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## CONSTRUCTING TESTS CONSISTING OF GROUPING AND VARIATIONS OF QUESTIONS

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**Abstract:** The paper presents a way of creating tests which are used to evaluate student's knowledge. The method of planning the examination is based on dividing questions into categories and making them randomized in every attempt. Prepared tests are implemented into Moodle learning management system and can be easily solved by participants. In this paper one can look at more complicated tasks which can be put in student's assignments. One of the plugins used to create tests is called Moodle Formulas, and the way of creating tasks using it is described in more detail. There are two examples of how one can form, construct, and implement tests with the use of presented tools.

**Keywords:** test planning, constructing tests, Moodle Formulas, randomized questions.

## **1. INTRODUCTION**

Education is a long process in which students broaden their knowledge constantly, no matter if they are in a primary school or at the university. Throughout all that time spent on learning, students find out about different issues from numerous courses. The requirement of getting a promotion to the next education level is always connected with positive grades, and it is popular to evaluate the level of conducted knowledge by examination. The most famous way of checking whether a student got to know required material or not is testing. However, not only students need to prepare for an exam, instructors need to think of a suited way of testing too. Every time when lecturers plan how to evaluate students' knowledge, their duty is to construct relevant tests or exams, as well as to focus on the methods and ways they use to prepare them.

One of the basic methods of research that is used to analyze students' progress and achievements is the form of a test. It allows instructors to obtain detailed information about what type of material was captured by students. Testing methods are more accurate and more objective in comparison to other research methods like observation, experiment, or oral answers because they do not deform information during checking someone's knowledge by the charisma of the speaker, or their language proficiency [1]. Tests which are responsible for checking student's abilities and knowledge belong to the group of didactic tests [2]. These types of tests are divided into standardized and nonstandardized. The first type of tests have been being arranged, improved and modified through the years because they need to cover specialist domains of research. They are tested on large groups of people. On the other hand, nonstandardized tests are designed to be used by tutors and they

can only check material which was absorbed by students. Whether the type of testing, every one of them needs to be reliable and accurate [3], which means, it should always measure the required information in a clearly defined manner to allow the evaluation based on the results of the test performed. To create a proper form of checking students' knowledge and understanding, non-standardized tests will be used.

#### 2. METHODS

#### 2.1. Preconditions

In case when a detailed analysis about students' achievement is desired, the tutor's duty is to prepare a test based on part of the material. The best practice is to create the tests that are to be assigned during the semester before starting the first lecture and proposing conditions to pass the course. Before preparing a test, it is needed to consider what should be included in such a way of checking knowledge to obtain meaningful results [4]. The goal of examining students is to evaluate how much part of teaching material was acquired by students. The best possible situation is to verify all points from the course description [5], yet it is almost impossible. Therefore, every tutor needs to think about a kind of representation of questions which cover the most detailed material. Representation group of questions should be balanced in that way to check knowledge from every topic from a given course for every level of requirements [1].

#### 2.2. Construction of tests

Being aware of what teaching material contains, and knowing first perception for the representation group, it is time to create questions. We should follow the principle that it is better to create more questions than it is needed. While making questions or building a test, it is often encountered that part of prepared questions are unsuitable, because the type of a test, or the task level is not adjusted to the overall level of the test-questions can be too easy or too difficult to solve. One of the basic rules of creating a test is to avoid memory tasks. In some teaching fields it is impossible, however, for mathematical courses, and especially computational tasks, it is available. If there is a choice between a theoretical question and a practical one – choosing the second option is more efficient [4]. An assignment, which describes real problems, forces students to use their theoretical knowledge to match types of questions with

possible models, use proper formulas, or derive the good ones is always better and more meaningful than tasks with all symbols and their values written. To understand the concept, it is good to think about simple questions such as calculating the area of a triangle. Writing all values symbolically causes the student to only substitute values to the formula, but making up a narration to this problem demands greater knowledge and understanding the case. Without a doubt, it requires more effort from both the lecturer, who needs to construct a proper text to a problem, and from the students' side-such a task is more timeconsuming than the basic one. Moreover, while creating a test it is needed to create both easier and harder tasks, however, a good balance of a level of difficulty makes this test more reliable. Dominance of one type of question destroys overall results. A good practice is to start the test with easier tasks to get students ready to solve more difficult questions [4].

