$ in time interval $\tau \in (20000{,}22000)$

Another character of motion is detected for $v_0 = 0.05$. Fig. 4 shows the transient states for chosen time interval, which indicate that the trajectories of the system go to the fixed points. After avoiding the mentioned transient states, the Poincaré sections are also obtained and presented in Fig. 5, and they prove that the system goes to steady state.

Table 1. Lyapunov exponents for different parameter v_0

v_0	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8
0.005	0.0069	0.0027	0.0001	-0.0010	-0.0030	-0.0077	-0.0206	-33.00
0.025	0.0000	-0.0026	-0.0027	-0.0095	-0.0358	-0.0384	-0.0960	-19.52
0.032	0.0002	-0.0004	-0.0011	-0.0080	-0.0149	-0.0324	-0.1333	-15.81
0.04	0.0000	-0.0022	-0.0026	-0.0160	-0.0198	-0.0421	-0.0850	-7.8248
0.05	-0.0043	-0.0045	-0.0100	-0.0102	-0.1195	-0.1197	-0.1764	-2.2371

Our numerical investigations are also conducted by calculations of the max. Lyapunov exponents, which are depicted in Tab. 1. Moreover, as an example, time histories of max. Lyapunov exponents for two different parameter v_0 are reported in Fig. 6. The

Lyapunov exponents for each values of v_0 has been obtained using the Gram-Schmidt reorthonormalization time $\Delta T = 0.5$, after avoiding the transition state and starting numerical computations from zeros initial conditions. Chaotic characters of motions are detected for v_0 equal to 0.005 and 0.032, while the periodic behavior are detected for v_0 equal to 0.025, 0.04 and 0.05. For $v_0 = 0.05$ the trajectories goes to the fixed points.

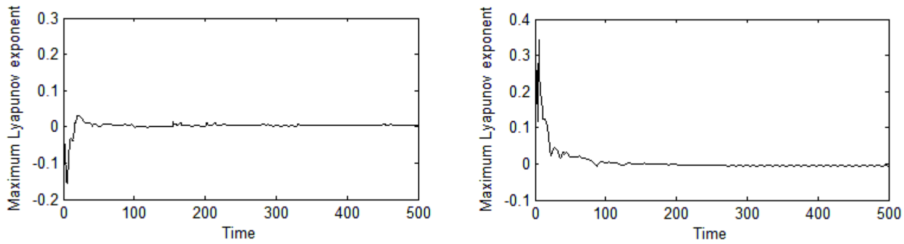

Figure 6. Time histories of max. Lyapunov exponents of the system for different values of velocity v_0 equal to: (a) 0.005 and (b) 0.05

5. Conclusions

Mathematical model of 4-DOF mechanical sliding systems with dry friction is considered. From a mathematical point of view the mentioned system is presented as a nonlinear system of equations of motion. Dynamics of the analyzed system is carried out for a set of system parameters and various non-dimensional control parameter. Interesting dynamics behaviors of the considered system are reported using standard tools dedicated to the both qualitative and quantitative theories of nonlinear differential equations. There are many technical devices in engineering applications, where we deal with stick-slip induced vibrations. The considered in this paper system can be treated as a model, which corresponds to a row of carriage laying on a guideway and moved at constant velocity with respect to the guideway as a foundation. As this paper shows, there are many possible behaviors of this system, and also it is very sensitive to the changes of the belt (foundation) velocity. It is therefore can be anticipated that also the movement of the real system of this type with various velocities of foundation, may vary considerably. In result, it can cause strongly nonlinear vibrations (regular or chaotic) that moving to the various components of the system may lead to its damage. Therefore the considered system can be used in engineering practice to predict its vibrations, and consequently to its protection.

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