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SOLUTIONS OF BENJAMIN-BONA-MAHONY, MODIFIED CAMASSA-HOLM AND DEGASPERIS-PROCESI EQUATIONS USING AN ITERATIVE METHOD

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Abstract. In the present paper, we solve the non-linear Benjamin-Bona-Mahony, modified Camassa-Holm, and Degasperis-Procesi equations using an iterative method introduced by Daftardar-Gejji and Jafari. Results are compared with those obtained by other iterative methods such as the Adomian decomposition method and homotopy perturbation method. It is observed that the proposed method is computationally inexpensive and yields more accurate solutions than the Adomian decomposition method and the homotopy perturbation method.

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1. Introduction

Nonlinear partial differential equations are important due to their vital role in the modeling of natural phenomena. Several analytical and numerical methods such as Laplace transform, Sumudu transform, Riccati transformation [1], Adomian decomposition method (ADM) [2], homotopy perturbation method (HPM) [3], homotopy analysis method [4], Daftardar-Gejji and Jafari method (DGJM) [5], variational iteration algorithms [6], exponential-expansion algorithm [7], meshless techniques [8], modified (G'/G)-expansion method [9], generalized Kudryashov technique [10], sine-Gordon expansion method [11], Lie symmetry method [12], Hirota bilinear method [13], invariant subspace method [14], Darboux transformation method [15], rational sine-cosine method [16], spectrum function method [17], residual power series method [18], generalized Taylor power series method [19] and so on have been developed in the literature. In spite of so many methods, getting an exact solution that exists in real-life models is still awaited.

Each method has its advantages and limitations. Transform methods are suitable only for linear equations. However, iterative/decomposition methods do not involve discretization and are free from rounding errors. In particular, the ADM introduced by G. Adomian is heavily used by researchers as it gives a solution in terms of a rapidly convergent series. Despite that, computation of the Adomian polynomials is a tedious task. Similarly, in the HPM introduced by Ji-Huan He, the choice of the small parameter homotopy is an art rather than a solution procedure, and an appropriate choice of such a parameter leads to an inaccurate solution or even a wrong one. Whereas the main advantage of DGJM introduced by Daftardar-Gejji and Jafari is that it does not include any such tedious calculations as required in ADM, HPM, and other decomposition methods. Besides, DGJM is easily implementable using mathematical software such as Mathematica, Maple, Python, etc. Moreover, DGJM has been utilized for solving a variety of non-linear equations successfully as well as for developing several new hybrid methods [20]. For more details of DGJM, we refer the reader to a review article [21] and references cited therein. In the present paper, DGJM is used for finding the solutions of the non-linear Benjamin-Mahony equation [22], modified Camassa-Holm and Degasperis-Procesi equations [23].

The Benjamin-Bona-Mahony equation (BBME) [24] has also been known as the regularized long-wave equation. This equation was studied by Benjamin, Bona and Mahony [25] as an improvement of the Korteweg-de Vries equation (KdV equation) for modelling long surface gravity waves with small amplitude – propagating unidirectionally in (1+1) dimensions, and the general form of BBME in [26] is given as

$$u_t - \delta u_{xtx} + \alpha u_x + \beta u u_x = 0, \tag{1}$$

where α, β and δ are constants with the nonlinear and dispersion coefficients $\beta \neq 0$ and $\delta > 0$. This equation has been solved by ADM and HPM in [24]. Further, we consider a family of third-order non-linear dispersive partial differential equations:

$$u_t - u_{xxt} + (b+1)u^2 u_x = b u_x u_{xx} + u u_{xxx}, \tag{2}$$

which is called b-equation [27], where b is a positive integer. Equation (2) is called the modified Camassa-Holm (mCH) equation for $b = 2$ and the modified Degasperis--Procesi (mDP) equation for $b = 3$. Both equations arise in the modeling of the propagation of shallow water waves over a flat bed [28]. The solitary wave solutions of (2) are defined as follows [29]

$$u(x,t) = -2\operatorname{sech}^2\left(\frac{x-t}{2}\right), \tag{3}$$

$$u(x,t) = -\frac{15}{8}\operatorname{sech}^2\left(\frac{x}{2} - \frac{5t}{4}\right). \tag{4}$$

In this work, we solve BBM (1), mCH, and mDP (2) equations using DGJM and compare the results with ADM and HPM. The obtained solutions are in agreement with exact solutions and more accurate as compared to ADM and HPM. Mathematica 10.0 has been used for plotting the graphs.

