PROBLEMY MECHATRONIKI
UZBROJENIE, LOTNICTWO, INŻYNIERIA BEZPIECZEŃSTWA

ISSN 2081-5891 **10, 2 (36), 2019, 89-102**

PROBLEMS OF MECHATRONICS
ARMAMENT, AVIATION, SAFETY ENGINEERING

A Concept of Live Fire Testing to Identify the Aerodynamic Coefficients of a 35 mm Anti-Aircraft Projectile

Leszek BARANOWSKI[1*], Błażej GADOMSKI[2],
Przemysław MAJEWSKI[2], Jacek SZYMONIK[2]

[1]Military University of Technology, Faculty of Mechatronics and Aerospace,
2 gen. Sylwestra Kaliskiego Str., 00-908 Warsaw, Poland
[2]PIT-RADWAR S.A., 30 Poligonowa Str., 04-051 Warsaw, Poland
[*]Corresponding author's e-mail address and ORCID:
leszek.baranowski@wat.edu.pl; https://orcid.org/0000-0002-5239-0659

Received by the editorial staff on 23 December 2016
The reviewed and verified version was received on 10 June 2019

DOI 10.5604/01.3001.0013.2118

Abstract. This paper presents a generic algorithm designed to identify aerodynamic coefficients among the data specified in firing tables and projectile flight parameter data recorded during live fire tests. The algorithm and the concept of live fire testing shown here allow developing a mathematical model of projectile trajectory in the form of a modified motion model of a point mass. Potential applications of the model include fire control systems.
Keywords: ballistics, firing tables, identification, aerodynamics

This work has been compiled from the paper presented during the 9th Scientific Conference on Fire Control of Air Defense (Anti-aircraft) Systems. Ustka, Poland, June 22, 2016.

1. INTRODUCTION

According to NATO standardisation specifications (STANAG 4144 [1] and STANAG 4119 [2]), the set-up of firing tables for a cannon and projectile system requires that a correct mathematical model is developed for the projectile flight. It is recommended to model the projectile trajectory as the motion of a point mass with four degrees of freedom, which is an MPMT (*Modified Point Mass Trajectory*) model. If a mathematical model of a projectile trajectory exists, it can be applied in automatic fire control systems. However, an application of the model to simulate a projectile flight (trajectory) requires that correct mass-inertial and aerodynamic characteristics are determined for the projectile (and being the aerodynamic force and moment coefficients dependent on the Mach number). Unfortunately, munitions manufacturers only provide the physical specifications of their shells (mass, dimensions, nominal muzzle velocity, etc.) and the firing tables which specify a set of projectile trajectory values, including shell range, sight elevation, flight duration, drift offset, terminal velocity, and terminal altitude[1].

Given the foregoing, this paper is a discussion of the methodology of identifying the aerodynamic characteristics applied in shell / projectile flight (trajectory) simulation according to MPMT. The focus of this paper includes indirect identification methods from the data in firing tables and the shell trajectory parameters recorded during live fire tests. The methods provide the best accuracy of determination for the aerodynamic characteristics.

The aerodynamic characteristics used in the shell trajectory model applied as discussed herein were expressed with dedicated polynomials, which were functions of the Mach number [3, 4], the identification of the aerodynamic characteristics was reduced to the determination of the factors of the polynomials. The paper also presents a concept for a test station designed to determine the Earth's atmosphere parameters required in the identification process.

2. THE PROJECTILE TRAJECTORY MATHEMATICAL MODEL

The Modified Point Mass Trajectory model (MPMT) is a mathematical (computational) model of a shell trajectory which includes shell trajectory drift caused by the dynamic equilibrium angle [5].

---

[1] [2] defines the "point of impact", which is the physical point in space at which a shell/projectile hits the target. In simulated flight trajectories, the shell trajectory equations are integrated until the condition of time to shell impact, t_{end}, is satisfied. Hence, it is necessary to introduce the term of shell terminal altitude, $h(t_{end})$, which is a vertical coordinate value at the time when the shell flight equations are solved to an end.

