

*matching pursuit, bearing faults, energy error*

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## **MATCHING PURSUIT ALGORITHM IN ASSESSING THE STATE OF ROLLING BEARINGS**

### **Abstract**

*In this paper the results of Matching Pursuit (MP) Octave algorithm applied to noise, vibration and harness (NVH) diagnosis of rolling bearings are presented. For this purpose two bearings in different condition state were examined. The object of the analysis was to calculate and present which energy error values of MP algorithm give the most accuracy results for different changes in bearing structures and also how energy values spread in time-frequency domain for chosen energy error value.*

### **1. INTRODUCTION**

With the development of industry and the entry of a vast number of everyday devices containing from a few to dozens of moving parts, the need for production and installation of bearings significantly increased. Although the technology of bearings is improving from year to year, it is not possible to produce a reliable bearing, not liable to damage in an infinitely long time. The operation of the bearing greatly reduces its operating life. To avoid the costs generated by the device closed to traffic due to damage to the bearings began to use a wide range of diagnostic tools enable ongoing assessment of its condition. By using diagnostic methods it can be early detection of bearing damage and its replacement at a convenient time for the user (Cempel, 1989).

One of the most commonly used methods of diagnosis of bearings is the study of NVH (Nguyen-Schafer, 2016). Vibro-acoustic measurements are based

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on the study of dynamic noise generated by operating machines or their components. With the development of computer analysis vibro-acoustic techniques became much more accurate and the time required for a complete analysis of a single item decreased significantly. In the case of a diagnosis of bearings, studied interference dynamic vibration components are working bearings. The main reason for using this method for diagnostics of the bearing is its non-invasive examination of their condition and the possibility of accurate detection of the type of damage occurring.

There are many methods of signal analysis which can be implemented to estimate the working bearing condition. For the most precise analysis of bearing state Time-frequency signal analysis method will be the best due to fact that this kind of analysis shows both time and the frequency domain. As vibration signals are non-stationary, the Time-frequency analysis features are very useful tools to describe processes that occur in the bearing (Gao & Yan, 2011). For many years to monitor machinery condition people have been using Windowed Fourier Transform (WFT) and Short Time Fourier Transform (STFT), but those algorithms show weak performance. To improve time resolution of STFT analysis the Continuous Wavelet Transform (CWT) was developed (Chandra & Sekhar 2016; Yan, Gao & Chen, 2014). The main disadvantage of CWT is that the algorithm needs a lot of time to be implemented. Matching Pursuit (MP) is the next step in algorithm quality improvement (Chandran et al., 2016). The main difference between MP and CWT is that MP uses a package of wavelets, called wavelet dictionaries compared to a single wave form in CWT, which allows to perform better analysis of vibration signals. Thanks to using wavelet dictionaries, the results of Time-frequency analysis show the best time resolution from among the rest of Time-frequency algorithms. For many recent years Matching Pursuit algorithm has frequently been used for fault detection in rolling bearings (Liu, Ling & Gribonval, 2002). In that time many of MP features were upgraded or changed (Tang et al., 2012). Some of them proposed to use impulse dictionaries (Cui, Wang & Lee, 2014), which provide higher efficiency and better stability. In a paper by He (2016), the dictionary applied for diagnostics was over-completed, which gives a low signal to noise ratio values for large rolling elements sliding. In many other papers (Cui, Gong, Zhang & Wang, 2016) changes involving the MP algorithm were focused on dictionary (Cui et al., 2016).

The main goal of this paper is an implementation of an MP algorithm based on an Octave dictionary (Kuś, Róžański & Durka, 2013) for bearings fault diagnosis. The main differences between traditional dictionaries and the Octave dictionary is an ability to control maximum error in every single MP iteration. With the ability to use error control there is a possibility to choose the appropriate size of dictionary redundancy when the energy error value is being decreased, the size of the dictionary increases. Constant MP energy value error will not have any impact on atoms' location, so the energy density distribution

will not be incorrect in different signal parts. Thanks to an appropriate selection of the dictionary size, MP decomposition will take less time and will be more accurate, which will have significant impact on diagnostic results.

