Computer Aided Methods for Stability Analysis of 2D Linear Systems Described by the First Fornasini-Marchesini Model

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Andrzej Ruszewski, Mikołaj Busłowicz

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Abstract:
Computer aided methods for investigation of the asymptotic stability of 2D discrete linear systems described by the first Fornasini-Marchesini model are given. The methods require computation of eigenvalues of complex matrices or values of complex functions. Effectiveness of the stability tests are demonstrated on numerical examples.

Keywords: *2D system, linear, discrete, stability, computational method*

1. Introduction

There are several models of 2D discrete linear system [9, 11, 12]. The most popular is the Fornasini-Marchesini model introduced in [9].

The problem of asymptotic stability of linear 2D systems has considerable attention since about 40 years. For the stability analysis of these systems various methods can be applied: analytical (similar to the Schur stability test of 1D systems) [1], based on Lyapunov stability theory [21, 22], based on LMI [8, 13, 23, 24], based on spectral radius [10, 17, 20, 25, 26], frequency domain methods [23] or algebraic methods for positive systems [12, 13, 14, 15, 16, 19]. The analytical methods require symbolic computations whereas the methods based on Lyapunov stability theory, LMI or spectral radius give only sufficient but not necessary conditions for stability of standard systems.

The main purpose of this paper is to present new frequency domain necessary and sufficient conditions for investigation of asymptotic stability of the first Fornasini-Marchesini model of 2D standard linear systems.

The following notation will be used: Z_+- the set of non-negative integers; $\Re^{n\times m}$ - the set of $n\times m$ real matrices; $\lambda_i\{X\} - i$-th eigenvalue of X.

2. Problem Formulation

Consider the state equation of the first Fornasini-Marchesini model of 2D linear system [9, 11, 12]

$$\begin{aligned} x(i+1,j+1) &= A_0x(i,j)+A_1x(i+1,j) \\ &+A_2x(i,j+1)+Bu(i,j), \quad i,j\in Z_+, \end{aligned} \tag{1}$$

where $x(i,j)\in\Re^n$, $u(i,j)\in\Re^m$ and $A_0,\ A_1,\ A_2\in\Re^{n\times n}$, $B\in\Re^{n\times m}$.

The boundary conditions for (1) are as follows

$$x(i,0)=x_{i0},\quad x(0,j)=x_{0j},\quad i,j\in Z_+. \tag{2}$$

The characteristic matrix of the model (1) has the form

$$H(z_1,z_2)=z_1z_2I_n-A_0-z_1A_1-z_2A_2, \tag{3}$$

where z_1 and z_2 are complex variables.

The characteristic function

$$\begin{aligned} w(z_1,z_2) &= \det H(z_1,z_2) \\ &= \det[z_1z_2I_n-A_0-z_1A_1-z_2A_2] \end{aligned} \tag{4}$$

of the model (1) is a polynomial in two independent variables z_1 and z_2, of the form

$$w(z_1,z_2)=\sum_{k=0}^{n}\sum_{l=0}^{n}a_{kl}z_1^kz_2^l,\quad a_{nn}=1. \tag{5}$$

The model (1) is called asymptotically stable (Schur stable) if for $u(i,j)\equiv 0$ and bounded boundary conditions (2) the condition $x(i,j)\to 0$ holds for $i,j\to\infty$.

From [1, 11] we have the following theorem.

Theorem 1. The model (1) is asymptotically stable if and only if

$$w(z_1,z_2)\neq 0,\quad \forall|z_1|\geq 1 \quad and \quad \forall|z_2|\geq 1. \tag{6}$$

The polynomial (5) satisfying the condition (6) is called discrete stable or Schur stable. Several algebraic methods for asymptotic stability checking of such bivariate polynomials were given in [1].

Computational method for investigation of asymptotic stability of the Fornasini-Marchesini model (1) has been given in [2]. This method requires computation of eigenvalue-loci of complex matrices.

The main purpose of this paper is to present new computational methods for checking the condition (6) of asymptotic stability of the model (1) which do not require a priori knowledge of the characteristic bivariate polynomial (5).