The organization of this paper is as follows: In Section 2, we give the basic idea of the proposed method and also prove its convergence. In Section 3, the proposed method is applied for getting the numerical solutions of BBME, mCH, and mDP equations. Further, the obtained results are compared with the HPM and the ADM solutions. Conclusions are summarized in Section 4.

2. Daftardar-Gejji and Jafari Method (DGJM)

The basic idea of DGJM is described below. We consider the following general functional equation

$$u(x,t) = f(x) + N(u(x,t)), \tag{5}$$

where f is a given function and N a given non-linear operator from a Banach space $B \to B$. In DGJM a solution $u(x,t)$ of equation (5) is expressed in terms of the following infinite series:

$$u(x,t) = \sum_{i=0}^{\infty} u_i. \tag{6}$$

The non-linear operator N can be decomposed as

$$N\left(\sum_{i=0}^{\infty} u_i\right) = N(u_0) + \sum_{i=1}^{\infty}\left(N\left(\sum_{j=0}^{i} u_j\right) - N\left(\sum_{j=0}^{i-1} u_j\right)\right). \tag{7}$$

Using the above equations (6) and (7), in equation (5)

$$\sum_{i=0}^{\infty} u_i = f + N(u_0) + \sum_{i=1}^{\infty}\left(N\left(\sum_{j=0}^{i} u_j\right) - N\left(\sum_{j=0}^{i-1} u_j\right)\right). \tag{8}$$

We define the recurrence relation in the following way:

$$\begin{aligned} u_0 &= f, \\ u_1 &= N(u_0), \\ u_2 &= N(u_0+u_1) - N(u_0), \\ u_{n+1} &= N(u_0+u_1+\ldots+u_n) - N(u_0+u_1+\ldots+u_{n-1}),\ n=1,2,\ldots \end{aligned} \tag{9}$$

Then,

$$(u_1+u_2+\ldots+u_{n+1}) = N(u_0+u_1+\ldots+u_n),\ n=1,2,\ldots \tag{10}$$

and

$$\sum_{i=0}^{\infty} u_i = f + N\left(\sum_{j=0}^{\infty} u_j\right). \tag{11}$$

The m-term approximate solution of equation (5) is given by $u(x,t) = u_0 + u_1 + \ldots + u_{m-1}$. For the convergence of this method, we prove the following theorem.

Theorem 1 *Let* $u(x,t) = \sum_{i=0}^{\infty} u_i(x,t)$ *be a function of two variables x and t, which is defined in the Banach space* $(C[0,1], \|.\|)$, *such that the nonlinear operator* $N(u)$ *in* (5) *is contraction i.e.* $\|N(v) - N(w)\| \leq \lambda \|v - w\|$, *where* $0 < \lambda < 1$. *Then* $\|u_{n+1}\| \leq \lambda^{n+1}\|u_0\|, \forall n \in \{0\} \cup \mathbb{N}$. □

Proof: It is clear that $u_0 = f, \|u_1\| = \|N(u_0)\| \leq \lambda \|u_0\|$,

$$\begin{aligned}
\|u_2\| &= \|N(u_0 + u_1) - N(u_0)\| \leq \lambda \|u_1\| \leq \lambda^2 \|u_0\|, \\
\|u_3\| &= \|N(u_0 + u_1 + u_2) - N(u_0 + u_1)\| \leq \lambda \|u_2\| \leq \lambda^3 \|u_0\|, \\
&\vdots \\
\|u_{n+1}\| &= \|N(u_0 + u_1 + \cdots + u_n) - N(u_0 + u_1 + \cdots + u_{n-1})\| \leq \lambda \|u_n\| \\
&\leq \lambda^{n+1}\|u_0\|, \; n = 0, 1, 2, \ldots,
\end{aligned} \tag{12}$$

which is the required result.

Remark: If u and $\bar{u}$ are the exact and perturbed solutions of (5) and $e = u - \bar{u}$ denotes the error. Then in view of (12), we have $\|e_{n+1}\| \leq \lambda^{n+1}\|e_0\|$. Further as $n \to \infty$, $e_{n+1} \to 0$, which proves the convergence of DGJM for solving (5). For more details on the convergence of DGJM, we refer to the reader [30].

3. Applications

In this section, we employ DGJM to find the analytic approximate solutions of equations (1) and (2). Besides, we compare the DGJM solutions with the existing results obtained by HPM and ADM. Here onwards we denote $I_t = \int_0^t ()dt$.