## 2. MATCHING PURSUIT ANALYSIS

MP is algorithm based on an iterative decomposition of a signal by waveforms called atoms ( $g_n$ ) (Mallat & Zhang, 1993). Atoms in the Time-frequency domain can be obtained from modulation, translation and scaling of a single window function:

$$g_\gamma(t) = \frac{1}{\sqrt{s}} g\left(\frac{t-u}{s}\right) e^{\xi\omega t} \quad (1)$$

where:  $u$  – translation,  
 $s$  – duration,  
 $\omega$  – modulation,  
 $\xi$  – frequency,  
 $\gamma$  – set of parameters  $\{u, \omega, s, \phi\}$ .

Atom functions are usually modulated Gaussians so they can be called “Gabor Atoms” (GA) and can be defined:

$$g_\gamma(t) = K(\gamma, \phi) e^{-\pi\left(\frac{t-u}{s}\right)^2} \sin(\omega(t-u) + \phi) \quad (2)$$

where:  $K(\gamma, \phi)$  – is normalization of  $\|g_{\gamma, \phi}\| = 1$ ,

A large number of GA creates a dictionary of waveforms described as  $D = \{g_1, g_2, \dots, g_n\}$  where  $\|g_i\| = 1$ , which allows to start the iteration procedure. The first iteration step chose atom  $g_{\gamma_0}$  which values describe signal parameters accurately (Durka, Ircha & Blinowska, 2001). In the next steps other atoms will be matched to signal residuum,  $R^n s$  which will remain after the values of the previous iteration will be cut off.

$$\begin{cases} R^0 s = s; \\ R^n s = \langle R^n s, g_{\gamma_n} \rangle g_{\gamma_n} + R^{n+1}; \\ g_{\gamma_n} = \arg \max_{g_{\gamma_i} \in D} |\langle R^n s, g_{\gamma_i} \rangle| \end{cases} \quad (3)$$

As mentioned in the introduction, Octave dictionary has a unique option of energy error control, which can be changed at the beginning of the dictionary construction task.

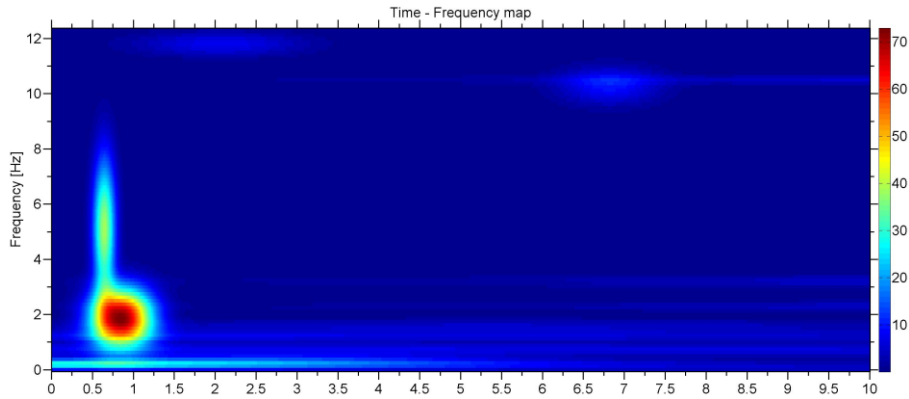
Energy error ( $\varepsilon$ ) will correspond to (Kuś, Róžański & Durka, 2013):

$$\begin{aligned} d(g_{(s,\omega,u,\phi)}, g_{(as,\omega,u,\phi)}) &\leq \varepsilon \\ d(g_{(s,\omega,u,\phi)}, g_{(as,\omega+\Delta\omega,u,\phi)}) &\leq \varepsilon \\ d(g_{(s,\omega,u,\phi)}, g_{(as,\omega,u+\Delta u,\phi)}) &\leq \varepsilon \end{aligned} \quad (4)$$

The final result of MP is Time-frequency spectrogram (Fig.1). The spectrogram shows signal energy distribution in the Time-frequency domain. Signal energy was obtained by the use of Wigner distribution for one  $g_\gamma$  in combination with conservation of MP energy:

$$\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} Es(t, \omega) dt d\omega = \|s\|^2 \quad (5)$$

where:  $Es(t, \omega)$  – translation,  
s – Signal energy density,



**Fig. 1. Time-frequency spectrogram with energy density distribution – atom placed in bottom left corner represents biggest concentration of energy (red color)**

### 3. EXPERIMENT

Researches were conducted on a test stand (Fig. 2) equipped with AC motor, jaw coupling and a shaft with two bearings and one of them was the test bearing. In this setup a magnetic break placed at the end of the shaft with breaking torque 1-10Nm was a load. An accelerometer, Brüel&Kjaer nr.4394, is a miniature device created for shock and vibration measurement with frequency range 1-25000Hz. The accelerometer was placed on the bearing case and connected to a signal amplifier PCB Model 482A20. A National Instruments PXI-1044 computer equipped with NI PXI-4472B data acquisition card was used to convert analogue signals to digital signals. For data acquisition LabVIEW was used. Sampling frequency was set at 20 KHz. Signal analyses were done in Matlab.