The shell trajectory equations compliant with MPMT are specified in STANAG 4355 [6]. The standard also includes a model in a confounded form which increases the time to compute the shell trajectories. Given this, a novel form of MPMT was applied in the identification process explained herein. The form is called ‘explicit’ and it significantly reduced the computing time, while eliminating the issues with algorithm convergence, which would otherwise occur in the confounded form of MPMT [7]. The final form of the explicit form of shell trajectory equations is shown below, while the full derivation process is explained in [8].

$$\dot{\boldsymbol{x}} = \boldsymbol{v} + \boldsymbol{w} \tag{1}$$

$$\dot{p} = \frac{\rho\, v^2}{2\, I_x} S\, d\, C_{Spin}\, \hat{p} \tag{2}$$

$$S = \frac{\pi d^2}{4} \tag{3}$$

$$\dot{v} = -\frac{\rho\, v^2}{2\, m} S \left(C_{D_0} + \hat{C}_{D_{\alpha^2}} \left(\frac{2\, m\, g}{\rho\, v^2\, S}\right)^2 \frac{\hat{I}_x^2 \hat{p}^2 cos^2(\gamma_a)}{(1 - \hat{I}_x \hat{p}^2 \hat{C}_{mag-f})^2 + (\hat{I}_x \hat{p} \hat{C}_{L\alpha})^2} \right) - g\, sin(\gamma_a) \tag{4}$$

$$\begin{bmatrix} \dot{\gamma}_a \\ \dot{\chi}_a\, cos(\gamma_a) \end{bmatrix} = -\frac{g}{v} \frac{cos(\gamma_a)}{(1 - \hat{I}_x \hat{p}^2 \hat{C}_{mag-f})^2 + (\hat{I}_x \hat{p} \hat{C}_{L\alpha})^2} \begin{bmatrix} 1 - \hat{I}_x \hat{p}^2 \hat{C}_{mag-f} \\ \hat{I}_x \hat{p} \hat{C}_{L\alpha} \end{bmatrix} \tag{5}$$

The dimensionless factors applied in the equations are:

$$\hat{I} = \frac{I_x}{md^2}, \qquad \hat{p} = \frac{pd}{v}, \tag{6}$$

$$\widehat{C}_{D_{\alpha^2}} = \frac{C_{D_{\alpha^2}}}{\left(C_{M_\alpha}\right)^2}, \widehat{C}_{L\alpha} = \frac{C_{L\alpha}}{C_{M\alpha}}, \widehat{C}_{mag-f} = \frac{C_{mag-f}}{C_{M\alpha}} \tag{7}$$

with:

- $\boldsymbol{x}$ – 3-D vector of shell position;
- $\boldsymbol{v}$ – shell velocity vector relative to air;
- $\boldsymbol{w}$ – wind velocity vector;
- p – axial spin;
- I_x – shell moment of inertia relative to lengthwise axis,
- C_{D_0}– zero-yaw drag coefficient;
- $C_{D\alpha^2}$ – yaw drag coefficient;
- C_{Spin}– spin attenuating moment coefficient;

C_{M_α} – overturning moment coefficient;
C_{mag-f} – Magnus force coefficient;
C_{L_α} – lift force coefficient;
g – gravitational acceleration;
ρ – air density;
d – shell calibre;
m – shell mass.

The form of the function which approximates the aerodynamic coefficients present in MPMT was significant to the identification process. The coefficients of drag C_{D_0}, lift force C_{L_α} and spin damping moment C_{Spin} were expressed as the following polynomials [3, 4]:

$$C(Ma) = (1+s)A(r) + (1-s)B(r), \tag{8}$$

$$A(r) = a_0 + a_1 r + a_2 r^2 \tag{9}$$

$$B(r) = b_0 + b_1 r + b_2 r^2 \tag{10}$$

$$r = (Ma^2 - K)/(Ma^2 + K) \tag{11}$$

$$s = \frac{(Ma^2 - K)/(Ma^2 + K)}{\sqrt{(1-L^2)r^2 + L^2}} \tag{12}$$

with:

$C(Ma)$ – the correct aerodynamic coefficient dependent on the Mach number;
a_0, a_1, a_2, b_0, b_1, b_2, K, L – the polynomial parameters to be identified. The set of the parameters was designated as vector $\vec{w}_i$.