3.1. Benjamin-Bona-Mahony Equation

Consider the following simplest form of BBME [25] (for $\alpha = \beta = \delta = 1$ (1)).

$$u_t - u_{xtx} + u_x + uu_x = 0, \tag{13}$$

with initial condition

$$u(x,0) = \operatorname{sech}^2\left(\frac{x}{4}\right). \tag{14}$$

Equations (13)-(14) possess the solitary wave solution [31] of the following form

$$u(x,t) = \operatorname{sech}^2\left(\frac{t}{3} - \frac{x}{4}\right). \tag{15}$$

Integrating the equation (13) with respect to t and using the initial condition (15), we get

$$u(x,t) = \operatorname{sech}^2\left(\frac{x}{4}\right) + I_t\Big(u_{xtx} - u_x - uu_x\Big) = f + N(u), \tag{16}$$

where $f = \operatorname{sech}^2\left(\frac{x}{4}\right)$ and $N(u) = I_t\Big(u_{xtx} - u_x - uu_x\Big)$. In view of the recurrence relation (9), we get

$$\begin{aligned}
u_0 &= f = \operatorname{sech}^2\left(\frac{x}{4}\right), \\
u_1 &= N(u_0) = I_t\Big(u_{0xtx} - u_{0x} - u_0u_{0x}\Big) \\
&= t\left(\frac{1}{2}\operatorname{sech}^2\left[\frac{x}{4}\right]\tanh\left[\frac{x}{4}\right] + \frac{1}{2}\operatorname{sech}^2\left[\frac{x}{4}\right]\tanh\left[\frac{x}{4}\right]\right), \\
u_2 &= N(u_0+u_1) - N(u_0) \\
u_2 &= \frac{1}{2048}\operatorname{sech}^{11}\left(\frac{x}{4}\right)\Bigg[-\frac{1}{3}700t^3\sinh\left(\frac{x}{4}\right) - \frac{20}{3}t^3\sinh\left(\frac{3x}{4}\right) \\
&\quad + \frac{44}{3}t^3\sinh\left(\frac{5x}{4}\right) + \frac{4}{3}t^3\sinh\left(\frac{7x}{4}\right) - 516t^2\cosh\left(\frac{x}{4}\right) \\
&\quad - 90t^2\cosh\left(\frac{3x}{4}\right) + 74t^2\cosh\left(\frac{5x}{4}\right) + 19t^2\cosh\left(\frac{7x}{4}\right) \\
&\quad + t^2\cosh\left(\frac{9x}{4}\right) - 70t\sinh\left(\frac{x}{4}\right) - 40t\sinh\left(\frac{3x}{4}\right) \\
&\quad + 80t\sinh\left(\frac{5x}{4}\right) + 55t\sinh\left(\frac{7x}{4}\right) + 5t\sinh\left(\frac{9x}{4}\right)\Bigg] \\
&\quad - t\left(\frac{1}{2}\tanh\left(\frac{x}{4}\right)\operatorname{sech}^4\left(\frac{x}{4}\right) + \frac{1}{2}\tanh\left(\frac{x}{4}\right)\operatorname{sech}^2\left(\frac{x}{4}\right)\right).
\end{aligned}$$

Hence, the three-term solution of (13)-(14) in series form is given by

$$\begin{aligned}
u(x,t) &= \frac{1}{2048}\operatorname{sech}^{11}\left(\frac{x}{4}\right)\Bigg[-\frac{1}{3}700t^3\sinh\left(\frac{x}{4}\right) - \frac{20}{3}t^3\sinh\left(\frac{3x}{4}\right) \\
&\quad + \frac{44}{3}t^3\sinh\left(\frac{5x}{4}\right) + \frac{4}{3}t^3\sinh\left(\frac{7x}{4}\right) - 516t^2\cosh\left(\frac{x}{4}\right) \\
&\quad - 90t^2\cosh\left(\frac{3x}{4}\right) + 74t^2\cosh\left(\frac{5x}{4}\right) + 19t^2\cosh\left(\frac{7x}{4}\right)
\end{aligned}$$

$$+t^2\cosh\left(\frac{9x}{4}\right)-70t\sinh\left(\frac{x}{4}\right)-40t\sinh\left(\frac{3x}{4}\right)$$
$$+80t\sinh\left(\frac{5x}{4}\right)+55t\sinh\left(\frac{7x}{4}\right)+5t\sinh\left(\frac{9x}{4}\right)\Bigg]+\operatorname{sech}^2\left(\frac{x}{4}\right).$$

The exact and three-term DGJM solutions of the BBME (13)-(14) are depicted in Figures 1-3, where green and purple color represents the exact and DGJM solutions respectively. Further, for $x = 0.03, 0.04, 0.05$ and $t = 0.01, 0.02, ..., 0.05$; the three-term DGJM solutions are compared with four-term ADM and HPM [24] solutions in Tables 1-3. It is clear that DGJM gives more accurate results as compared to ADM and HPM.