3. Solution of the Problem

Theorem 2. The model (1) is asymptotically stable if and only if the following two conditions hold:

$$w(e^{jy},z_2)\neq 0,\ |z_2|\geq 1,\ \forall y\in[0,\ 2\pi],\ j^2=-1, \tag{7}$$

$$w(z_1,e^{j\omega})\neq 0,\quad |z_1|\geq 1,\quad \forall\omega\in[0,\ 2\pi]. \tag{8}$$

Proof. From [1, 2] it follows that (6) is equivalent to the conditions

$$w(z_1,z_2)\neq 0,\quad |z_1|=1,\quad |z_2|\geq 1, \tag{9}$$

$$w(z_1, z_2) \neq 0, \quad |z_1| \geq 1, \quad |z_2| = 1. \tag{10}$$

It is easy to see that conditions (9) and (10) can be written in the forms (7) and (8), respectively. ■

Lemma 1. If the model (1) is asymptotically stable then

$$|\lambda_i(A_1)| < 1, \quad i = 1, 2, ..., n, \tag{11}$$

and

$$|\lambda_i(A_2)| < 1, \quad i = 1, 2, ..., n. \tag{12}$$

Proof. From (1) for $A_0 \equiv 0$, $A_2 \equiv 0$ and $B \equiv 0$ one obtains the homogeneous state equation of the discrete-time linear system

$$x(i+1, j+1) = A_1 x(i+1, j). \tag{13}$$

The system (13) is asymptotically stable if and only if the condition (11) holds, i.e. the matrix A_1 is Schur stable (is a Schur matrix).

Similarly, substitution of $A_0 \equiv 0$, $A_1 \equiv 0$ and $B \equiv 0$ in (1) gives the homogeneous state equation of discrete-time linear system

$$x(i+1, j+1) = A_2 x(i, j+1), \tag{14}$$

which is asymptotically stable if and only if the condition (12) holds, i.e. the matrix A_2 is Schur stable (is a Schur matrix).

Asymptotic stability of the model (1) with any fixed triple of matrices A_0, A_1 and A_2 means that the condition (6) holds for this triple. In particular, asymptotic stability of the system with $A_0 \equiv 0$ and $A_2 \equiv 0$ (or $A_0 \equiv 0$ and $A_1 \equiv 0$) is equivalent to satisfaction of the condition (6) for $A_0 \equiv 0$ and $A_2 \equiv 0$ (or $A_0 \equiv 0$ and $A_1 \equiv 0$). Hence, the conditions (11) and (12) are necessary for asymptotic stability of the model (1). ■

To show that the conditions (11) and (12) are not sufficient, we consider the scalar system (1) with $A_1 = 0$, $A_2 = 0.5$ ((11) and (12) hold) and $A_0 = 1$. In this case the characteristic equation has the form $z_1 z_2 - 0.5 z_2 - 1 = 0$. From this equation we have that if, for example, $z_1 = 0$ then $z_2 = -2$ and if $z_2 = 0.5$ then $z_1 = 2.5$. This means that there exist such values of roots of the characteristic equation which do not satisfy the condition (6) and the system is unstable.

Using the rules for computing the determinant of block matrices, we obtain that the characteristic matrix (3) of the model (1) can be computed from one of the following equivalent formulae

$$H(z_1, z_2) = [z_1 I_n - A_2][z_2 I_n - S_1(z_1)], \tag{15}$$

$$H(z_1, z_2) = [z_2 I_n - A_1][z_1 I_n - S_2(z_2)], \tag{16}$$

where

$$S_1(z_1) = (z_1 I_n - A_2)^{-1}(A_0 + z_1 A_1), \tag{17}$$

$$S_2(z_2) = (z_2 I_n - A_1)^{-1}(A_0 + z_2 A_2). \tag{18}$$

Using (4) and (15), (16) we can write

$$w(z_1, z_2) = \det[z_1 I_n - A_2] \det[z_2 I_n - S_1(z_1)], \tag{19}$$

$$w(z_1, z_2) = \det[z_2 I_n - A_1] \det[z_1 I_n - S_2(z_2)]. \tag{20}$$

From (15) for $z_1 = e^{jy}$ we have

$$H(e^{jy}, z_2) = [e^{jy} I_n - A_2][z_2 I_n - S_1(e^{jy})], \tag{21}$$

where

$$S_1(e^{jy}) = (e^{jy} I_n - A_2)^{-1}(A_0 + e^{jy} A_1). \tag{22}$$

Lemma 2. Let the necessary condition (12) be satisfied. The condition (7) holds if and only if all eigenvalues of the complex matrix (22) have absolute values less than one for all $y \in Y = [0,\ 2\pi]$.