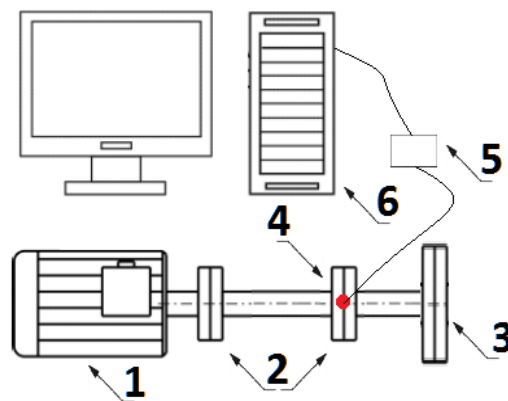
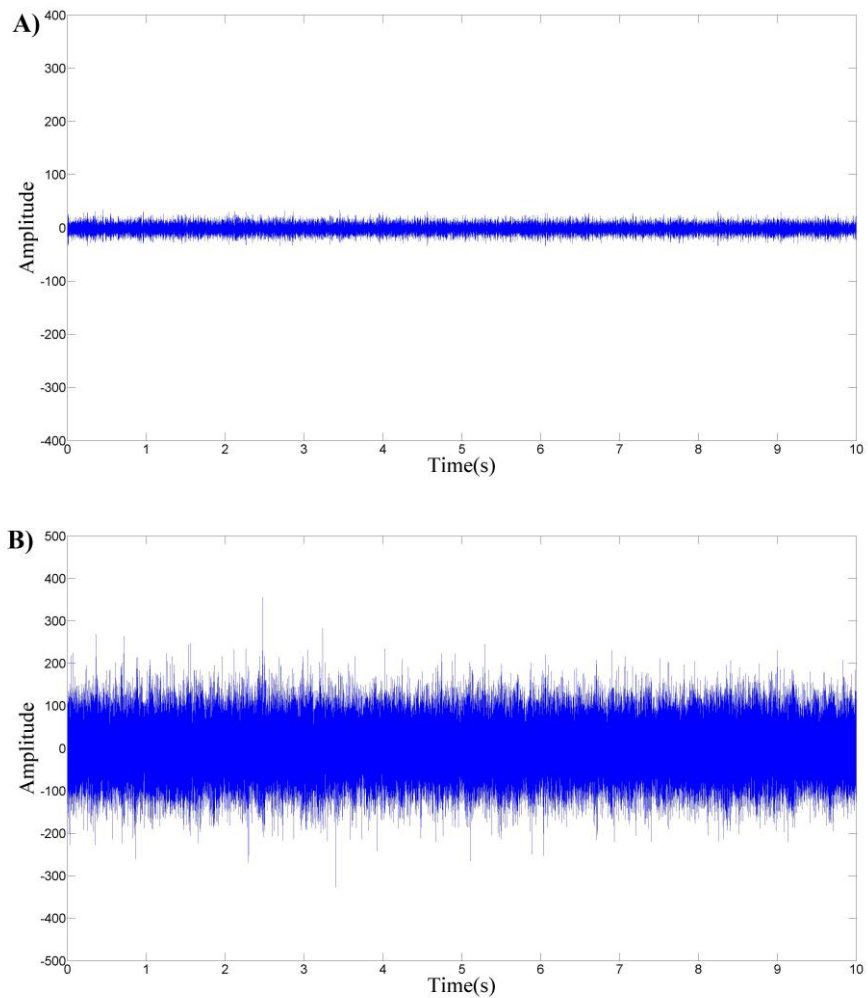


Fig. 2. Test Stand: 1 – AC Motor, 2 – Bearings, 3 – Magnetic break, 4 – Accelerometer, 5 – Amplifier, 6 – Computer

### 4. RESULTS

In order to verify the usefulness of MP algorithm based on Octave dictionary in rolling bearing fault detection, two bearings in different wear conditions were tested. As presented in Fig.2 bearings worked in pairs, to avoid propagation of oscillations from the damaged bearing to the healthy one, after one measurement cycle the damaged bearing that worked under an accelerometer was switched with the health one. Time duration of one measurement cycle was set into 10s. During the whole cycle velocity was constant at 100Hz. The next set of data collected from the experiment was analyzed by an MP algorithm with Octave dictionary. To show energy distributions in signals precisely the number of iterations was set into 20 for all cases.