Table 1. Comparison of absolute errors in the solutions of (13)-(14) ($x = 0.03$)

t	$\lvert u_{Exact} - u_{HPM}\rvert$	$\lvert u_{Exact} - u_{ADM}\rvert$	$\lvert u_{Exact} - u_{DGJM}\rvert$
0.01	2.26646×10^{-4}	2.26646×10^{-4}	4.04367×10^{-5}
0.02	6.03525×10^{-4}	6.03525×10^{-4}	1.08637×10^{-4}
0.03	1.13061×10^{-3}	1.13061×10^{-3}	2.04603×10^{-4}
0.04	1.80786×10^{-3}	1.80786×10^{-3}	3.28339×10^{-4}
0.05	2.63524×10^{-3}	2.63524×10^{-3}	4.79852×10^{-4}

Table 2. Comparison of absolute errors in the solutions of (13)-(14) ($x = 0.04$)

t	$\lvert u_{Exact} - u_{HPM}\rvert$	$\lvert u_{Exact} - u_{ADM}\rvert$	$\lvert u_{Exact} - u_{DGJM}\rvert$
0.01	2.77073×10^{-4}	2.77073×10^{-4}	4.92752×10^{-5}
0.02	7.04304×10^{-4}	7.04304×10^{-4}	1.26302×10^{-4}
0.03	1.28165×10^{-3}	1.28165×10^{-3}	2.31083×10^{-4}
0.04	2.00908×10^{-3}	2.00908×10^{-3}	3.63622×10^{-4}
0.05	2.00908×10^{-3}	2.88653×10^{-3}	5.23925×10^{-4}

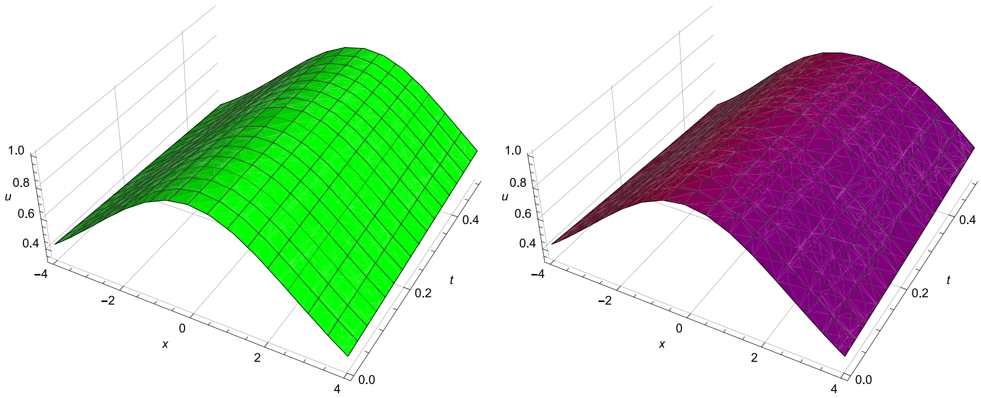
Table 3. Comparison of absolute errors in the solutions of (13)-(14) ($x = 0.05$)

t	$\lvert u_{Exact} - u_{HPM}\rvert$	$\lvert u_{Exact} - u_{ADM}\rvert$	$\lvert u_{Exact} - u_{DGJM}\rvert$
0.01	3.27453×10^{-4}	3.27453×10^{-4}	5.81056×10^{-5}
0.02	8.04969×10^{-4}	8.04969×10^{-4}	1.43948×10^{-4}
0.03	1.43250×10^{-3}	1.43250×10^{-3}	2.5753×10^{-4}
0.04	2.20999×10^{-3}	2.20999×10^{-3}	3.98853×10^{-4}
0.05	3.13739×10^{-3}	3.13739×10^{-3}	5.67926×10^{-4}