Proof. From (21) we have

$$w(e^{jy}, z_2) = \det[e^{jy} I_n - A_2] \det[z_2 I_n - S_1(e^{jy})]. \tag{23}$$

If (12) holds then the matrix $I_n e^{jy} - A_2$ is nonsingular for all $y \in Y$. Hence, from (23) it follows that the condition (7) is satisfied if and only if

$$\det[z_2 I_n - S_1(e^{jy})] \neq 0, \quad |z_2| \geq 1, \quad \forall y \in Y. \tag{24}$$

Satisfaction of (24) means that all eigenvalues of the complex matrix (22) have absolute values less than one for all $y \in Y$. ■

Eigenvalue-loci of $S_1(e^{jy})$ for $y \in [0,\ \pi]$ and for $y \in [\pi,\ 2\pi]$ are symmetric respect to the real axis of the complex plane. Therefore, we can equivalently consider in (24) the interval $Y = [0,\ \pi]$ instead of the interval $Y = [0,\ 2\pi]$.

From (16) for $z_2 = e^{j\omega}$ we have

$$H(z_1, e^{j\omega}) = [e^{j\omega} I_n - A_1][z_1 I_n - S_2(e^{j\omega})] \tag{25}$$

and

$$w(z_1, e^{j\omega}) = \det[e^{j\omega} I_n - A_1] \det[z_1 I_n - S_2(e^{j\omega})], \tag{26}$$

where

$$S_2(e^{j\omega}) = (e^{j\omega} I_n - A_1)^{-1}(A_0 + e^{j\omega} A_2). \tag{27}$$

Lemma 3. Let the necessary condition (11) be satisfied. The condition (8) holds if and only if all eigenvalues of the complex matrix (27) have absolute values less than one for all $\omega \in \Omega = [0,\ 2\pi]$.

Proof. If (11) holds then the matrix $e^{j\omega} I_n - A_1$ is nonsingular for all $\omega \in \Omega$. From (26) we have that the condition (8) is satisfied if and only if

$$\det[z_1 I_n - S_2(e^{j\omega})] \neq 0, \quad |z_1| \geq 1, \quad \forall \omega \in \Omega, \tag{28}$$

i.e. all eigenvalues of the matrix (27) have absolute values less than one for all $\omega \in \Omega$. ■

Similarly as in Lemma 2, we can equivalently consider the interval $\Omega = [0,\ \pi]$ instead of the interval $\Omega = [0,\ 2\pi]$.

The conditions of Lemmas 2 and 3 can be written in the following forms

$$|\lambda_i\{S_1(e^{jy})\}| < 1, \quad \forall y \in Y, \quad i = 1, 2, ..., n \tag{29}$$

and

$$|\lambda_i\{S_2(e^{j\omega})\}| < 1, \quad \forall\omega \in \Omega, \quad i = 1, 2, ..., n, \quad (30)$$

respectively.

Theorem 3. The model (1) is asymptotically stable if and only if the conditions (11), (12), (29) and (30) are satisfied.

Proof. The proof follows directly from Theorem 2 and Lemmas 1, 2 and 3. ■

Computational methods for checking the conditions (29) and (30) for the Fornasini-Marchesini model (1), based on the eigenvalues-loci of the matrices (22) and (27), are given in [2].

It is easy to see that the conditions (29) and (30) can be written in the forms: $\eta(y) > 0$ for all $y \in Y$ and $\mu(\omega) > 0$ for all $\omega \in \Omega$, where

$$\eta(y) = 1 - \max_{i=1,...,n} |\lambda_i\{S_1(e^{jy})\}|, \quad (31)$$

$$\mu(\omega) = 1 - \max_{i=1,...,n} |\lambda_i\{S_2(e^{j\omega})\}|. \quad (32)$$

Hence, from Theorem 3 one obtains the following lemma.