Raw signals acquired from the experiment are presented as (Fig. 3) time-amplitude plots. The first graph (A) presents the signal registered for a bearing in normal condition state. The second graph (B) presents the recording for a bearing with a rolling element fault (REF).



**Fig. 3. Raw signal of bearing vibrations under conditions:  
A) Health condition B) Rolling element fault**

Analyzing amplitude values on both plots it can be seen that a bearing in health state (A) has lower amplitude values in comparison to amplitude values from a damaged bearing (B).

After gathering the data was processed by an MP algorithm. The first step of the MP analysis was to select 25 appropriate signal episodes for future decomposition. Due to long calculation time for small energy error values and to illustrate differences between single reconstruction function, the analyzed time was set up for 0.16s. Octave dictionary implemented to analysis was constructed separately for three energy error values 0.1, 0.01 and 0.001. Averaged Matching Pursuit coefficients from 25 segments were placed in Tab.1 for the health state bearing and in Tab.2 for bearing with REF. Values are presented according to iterations “I” and energy error values “ER”. The number of Gabor atoms for each error value are presented in the “A” row. Energy values are placed in “E” rows separately for different energy error values (E1, E2, E3). Individual differences (in percent) between energy values for different energy errors were placed in C1 as E1-E2 difference and C2 as E2-E3 difference.

**Tab. 1. Matching Pursuit coefficients for HC bearing.**  
**ER** – energy error value; **A** – Number of functions in dictionary (in millions); **I** – iteration; **E(1,2,3)** – energy value for single Gabor Atom; **C1** – percentage difference between E1 and E2; **C2** – percentage difference between E2 and E3 [source: own study]

ER	0.1	0.01	0.001		
A	114	4,008	128,009	C1	C2
I	E1	E2	E3	%	%
0	196	220,1	220,7	12,29	0,27
1	139	141,4	141,5	1,70	0,10
2	124	123,5	123,1	0,44	0,27
3	115	118,1	118,1	2,68	0,04
4	114	107,1	106,9	6,02	0,20
5	106	94,2	97,5	11,13	3,47
6	77	80,9	81,3	5,12	0,48
7	75	79,7	79,8	6,24	0,13
8	72	73,4	73,3	1,94	0,13
9	70	55,1	54,6	21,33	0,85