The foregoing form of polynomials was chosen since they represent functions which described well the nature of change of the aerodynamic coefficients vs. the Mach number, both in supersonic and subsonic ranges. Here, examples were the drag force coefficient and the lift force coefficient for the TP-T 35-mm AA shells generated in a commercial software environment PRODAS v3.5.3 from Arrow Tech and approximated with the foregoing functions (see Fig. 1 and 2).

The Magnus force coefficient C_{mag-f} and the yaw drag coefficient $C_{D\alpha^2}$ were identified as constants, according to the guidelines in STANAG 4144.

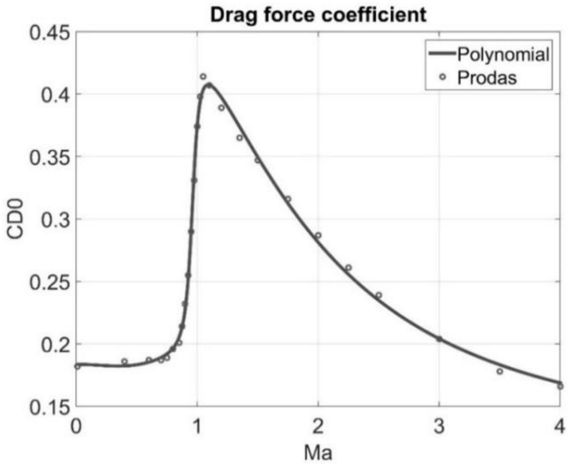

Fig. 1. Result of the approximation of the zero-yaw drag coefficient C_{D_0} values

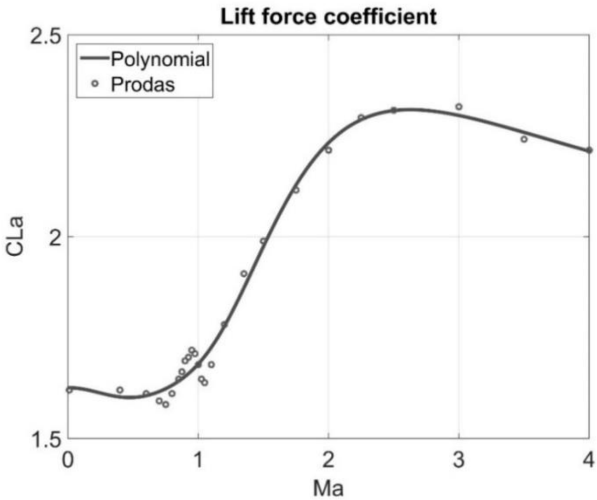

Fig. 2. Result of the approximation of the lift force coefficient C_{L_α} values

3. AERODYNAMIC COEFFICIENTS IDENTIFICATION ALGORITHM

Figure 3 shows a process flow of a universal aerodynamic coefficient's identification algorithm operating on the data from firing tables and from the shell trajectory parameters recorded during live fire testing.

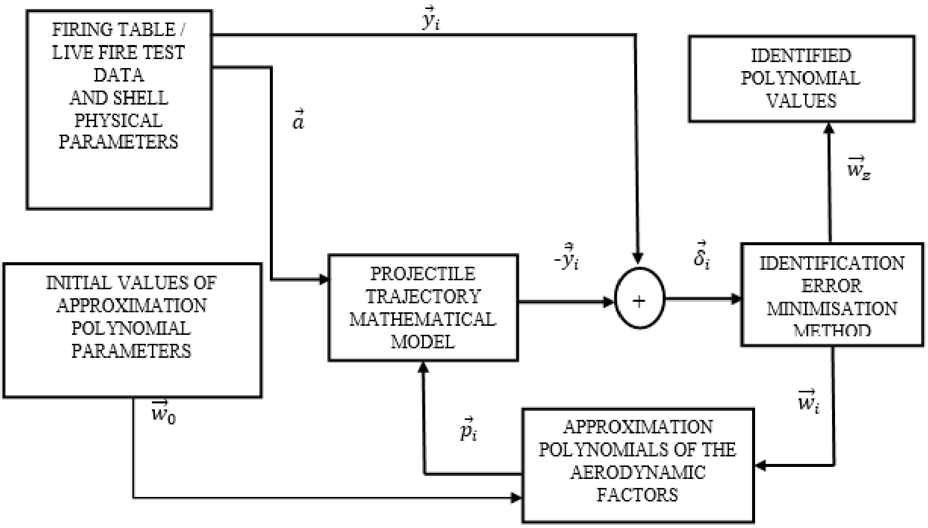

Fig. 3. Process flow of the aerodynamic coefficient's identification algorithm

List of designations in Fig. 3:

- $\vec{y}_i$ – vector which includes the shell trajectory parameters from the firing tables (i.e. shell range, sight elevation, shell terminal velocity, shell terminal altitude, and shell drift) or the measurements of a single complete shell trajectory;
- $\hat{\vec{y}}_i$ – vector which includes shell trajectory parameters and was calculated with MPMT in iteration number *i*;
- $\vec{a}$ – vector of shell physical parameters (initial velocity, mass, diameter, and axial moment of inertia);
- $\vec{p}_i$ – vector of aerodynamic force and moment coefficients in iteration number *i*;
- $\vec{\delta}_i$ – minimised relative identification error;
- $\vec{w}_0$ – vector of initial parameters of the approximation polynomials applicable to the aerodynamic characteristics (and the parameters were the objects of identification);
- $\vec{w}_i$ – vector of initial parameters of the approximation polynomials applicable to the aerodynamic characteristics in iteration number *i*;
- $\vec{w}_z$ – vector of identified parameters of the approximation parameters applicable to the aerodynamic characteristics.

The idea behind the algorithm was that the data of these shell trajectory parameters:

- shell range, sight elevation, shell flight duration, shell drift, shell terminal velocity and shell terminal altitude in the identification based on the general anti-aircraft firing tables;
- shell location in successive points of time for the identification based on the measurement of a section of or the whole shell trajectory;

were compared with the results from the shell flight simulation parameters to determine the identification error. The condition of ending the integration of the equations of trajectory and computation of the shell flight trajectory parameters was the expiry of the time to shell impact, t_{end}, specified in the firing tables. The identified values (the aerodynamic force and moment coefficients) were changed iteratively until the identification error achieved a preset level. The initial values of the aerodynamic coefficients $\vec{w}_0$, and thus the values of approximation function factors, were generated in the PRODAS software environment[2]. PRODAS was capable of computing the aerodynamic factors for a preset and dimensioned shell geometry.

A significant problem in the process of aerodynamic coefficients identification was to properly define the identification error to be minimised. The relative measure of identification error was the elapsed shell flight distance[3]. The elapsed shell flight distance was calculated by inclusion of a differential equation:

$$\dot{s} = v \tag{13}$$

with v – shell velocity and s – elapsed shell flight distance, with the initial condition of $s(0) = 0$ metres.

The relative errors for shell range, shell trajectory drift, shell terminal altitude and shell terminal velocity were expressed as follows:

$$\delta_x = \frac{x - x_m}{s(t_{end})} \qquad \delta_y = \frac{y - y_m}{s(t_{end})} \qquad \delta_h = \frac{h - h_m}{s(t_{end})} \qquad \delta_v = \frac{(v - v_m)t_{end}}{s(t_{end})} \tag{14}$$

with:

δ_x – shell range relative error;
δ_y – shell trajectory drift relative error;
δ_h – shell terminal altitude relative error;
δ_v – shell terminal velocity relative error;
x – shell range from the firing tables;
y – shell derivation (flight drift) from the firing tables (the angular value in radians was converted into metres);

[2] http://www.prodas.com/Documents/Arrow%20Tech%20Software%20Products%20Catalog%20June%202013.pdf

[3] Note that 'elapsed shell flight distance' should not be construed as the distance to target along the horizontal plane (as in ground to ground firing tables) or a radial distance (as in AA firing tables). It was the elapsed distance along the shell trajectory.

h – shell terminal altitude from the firing tables (for ground to ground fire, the shell terminal altitude was 0 m);

$s(t_{end})$ – shell trajectory elapsed in the duration from the firing tables;

v – shell terminal velocity specified in the firing tables;

x_m, y_m, h_m, v_m – shell flight range, trajectory drift, terminal altitude and terminal velocity derived by MPMT computing. The shell trajectory equations were integrated until the shell flight duration t_{end} specified in the firing tables expired.