Lemma 4. Let the necessary conditions (11), (12) hold. The model (1) is asymptotically stable if and only if $\eta(y) > 0$ for all $y \in Y$ and $\mu(\omega) > 0$ for all $\omega \in \Omega$ or equivalently, the conditions

$$\eta_{\min} = \min_{y\in Y} \eta(y) > 0, \quad \mu_{\min} = \min_{\omega\in\Omega} \mu(\omega) > 0, \quad (33)$$

are satisfied.

Example 1. Consider the model (1) with the matrices

$$A_0 = \begin{bmatrix} -0.3 & 0.1 & -0.4 \\ 0.4 & -0.1 & 0 \\ 0 & 0.3 & -0.2 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0.1 & -0.2 & 0 \\ 0 & 0.4 & 0.3 \\ 0.1 & 0.3 & 0.1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.3 & 0.1 & -0.2 \\ 0 & 0.2 & 0.1 \\ -0.3 & -0.2 & 0.4 \end{bmatrix}. \quad (34)$$

Computing eigenvalues of A_1 and A_2 we obtain

- eigenvalues of A_1: -0.1233; 0.1577; 0.5656.
- eigenvalues of A_2: 0.1166; 0.2343; 0.5491.

Hence, the necessary conditions (11) and (12) hold, i.e. the matrices A_1 and A_2 are Schur stable.

Plots of the functions $\eta(y)$ ($y \in Y$) and $\mu(\omega)$ ($\omega \in \Omega$) are shown in Figure 1. By 'o' are denoted the endpoints of the plots. The ranges $Y = [0,\ 2\pi]$ and $\Omega = [0,\ 2\pi]$ were digitized with the steps $\Delta y = 0.01\pi$ and $\Delta\omega = 0.01\pi$.

From Figure 1 and also from the fact that $\eta_{\min} = 0.3012 > 0$ and $\mu_{\min} = 0.2737 > 0$ it follows that the conditions of Lemma 4 are satisfied and the model is asymptotically stable. ■

Checking the conditions of Theorem 3 and Lemma 4 require computation of eigenvalues of the matrices (22) and (27). This may be inconvenient with respect to computational problems, particularly in the case of ill conditioned matrices. Therefore, we present a new method for investigation of asymptotic stability of the model (1) which require computation only determinants of some matrices, not eigenvalues.

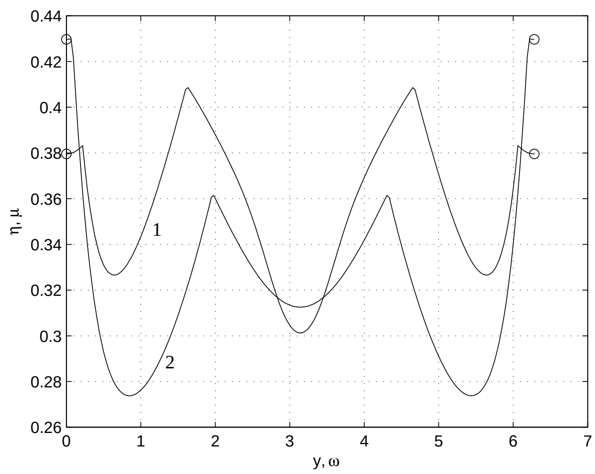

Fig. 1. Plots of the functions (31) (curve 1) and (32) (curve 2) for $y = \omega \in [0, 2\pi]$

Consider the polynomial

$$w_1(e^{jy}, z_2) = \det(z_2 I_n - S_1(e^{jy})), \quad (35)$$

where the matrix $S_1(e^{jy})$ is defined by (22). From the classical Mikhailov theorem (see for example [18]) it follows that the condition (24) holds if and only if for any fixed $y \in Y$ plot of $w_1(e^{jy}, e^{j\omega})$ for $\omega \in \Omega$ encircles in the positive direction n times the origin of the complex plane.