## 2.3. Placing questions into Moodle

Once we know the definition of a test, ways of planning and forming it, it is possible to create a quiz. Firstly, it is essential to create a sufficient number of tasks. They will be implemented in Moodle [6] which is an opensource learning management system (LMS). It helps to create virtual classes or courses, and to manage them. We choose Moodle because it offers a lot of possibilities to check knowledge in the form of quizzes. The basic ones are the following: true/false, multiple choice, short answer, and the most important in terms of mathematical courses numerical type of quizzes. Moreover, a lot of questions in these quizzes can be checked automatically after finishing an attempt. To create tests from different mathematical content we use an additional package to Moodle which is called Formulas [7]. It helps us to create randomized variables from a given interval in every task. Finally, we obtain the same text in every task with different values inside. Every time the test is open, variables in an exercise will differ. It ensures us that even if the test will be conducted a few times, there is a small possibility of having the same question with the same variables. This probability depends on the length of the interval, number of places after delimiter in a floating point number, and on the number of participants writing the test.

### **3. EXAMPLES**

Analyzing two examples of tests, which can be created using Moodle management learning system and its additional package, Moodle Formulas, we can present the way of generating such an exam. We use the possibility of randomization defined parameters in every task, which lets us make tests more different from each other. It prevents from possible cheating or cooperation between examined students. Moreover, we will use Moodle's categories to group prepared tasks by part of material. From each group of tasks, we will randomly choose one question with randomized parameters inside, which will be shown during the exam.

#### 3.1. Calculus

At first we plan an exam from the Calculus I course, which appears in almost every technical field of studies at every university. We need to begin with reading the program content which covers all material required to pass this course. Let us look at the exemplary and simplified program content of Calculus I, which will be used to create an exam:

- Functions: inverse functions, trigonometric functions, exponential functions, logarithm functions,
- Limits: one-sided limits, limit properties, ways of computing limits, infinite limits,
- Derivatives and its application: differentiation formulas, chain rule, critical points, minimum and maximum values, linear approximation, L'Hospital rule, tangent to the curve,
- Integrals and its application: indefinite integrals, substitution rule, integration by parts, definite integrals, area between curves, average function value.

The above four parts (which are called categories in Moodle) are selected from Paul Dawkins's online Calculus I course [8] from Lamar University. We grouped all of the material in a more understandable way and summarized it to only four sections. Obviously, Calculus I materials can differ from one another, so the number of categories and their descriptions can be longer, or divided into smaller parts i.e. making a separate category for infinity limits and limits in a point. Now that we are aware of what type of material should be covered during this course, we can start analyzing every category and make up a lot of questions related to the given part of the material. For legibility, all questions in each category are presented in the Table 1., and we limit task number to 2.

Table 1. Examples of questions for each category of a course Calculus I. All constant must be appropriate to the task.

Functions	Find the value of inverse function of
	$f(x) = a \log_b(cx + d)$ for $x = e$
	Solve $a + be^{1-cx} = d$
Limits	Determine $\lim_{x \to \infty} \left( \frac{ax+b}{cx+d} \right)^{ex+f}$
	Compute the value of the following limit
	$\lim_{x \to a} (bx^2 + cx - d)$
Derivatives	Using L'Hospital's Rule calculate
	$\lim_{x\to 0} \frac{a - \sqrt{a^2 - bx}}{cx}$
	Find the maximum of the function
	$f(x) = ax^3 + bx + c$ on the interval (d, e)
Integrals	Evaluate the integral $\int_{b}^{a} cx^{2} \sin(d + ex^{3})$
	Find the area bounded by $y = \frac{a}{x}$ , $y = bx$ , $x = c$

To create more varied tests in every category, we can add sub variations to every question. For the first category, which is called "Functions", there is a possibility to change the type of a function. Instead of using the logarithmic ones, we can use exponential or rational functions, which make our questions more diverse. Similarly to the category about derivatives, we can easily change function to different one, which can be calculated using L'Hospital's Rule. These changes in an easy way enlarge the number of variations in every main category, which leads to creating more alternatives and reducing the probability of repetition for every student.