3.2. Modified Camassa-Holm equation

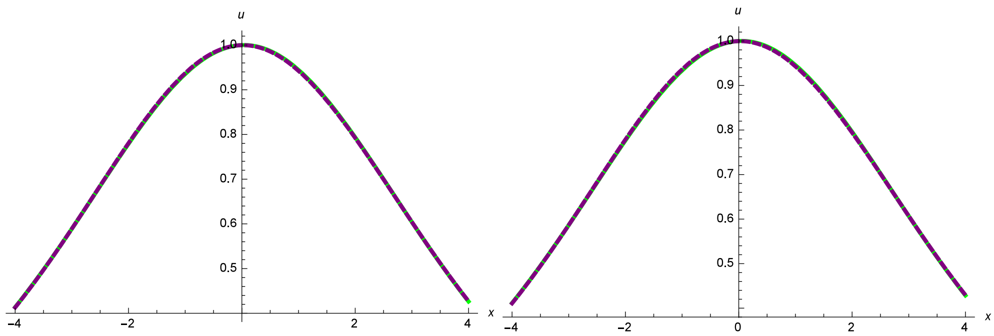
Consider the following modified Camassa-Holm equation

$$u_t - u_{xxt} + 3u^2u_x = 2u_xu_{xx} + uu_{xxx}, \tag{17}$$



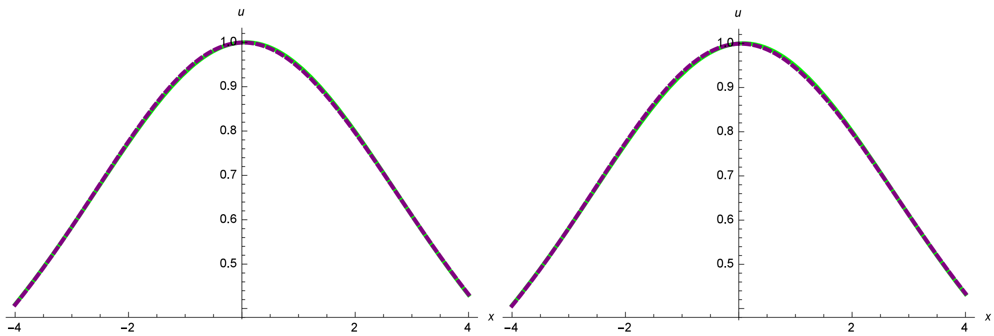
(a) (b)

Fig. 1. Exact (a) (green) and DGJM (b) (purple) solutions of BBME (13)-(14)



(a) (b)

Fig. 2. Exact and three-term DGJM solutions of (13)-(14) for: a) $t = 0.03$, b) $t = 0.04$



(a) (b)

Fig. 3. Exact and three-term DGJM solutions of (13)-(14) for: a) $t = 0.05$, b) $t = 0.06$

along with the initial condition

$$u(x,0) = -2\text{sech}^2\left(\frac{x}{2}\right). \tag{18}$$

Integrating the equation (17) with respect to t and using the initial condition (18), we get

$$u(x,t) = -2\text{sech}^2\left(\frac{x}{2}\right) + I_t\Big(u_{xxt} - 3u^2u_x + 2u_xu_{xx} + uu_{xxx}\Big) = f + N(u), \tag{19}$$

where $f = -2\text{sech}^2\left(\frac{x}{2}\right)$ and $N(u) = I_t\Big(u_{xxt} - 3u^2u_x + 2u_xu_{xx} + uu_{xxx}\Big)$. In view of the recurrence relation (9), we get

$$\begin{aligned} u_0 &= f = -2\text{sech}^2\left(\frac{x}{2}\right), \\ u_1 &= N(u_0) = I_t\Big(u_{0xxt} - 3u_0^2u_{0x} + 2u_{0x}u_{0xx} + u_0u_{0xxx}\Big) \\ &= -384t\sinh^6\left(\frac{x}{2}\right)\text{csch}^5(x). \end{aligned}$$

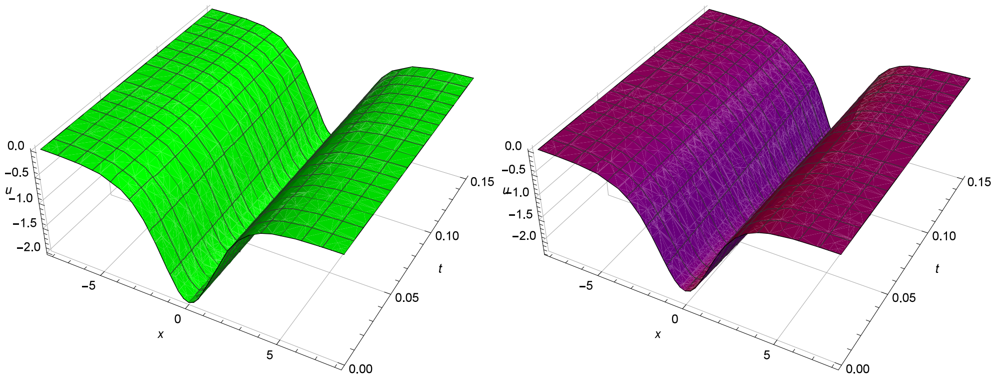
Hence, the two-term DGJM approximate solution of (17)-(18) is

$$\begin{aligned} u(x,t) &= u_0(x,t) + u_1(x,t) \\ &= -384t\sinh^6\left(\frac{x}{2}\right)\text{csch}^5(x) - 2\text{sech}^2\left(\frac{x}{2}\right). \end{aligned} \tag{20}$$