Direct application of the Mikhailov theorem to checking the condition (24) is not practically reliable for a large values of n. Therefore, we introduce the rational function

$$\phi_1(e^{jy}, e^{j\omega}) = \frac{w_1(e^{jy}, e^{j\omega})}{w_{1o}(e^{j\omega})}, \quad y \in Y, \quad (36)$$

instead of $w_1(e^{jy}, e^{j\omega})$, where $w_{1o}(z_2)$ is any Schur stable polynomial of degree n.

Lemma 5. The condition (29) holds if and only if for all fixed $\omega \in \Omega$ plot of the function (36) does not encircle or cross the origin of the complex plane.

Proof. If the reference polynomial $w_{1o}(z_2)$ is Schur stable then from the Argument Principle we have

$$\Delta \arg_{\omega\in\Omega} w_{1o}(e^{j\omega}) = n. \quad (37)$$

From (36) it follows that for any fixed $y \in Y$

$$\Delta \arg_{\omega\in\Omega} \phi_1(e^{jy}, e^{j\omega}) = \Delta \arg_{\omega\in\Omega} w_1(e^{jy}, e^{j\omega}) - \Delta \arg_{\omega\in\Omega} w_{1o}(e^{j\omega}). \quad (38)$$

The matrix (22) for any fixed $y \in Y$ is Schur stable if and only if

$$\Delta \arg_{\omega\in[0,2\pi]} w_1(e^{jy}, e^{j\omega}) = \Delta \arg_{\omega\in[0,2\pi]} w_{1o}(e^{j\omega}) = n,$$

which holds if and only if $\Delta \arg_{\omega\in\Omega} \phi_1(e^{jy}, e^{j\omega}) = 0$.

Taking into account all $y \in Y$, we obtain that the above holds if and only if for all fixed $\omega \in \Omega$ plot of (36) as a function of $y \in Y$ does not encircle or cross the origin of the complex plane. ∎

The reference polynomial $w_{1o}(z_2)$ can be chosen in the form

$$w_1(1, z_2) = \det(z_2 I_n - S_1(1)), \quad (39)$$

where $S_1(1) = (I_n - A_2)^{-1}(A_0 + A_1)$, which we get from (35) and (22) substituting $y = 0$. Schur stability of (39) is necessary for Schur stability of complex polynomial (35) for all $y \in Y$.

If $w_{1o}(z_2) = w_1(1, z_2)$, then

$$\phi_1(e^{jy}, e^{j\omega}) = \frac{w_1(e^{jy}, e^{j\omega})}{w_1(1, e^{j\omega})}, \quad y \in Y. \quad (40)$$

Plot of (40) as a function of $y \in Y$ (with any fixed $\omega \in \Omega$) is a closed curve. It begins with $y = 0$ and ends with $y = 2\pi$ in the point $\phi_1(1, e^{j\omega}) = 1$.

Now, we consider the complex polynomial

$$w_2(z_1, e^{j\omega}) = \det(z_1 I_n - S_2(e^{j\omega})), \quad (41)$$

where the matrix $S_2(e^{j\omega})$ is defined by (27).

Let $w_{2o}(z_1)$ be any Schur stable polynomial of degree n.

Proceeding similarly as in the case of Lemma 5, we obtain the following lemma.

Lemma 6. The condition (30) holds if and only if for all fixed $y \in Y$ plot of the function

$$\phi_2(e^{jy}, e^{j\omega}) = \frac{w_2(e^{jy}, e^{j\omega})}{w_{2o}(e^{j\omega})}, \quad \omega \in \Omega, \quad (42)$$

does not encircle or cross the origin of the complex plane, where $w_2(e^{jy}, e^{j\omega})$ has the form (41) for $z_1 = e^{jy}$.

The reference polynomial $w_{2o}(z_1)$ can be chosen in the form

$$w_2(z_1, 1) = \det(z_1 I_n - S_2(1)), \quad (43)$$

where $S_2(1) = (I_n - A_1)^{-1}(A_0 + A_2)$. Schur stability of (43) is necessary for Schur stability of the complex polynomial (41) for all $\omega \in \Omega$.