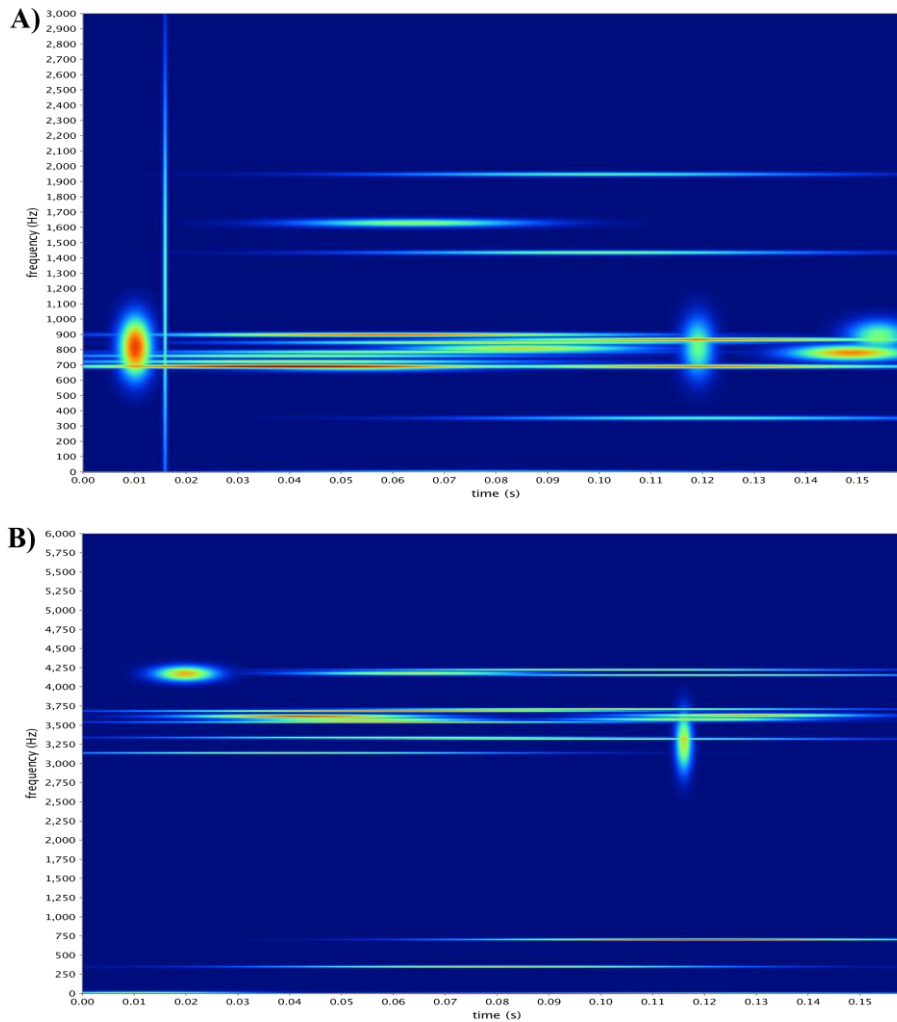
**Tab. 2. Matching Pursuit coefficients for REF bearing.**  
**ER** – energy error value; **A** – Number of functions in dictionary (in millions); **I** – iteration; **E(1,2,3)** – energy value for single Gabor Atom; **C1** – percentage difference between E1 and E2; **C2** – percentage difference between E2 and E3 [source: own study]

ER	0.1	0.01	0.001		
A	114	4,008	128,009	C1	C2
I	E1	E2	E3	%	%
0	694,96	781,24	783,57	12,42	0,30
1	518,66	531,14	532,08	2,41	0,18
2	511,43	513,97	514,21	0,50	0,05
3	488,49	495,91	497,08	1,52	0,24
4	340,33	382,31	382,91	12,33	0,16
5	339,90	369,38	369,99	8,67	0,17
6	334,33	362,01	362,46	8,28	0,12
7	334,23	349,63	350,28	4,61	0,19
8	324,58	326,87	327,71	0,70	0,26
9	314,03	321,18	321,54	2,28	0,11

Analyzing results presented in table one and two, it can be seen that the biggest differences in energy values are between E1 and E2 energy error values and even a different bearing state does not affect the results. According to the results it can be said that the highest accuracy with good calculation time (bigger dictionary size affects calculation time) has 0.01 energy error value. Energy values for the REF bearing are higher in every energy error case in comparison to HC which is normal for this bearing fault.

The last step of the analysis was the creation of an MP spectrogram to demonstrate the arrangement of Gabor atoms in the time-frequency system. Both spectrograms presented MP decomposition results for 0.01 energy error and correspond to results presented in tables 1 and 2.





**Fig. 4. Matching Pursuit results: A) Health condition B) Rolling ele. fault**

For the HC (Fig.4 A) bearing Gabor atoms were localized mostly at 600-1000Hz frequency band, through almost the whole time. The second spectrogram (B) contains energy values for REF bearing. In this case very high energy values were concentrated at 3K-4K Hz and some under the 1 kHz frequency band.

## 5. CONCLUSIONS

In this paper the results of the application of an MP algorithm with Octave dictionary for the diagnostics of the bearing condition are presented. Implementation of Octave dictionary provided an opportunity to choose an energy error value which is the most proper for the analyzed conditions. The results show that for bearing state diagnosis the best energy error value is 0.01 due to good energy value representation with decent Gabor atom quantity. Thanks to the balanced algorithm structure, energy values can easily show abnormalities contained in signals with good accuracy and decent calculation time. To show differences in frequency bands between the examined bearings, the spectrograms of signal energy density distribution had been created. As the spectrograms show, Gabor atoms' locations in time-frequency domain can be compared with characteristic estimates of different bearing faults. The coefficients acquired as the results of the MP algorithm show direct differences in energy level for each iteration according to bearing condition state. For future analysis regulation of energy error estimator values would be the best method for preposition the data to fed Machine Learning algorithms.

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