The primary component of the identification process was the method of minimising the relative errors defined above. In this work, the data minimised was the RMS error of the data range contained in the firing tables. The minimisation was processed with the function *lsqnonlin* in MATLAB, which used a trust region reflective algorithm by default [9, 10].

4. IDENTIFICATION BY FIRING TABLE DATA

The aerodynamic characteristic identification algorithm was tested with an example of the firing table data applicable to the ground-to-ground firing of AHEAD 35-mm artillery shells. The basic geometric characteristics of the shell are shown in Fig. 4. The shell physical parameters are shown in Table 1.

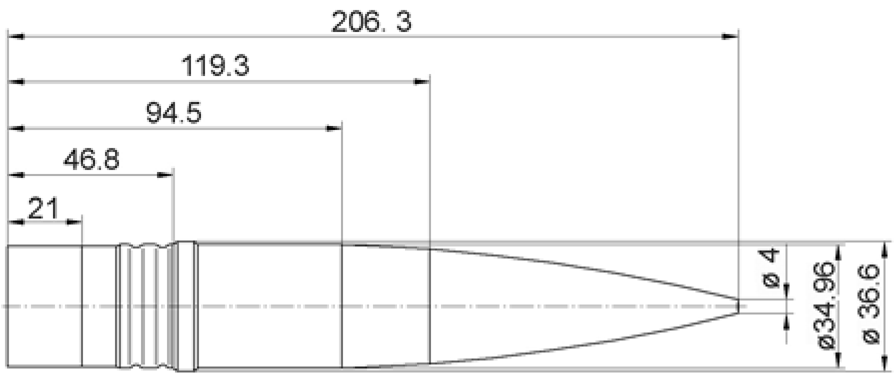

Fig. 4. Geometric characteristics of an AHEAD 35-mm artillery shell

The identification process inputs included the following data: shell flight range, x, from 100 to 4500 m in 100 m increments, the respective projection angles, Θ, the shell flight drift, y, and the shell terminal velocities, v. The initial values for the aerodynamic factors were taken from [11].

Table 1. Physical parameters of an AHEAD 35-mm artillery shell

Shell length	L_p	0.2063	m
Shell calibre	d	0.035	m
Shell weight w/fuse	m	0.745	kg
Centre-of-mass coordinate relative to the shell bottom	$x_{ś.m}$	0.0771	m
Axial moment of inertia	I_x	0.0000874	kgm^2
Polar moment of inertia	I_y	0.0007575	kgm^2
Initial velocity (firing table specified)	V_0	1050	m/s

Once the aerodynamic characteristics were identified, the firing tables were recomputed with MPMT model as the mathematical shell trajectory model (Eq. 1 to 7). The initial conditions for the calculations, velocity V_0 and projection angles Θ, were consistent with those specified in the firing tables.

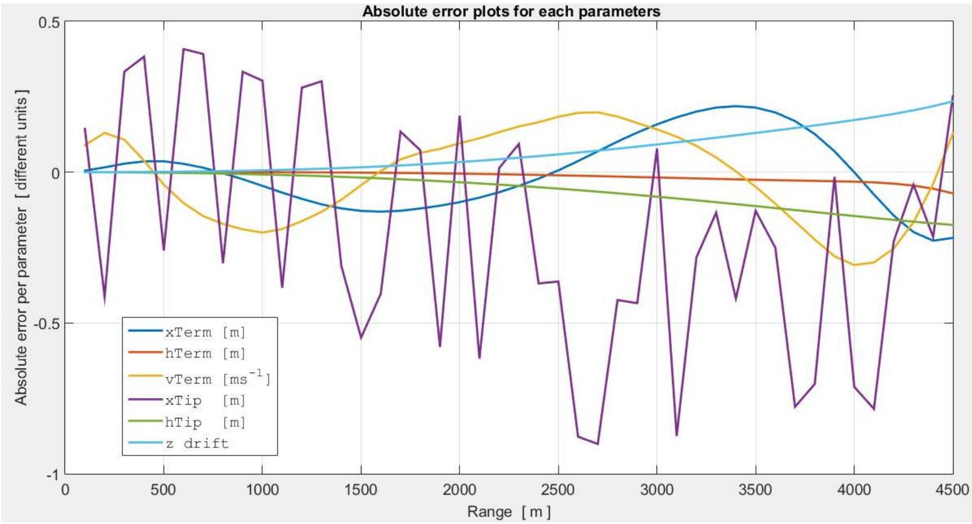

Fig. 5. Differences between the computed shell flight parameter values and their respective firing table data