If $w_{2o}(z_1) = w_2(z_1, 1)$, then

$$\phi_2(e^{jy}, e^{j\omega}) = \frac{w_2(e^{jy}, e^{j\omega})}{w_2(e^{jy}, 1)}, \quad \omega \in \Omega. \quad (44)$$

Plot of (44) as a function of $\omega \in \Omega$ with the fixed $y \in Y$ is a closed curve. It begins with $\omega = 0$ and ends with $\omega = 2\pi$ in the point $\phi_2(e^{jy}, 1) = 1$.

From Theorem 3 and Lemmas 5 and 6 we have the following theorem.

Theorem 4. Assume that the necessary conditions (11) and (12) are satisfied and the polynomials (39) and (43) are Schur stable. The model (1) is asymptotically stable if and only if the following two conditions hold:

1) plots of the function (40) do not encircle or cross the origin of the complex plane for all fixed $\omega \in \Omega$,
2) plots of the function (44) do not encircle or cross the origin of the complex plane for all fixed $y \in Y$.

Applying computational method given in Theorem 4 we can take into consideration the following remark.

Remark. Refer to point 1) of Theorem 4, one should set any fixed $\omega \in \Omega$, determined with appropriately small step $\Delta\omega$, and draw plots of the function (40) separately digitizing the range Y with a sufficiently small step Δy. For point 2) of Theorem 4 one should set any fixed $y \in Y$, determined with appropriately small step Δy, and draw plots of the function (44) separately digitizing the range Ω with a sufficiently small step $\Delta\omega$. Plots should be smooth especially near the origin of the complex plane so that the important parts have not been neglected.

Example 2. Using Theorem 4 check asymptotic stability of the model (1) with the matrices (34).

In Example 1 it has been shown that the necessary conditions (11) and (12) hold.

Computing eigenvalues of the matrices $S_1(1) = (I_n - A_2)^{-1}(A_0 + A_1)$ and $S_2(1) = (I_n - A_1)^{-1}(A_0 + A_2)$ we obtain respectively:

$$\begin{aligned} \lambda_{11} &= 0.4201 + j0.2872, \\ \lambda_{12} &= 0.4201 - j0.2872, \\ \lambda_{13} &= -0.6204, \end{aligned} \quad (45)$$

$$\begin{aligned} \lambda_{21} &= 0.4762 + j0.2152, \\ \lambda_{22} &= 0.4762 - j0.2152, \\ \lambda_{23} &= -0.5703. \end{aligned} \quad (46)$$

Moduli of all eigenvalues (45) and (46) are less than one and the reference polynomials (39) and (43) are Schur stable.

Plots of (40) and (44) are shown in Figures 2 and 3, respectively. The ranges $\Omega = [0, 2\pi]$ and $Y = [0, 2\pi]$ for all plots was digitized with the steps $\Delta y = 0.01\pi$ and $\Delta\omega = 0.01\pi$.

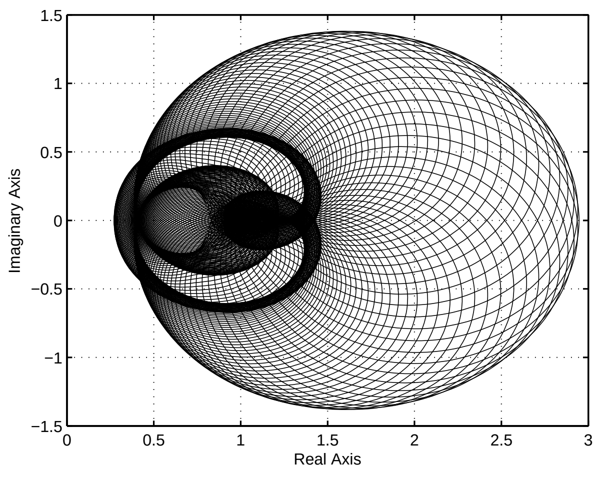

Fig. 2. Plots of (40)