In the analyzed Table 1., we have used some constants a, b, c, d... and no numbers. When a student opens a test and is able to see a single question, instead of these constants, they will see numbers which will be generated using Moodle LMS. It spawns a lot of similar questions in terms of the way

of solving them, but they differ from each other by a constant.

Finally, we obtain a test in which every participant has at least one question from a given category. Since the number of questions in the singular category is higher than the number of questions chosen for the test from the same material, this way of checking students' knowledge makes almost every quiz more reliable. The previously mentioned feature of the Moodle system (generating randomized variables) in questions lets us create numerous tests with various ways of examination of the same program content.

Presented exemplary tasks are implemented into the Moodle system. In this case, we can use the "Numerical" type of a single task, which means that every participant is forced to write an answer as a real number. This LMS system enables users to create multiple choice questions, in which the number of good answers can be equal or higher than 0. Obviously, the most desirable situation is to prepare tasks for individual calculations for students, and leave some space for them to write their own results of the solution, which prevents them from choosing accidental options.

Plugin Formulas to Moodle system adds an additional way of testing which allows to create a specified number of numeric fields. It broadens the possibility of creating tasks. Using this unusual feature lecturers can operate on more than one numeric field in a single question. Due to this upgrade, quizzes, which we can create, can be more advanced, and participants need to calculate more values. Lecturers can create sequential numeric fields in one question in which every next field is essential to calculate the following one. Instructor has a better understanding of a place where students make a mistake while solving a task. It may be helpful in the future because the lecturer can point out this type of problem in detail in later courses. Moreover, multiple numeric fields open opportunities to prepare more questions. As an example, we can consider operations on matrices. Now it is possible to create two matrices with the same size  $n \cdot m$  and ask students about  $n \cdot m$  values after some transformations, which is not possible using Moodle without Formulas plugin.

### 3.2. Algebra

Let us analyze the second example of such a way of creating tests. The key point of this test is to use Moodle Formulas's feature. In most of the questions we will use multiple numerical field text, which means that a student is obligated to calculate a few values. We propose the outline of an introduction to a linear system equation with algebra topics. Proposed in this paper outline is shortened and customized to our needs. The draft of the course Algebra is following:

- Complex numbers: arithmetic of complex numbers, modulus of a complex number, De Moivre's Theorem, roots of complex number, Euler's theorem,
- Matrices: transpose and trace of a matrix, addition and multiplication of matrices, calculating a determinant,
- System of equations: writing a system of linear equation in matrix form, solving by Cramer's rule, Gauss-Jordan elimination or using inverse matrix,
- Polynomials: dividing polynomials, roots of polynomials, fraction decomposition.

All topics were grouped in four categories from which a test can be created. It divides all potential questions into an applicable way. One more time, a lot of Algebra outlines courses can differ from one another, so the number of categories and questions inside them is flexible. Knowing the planned piece of material for this course, it is time to create a representation of questions. Again, for each category we write 2 examples of tasks, which can be easily implemented into the Moodle System. They are presented in the Table 2.

Table 2. Examples of questions for each category of a course Calculus I. All constant must be appropriate to the task.

Complex	Compute the absolute value of
numbers	$z = (a + bi)^c$
	Compute the cube root of
	z = a + bi
Matrices	Calculate the determinant of
	a $n \times m$ matrix
	Find inverse of a $n \times m$ matrix
System of	Use Cramer's rule to solve the system of
equations	equation
	Solve system of equations by Gauss-
	Jordan elimination
Polynomials	Find roots of $f(x) = ax^3 + bx^2 + cx$
	Determine the partial fraction
	decomposition $f(x) = \frac{ax+b}{cx^2+dx+e}$

We will analyze some questions from the presented Table 2. Let us consider the calculation of the determinant of a given matrix. In this task, the individual items in an quadratic matrix with 3 rows and 3 columns can be generated e.g. as an integer number from interval [-5,5] not to receive difficult calculations. It makes about  $11^9$  possibilities of matrices. We choose integers due to the simplicity of multiplying and adding. Calculating the determinant of a matrix returns only one real value, which means in this type of questions there is only one numeric field to fill.