The exact (green color) and DGJM approximate (purple color) solutions of (17)-(18) are plotted in Figures 4-6. It is noted that DGJM solutions are in good agreement with the exact solutions. Further, we compare the absolute errors in the solutions of (17)-(18) obtained by DGJM, ADM [32] and HPM [33] in Table 4. It is observed that DGJM gives more accurate results as compared to ADM and HPM.

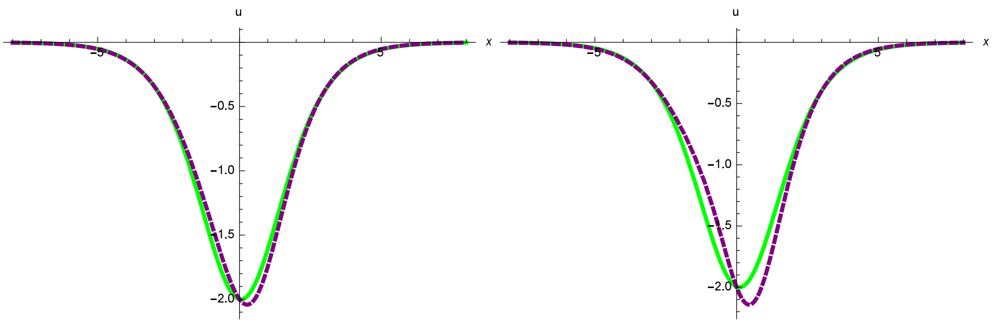
Table 4. Comparison of absolute errors in the solutions of (17)-(18)

(x,t)	$\lvert u_{Exact} - u_{ADM}\rvert$	$\lvert u_{Exact} - u_{HPM}\rvert$	$\lvert u_{Exact} - u_{DGJM}\rvert$
$-(8,0.05)$	2.8077×10^{-4}	2.80771×10^{-4}	1.36329×10^{-4}
$-(9,0.05)$	1.0363×10^{-4}	1.03633×10^{-4}	5.04471×10^{-5}
$-(10,0.05)$	3.816×10^{-5}	3.8170×10^{-5}	1.85984×10^{-5}
$-(8,0.10)$	5.9114×10^{-4}	5.91138×10^{-4}	2.79693×10^{-4}
$-(9,0.10)$	2.1817×10^{-4}	2.18174×10^{-4}	1.03487×10^{-4}
$-(10,0.10)$	8.015×10^{-5}	8.0357×10^{-5}	3.81512×10^{-5}
$-(8,0.15)$	9.3421×10^{-4}	9.34203×10^{-4}	4.30451×10^{-4}
$-(9,0.15)$	3.4477×10^{-4}	3.44769×10^{-4}	1.59252×10^{-4}
$-(10,0.15)$	1.2697×10^{-4}	1.26982×10^{-4}	5.87072×10^{-5}
$-(8,0.20)$	1.31340×10^{-3}	1.313390×10^{-3}	5.88981×10^{-4}
$-(9,0.20)$	4.8467×10^{-4}	4.84685×10^{-4}	2.17882×10^{-4}
$-(10,0.20)$	2.2074×10^{-4}	1.78511×10^{-4}	8.0318×10^{-5}



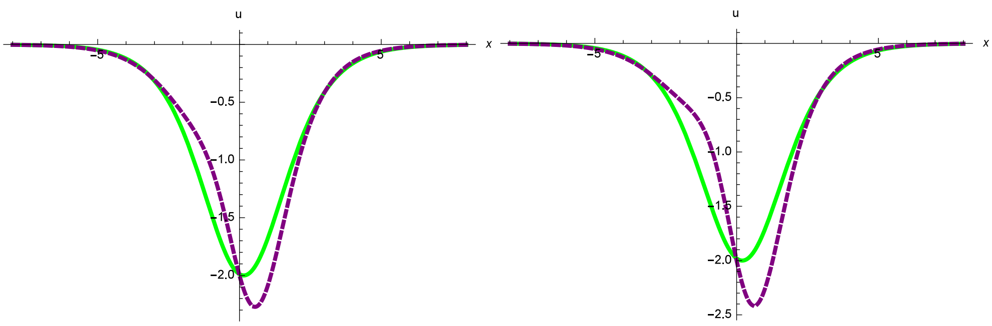
(a) (b)