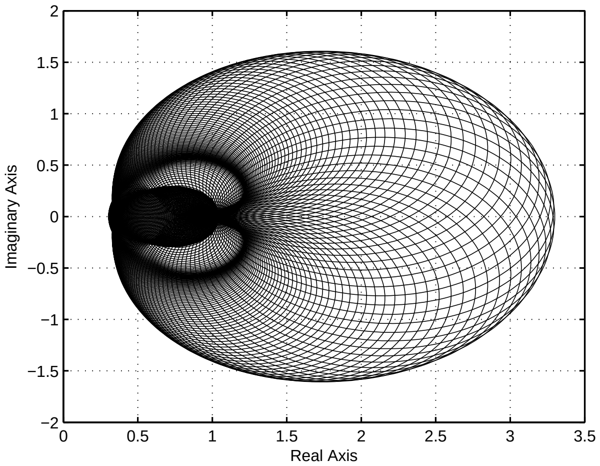

Fig. 3. Plots of (44)

From Figures 2 and 3 is follows that the plots do not encircle the origin of the complex plane for all $y \in Y$ and $\omega \in \Omega$. This means, according to Theorem 4, that the model (1), (34) is Schur stable. ■

Now we consider the 1st order Fornasini-Marchesini model described by the equation

$$x(i+1,j+1) = a_0 x(i,j) + a_1 x(i+1,j) + a_2 x(i,j+1) + bu(i,j), \quad (47)$$

where a_0, a_1, a_2 and b are real coefficients.

For the system (47) the necessary conditions (11) and (12) take the forms

$$|a_1| < 1, \quad |a_2| < 1. \quad (48)$$

The matrix (22) for the system has the form

$$S_1(e^{jy}) = \frac{a_0 + e^{jy} a_1}{e^{jy} - a_2}. \quad (49)$$

It is easy to check that plot of (49) for $y \in Y = [0,\ 2\pi]$ is a circle with the center at real axis. This circle crosses real axis in points

$$S_{10} = S_1(e^{j0}) = \frac{a_0 + a_1}{1 - a_2},\ S_{1\pi} = S_1(e^{j\pi}) = \frac{a_1 - a_0}{1 + a_2}.$$

Hence, the first condition (33) holds if and only if

$$\eta_{\min} = 1 - \max\{|S_{10}|, |S_{1\pi}|\} > 0. \quad (50)$$

Similarly, we can show that the second condition (33) holds if and only if

$$\mu_{\min} = 1 - \max\{|S_{20}|, |S_{2\pi}|\} > 0, \quad (51)$$

where

$$S_{20} = S_2(e^{j0}) = \frac{a_0 + a_2}{1 - a_1},\ S_{2\pi} = S_2(e^{j\pi}) = \frac{a_2 - a_0}{1 + a_1}.$$

From the above and Theorem 3 we have the following condition.

Lemma 7. The 1st order Fornasini-Marchesini model (47) is asymptotically stable if and only if the conditions (48) and (50), (51) are satisfied.

4. Concluding Remarks

Simple necessary conditions (Lemma 1) and two computational methods for investigation of asymptotic stability of the first Fornasini-Marchesini model (1) of 2D discrete linear systems have been given.

The first method (Theorem 3, Lemma 4) require computation of eigenvalues of complex matrices (22) and (27). Similar methods have been applied in [7, 23] to asymptotic stability analysis of the Roesser model of 2D systems and in [3] for the Fornasini-Marchesini and the Roesser type models of 2D continuous-discrete linear systems.

The second method (Theorem 4) require computation of values of functions (40) and (44) and therefore is simpler from the computation point of view. Similar methods have been applied in [3], [4], [5] and [6], respectively, to asymptotic stability analysis of 2D continuous-discrete linear systems described by the first and the second Fornasini-Marchesini type models and the Roesser type model.

The proposed methods can be applied to the stability checking of the second Fornasini-Marchesini model described by the state equation (1) with $A_0 \equiv 0$.

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AUTHORS

Andrzej Ruszewski* – Bialystok University of Technology, Faculty of Electrical Engineering, Wiejska 45D, 15-351 Białystok, Poland, e-mail: andrusz@pb.edu.pl, www: pb.edu.pl.
Mikołaj Busłowicz – Bialystok University of Technology, Faculty of Electrical Engineering, Wiejska 45D, 15-351 Białystok, Poland, e-mail: busmiko@pb.edu.pl, www: pb.edu.pl.
*Corresponding author

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