Let us focus on a type of question from the same type of category which is multiplying two matrices. We know that the number of columns in the first matrix must be equal to the number of rows in the second matrix. As a result, the final matrix has the number of rows of the first and the number of columns of the second matrix. In such a task, an instructor can implement in a Moodle system with Formulas plugin defined number of numeric fields to fill e.g. for final matrix  $2 \times 3$  we need to set 6 independent fields. This case is one of applications of Moodle Formulas' multiple numeric fields. It demands participants to calculate and fill more than one value.

Analyzing possible questions in every category, we can apply more than one numeric field to the following question: operating on complex numbers, computing roots of complex numbers, finding inverse matrices, determining values from a system of equations, finding roots of polynomial, or applying partial fraction decomposition. The numerous tasks with an opportunity to use Moodle Formulas show that this plugin is beneficial. Requirements of calculation e.g. every variable from a system of equations reminds of the usual examination in which participants are obligated to solve such a system.

### 4. CONCLUSIONS

Creating tests is an inseparable part of instructors' work. It lets them evaluate the level of knowledge conducted

by participants of a given course. To give a proper mark based on tests, they need to be well-prepared and, what is most important, reliable and accurate. We considered the way of preparation by instructors to be able to construct an efficient way of checking knowledge. The process of test generating contains the analysis of topics included, the choice of the way of conducting the test, the selection of a representative group of questions, and finally, creation of a test based on previous consideration. Sticking to the presented rules allows us to build a good tool to evaluate students' knowledge correctly. It is essential to analyze students' correct and wrong answers to be aware of what kind of questions are the most difficult. Due to the detailed investigation, instructors notice what topics should be discussed more precisely in the future, which enlarges the level of teaching.

In the Moodle system, lecturers can implement a prepared representative group of questions, which helps them to conduct an exam. Dividing every type of question in a specified category and then randomly choosing a finite number of questions. What is more, most of the questions have some variations due to the possibility of choosing randomized variables to tasks. It builds a lot of similar questions with a low probability of repetition, but the scheme of solutions is similar to one another. It enables LMS to set this system to evaluate the filled numeric values automatically. As a result, we can create self-sufficient tests, with a big number of variations inside them.

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# TWORZENIE TESTÓW SKŁADAJĄCYCH SIĘ Z GRUPOWANIA PYTAŃ I WYKORZYSTANIA WARIANCJI ZMIENNYCH

W artykule zaprezentowano proces konstruowania testów nauczycielskich, które służą do oceny postępów uczestników kursu. Przedstawiono definicje, sposoby konstruowania testów oraz najważniejsze ich cechy. Opisano metodę planowania egzaminu niestandaryzowanego, która polega na podzieleniu pytań na kategorie i znalezieniu wartości, które mogą zostać zmienione przy każdym kolejnym losowaniu pytania przez zdającego. Na metodę tworzenia składa się wybranie z każdej kategorii określonej liczby pytań i wstawieniu wygenerowanych wartości liczbowych Na podstawie dwóch przykładów przedyskutowano podejście do planowania, przygotowania pytań oraz finalne ułożenie testu. Wszystkie pytania zostały zaimplementowane do platformy Moodle, która ułatwia tworzenie, zarządzanie i automatyzowanie testów, oraz której najważniejsze funkcje zostały opisane w artykule. Pracując z rozszerzeniem Moodle Formulas zaproponowano sposoby tworzenia trudniejszych zadań z większą liczbą pól numerycznych do uzupełnienia. Opisano również, jak za pomocą platformy Moodle możliwe było stworzenie nawet kilkuset modeli testów z pytaniami z różnych kategorii oraz ze zmiennymi w każdym zadaniu.

Słowa kluczowe: planowaniu testów, konstruowanie testów, Moodle Formulas, system zarządzania nauczaniem.