Fig. 4. Exact (a) (green) and DGJM (b) (purple) solutions of (17)-(18)



(a) (b)

Fig. 5. Exact and DGJM solutions of (17)-(18) for: a) $t = 0.05$, b) $t = 0.10$



(a) (b)

Fig. 6. Exact and DGJM solutions of (17)-(18) for: a) $t = 0.15$, b) $t = 0.20$

3.3. Modified Degasperis-Procesi equation

Consider the following modified Degasperis-Procesi equation

$$u_t - u_{xxt} + 4u^2 u_x = 3u_x u_{xx} + u u_{xxx}, \tag{21}$$

with the initial condition

$$u(x,0) = -\frac{15}{8}\operatorname{sech}^2\left(\frac{x}{2}\right). \tag{22}$$

Equation (21) is equivalent to the following integral equation

$$u(x,t) = u(x,0) + I_t\Big(u_{xxt} - 4u^2 u_x + 3u_x u_{xx} + u u_{xxx}\Big) = f + N(u), \tag{23}$$

where $f = u(x,0)$ and $N(u) = I_t\Big(u_{xxt} - 4u^2 u_x + 3u_x u_{xx} + u u_{xxx}\Big)$.

In view of the recurrence relations (9), we get

$$\begin{aligned} u_0 &= f = -\frac{15}{8}\operatorname{sech}^2\left(\frac{x}{2}\right), \\ u_1 &= N(u_0) = I_t\Big(u_{0xxt} - 4u_0^2 u_{0x} + 3u_{0x} u_{0xx} + u_0 u_{0xxx}\Big) \\ &= -450t\sinh^6\left(\frac{x}{2}\right)\operatorname{csch}^5(x). \end{aligned}$$

Therefore, the two-term solution of (21) is

$$\begin{aligned} u(x,t) &= u_0 + u_1 \\ &= -450t\sinh^6\left(\frac{x}{2}\right)\operatorname{csch}^5(x) - \frac{15}{8}\operatorname{sech}^2\left(\frac{x}{2}\right). \end{aligned} \tag{24}$$

The exact (green color) and two-term DGJM (purple color) solutions of (21)-(22) are plotted in Figures 7-9. Further, absolute errors obtained by HPM [33], ADM [32] and DGJM in the solutions of (21)-(22) are compared in Table 5. It is noticed that DGJM provides more accurate results as compared to ADM and HPM.

Table 5. Comparison of absolute errors for the equations (21)-(22)

(x,t)	$\lvert u_{Exact} - u_{ADM}\rvert$	$\lvert u_{Exact} - u_{HPM}\rvert$	$\lvert u_{Exact} - u_{DGJM}\rvert$
$-(8,0.05)$	3.3326×10^{-4}	3.33255×10^{-4}	3.33255×10^{-4}
$-(9,0.05)$	1.2298×10^{-4}	1.23003×10^{-4}	1.23003×10^{-4}
$-(10,0.05)$	4.521×10^{-5}	4.5305×10^{-5}	4.5305×10^{-5}
$-(8,0.10)$	7.1089×10^{-4}	7.10978×10^{-4}	7.10978×10^{-4}
$-(9,0.10)$	2.6231×10^{-4}	2.62396×10^{-4}	2.62396×10^{-4}
$-(10,0.10)$	9.659×10^{-5}	9.6644×10^{-5}	9.66441×10^{-5}
$-(8,0.15)$	1.13903×10^{-3}	1.139070×10^{-3}	1.13907×10^{-3}
$-(9,0.15)$	4.2034×10^{-4}	4.20359×10^{-4}	4.20359×10^{-4}
$-(10,0.15)$	1.5476×10^{-4}	1.54820×10^{-4}	1.5482×10^{-4}
$-(8,0.20)$	1.62416×10^{-3}	1.624210×10^{-3}	1.62421×10^{-3}
$-(9,0.20)$	5.9927×10^{-4}	5.99362×10^{-4}	5.99362×10^{-4}
$-(10,0.20)$	2.2074×10^{-4}	2.20743×10^{-4}	2.20743×10^{-5}

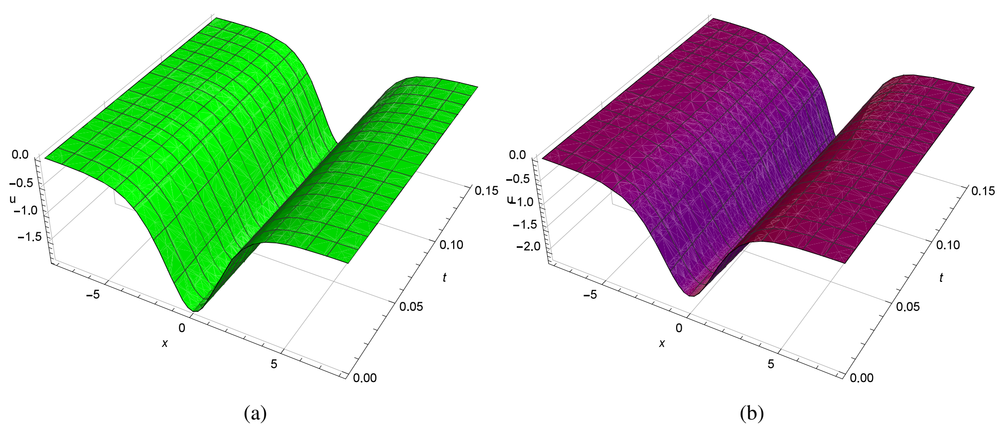


Fig. 7. Exact (a) (green) and DGJM (b) (purple) solutions of (21)-(22)

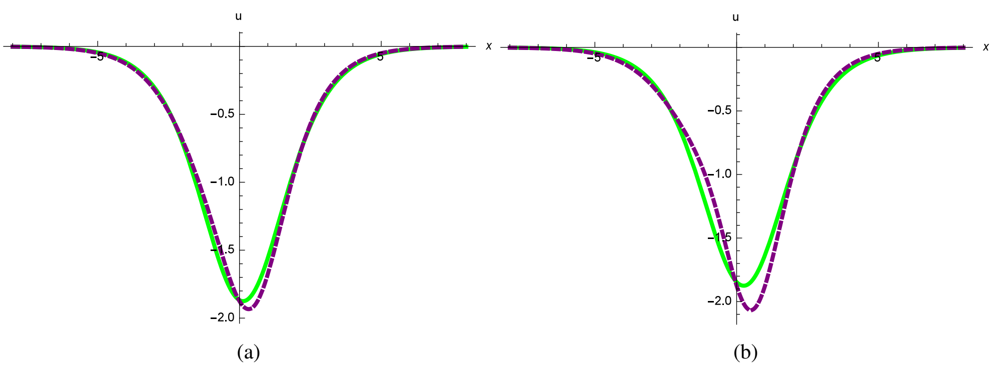
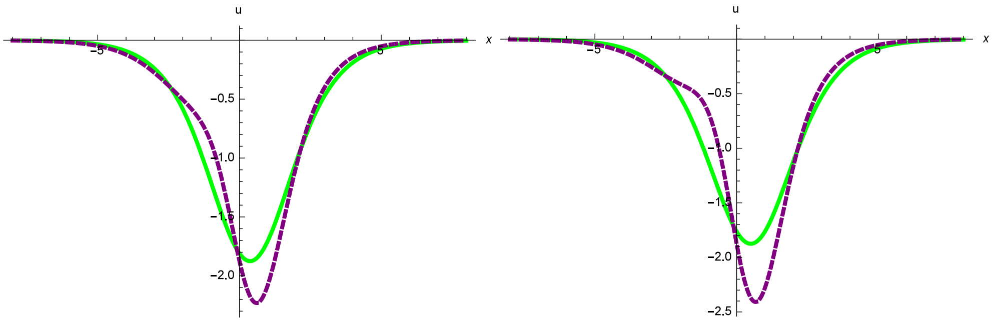


Fig. 8. Exact and DGJM solutions of (21)-(22) for: a) $t = 0.05$, b) $t = 0.10$



(a) (b)

Fig. 9. Exact and DGJM solutions of (21)-(22) for: a) $t = 0.15$, b) $t = 0.20$

4. Conclusion

In this paper, a reliable and efficient method introduced by Daftardar-Gejji and Jafari has been successfully employed to solve the Benjamin-Bona-Mahony equation, modified Camassa-Holm, and modified Degasperis-Procesi equations. Further, the DGJM solutions are compared with ADM and HPM numerically as well as graphically. We observed that DGJM agrees well with exact solutions and provides more accurate results than ADM and HPM. Besides, DGJM is simple in its principle and easily employable using a computer algebra system. Hence, the proposed method is suitable for solving these types of models.

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