Figure 5 shows a comparison of the computed shell flight trajectory characteristics to the original firing table data, and the following designations were applied: *xTerm* – shell flight range; *hTerm* – shell terminal altitude; *vTerm* – shell terminal velocity; *xTip* – shell tip horizontal coordinate; *yTip* – shell tip vertical coordinate; z *drift* – shell trajectory drift.

It is clear that for the firing tables included in the tests, the process of aerodynamic coefficients identification was very effective; the maximum differences between the computed results and the respective firing table values are within 1 metre.

5. IDENTIFICATION BY RECORDED LIVE FIRE TEST DATA

If no firing tables are available, it could be possible to identify the aerodynamic characteristics from live fire tests to develop a mathematical model of shell trajectory.

The idea behind this method is to do live firing at specific projection angles and measure the shell flight trajectory coordinates or the shell radial velocity [5]. A precondition of a valid identification of the aerodynamic characteristics is to know the weather conditions prevailing during live fire tests. This requires a test station which includes a radar to measure the shell trajectory and a network of sensors to monitor real-time atmospheric conditions.

To verify that the method is true, live fire tests are planned with TP-T 35-mm practice shells with a target located 2,500 m from the artillery unit. A Doppler radar will record the shell radial velocity. Atmospheric monitoring will be done by locating an array of twenty sensor towers along the tactical lane (as shown in Fig. 6) to form 20 weather monitoring units.

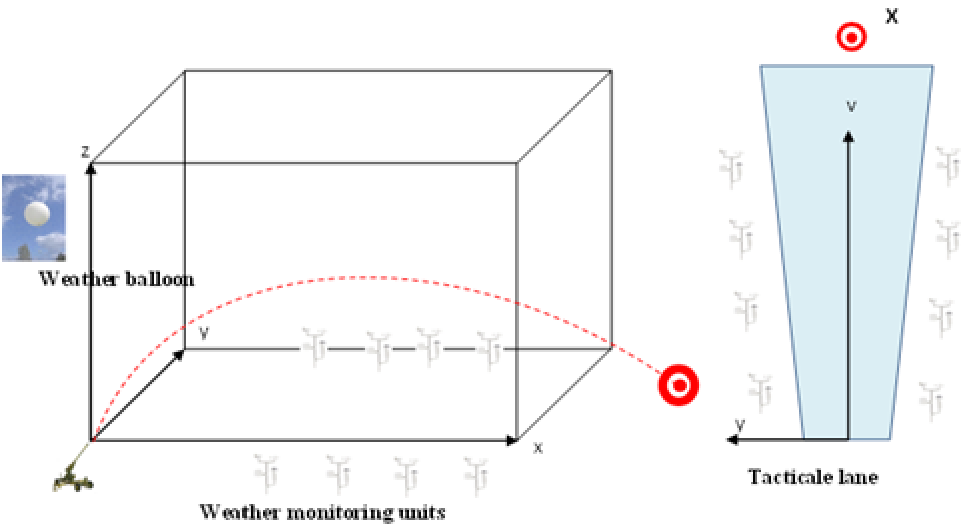

Fig. 6. Schematic layout of the 20 sensor towers along the tactical lane

Each sensor tower will accommodate a barometric pressure sensor, an ambient temperature sensor, a relative humidity sensor, and horizontal wind speed and direction sensors (see Fig. 7).

Each sensor tower will feature an SD memory card drive and a GPS module to enable precise positioning of the weather monitoring unit. The atmospheric conditions will be sampled by the sensors at 1 kHz.

All measured atmospheric parameters will be saved on the SD memory cards and relayed to a base station for recording and visual output of data

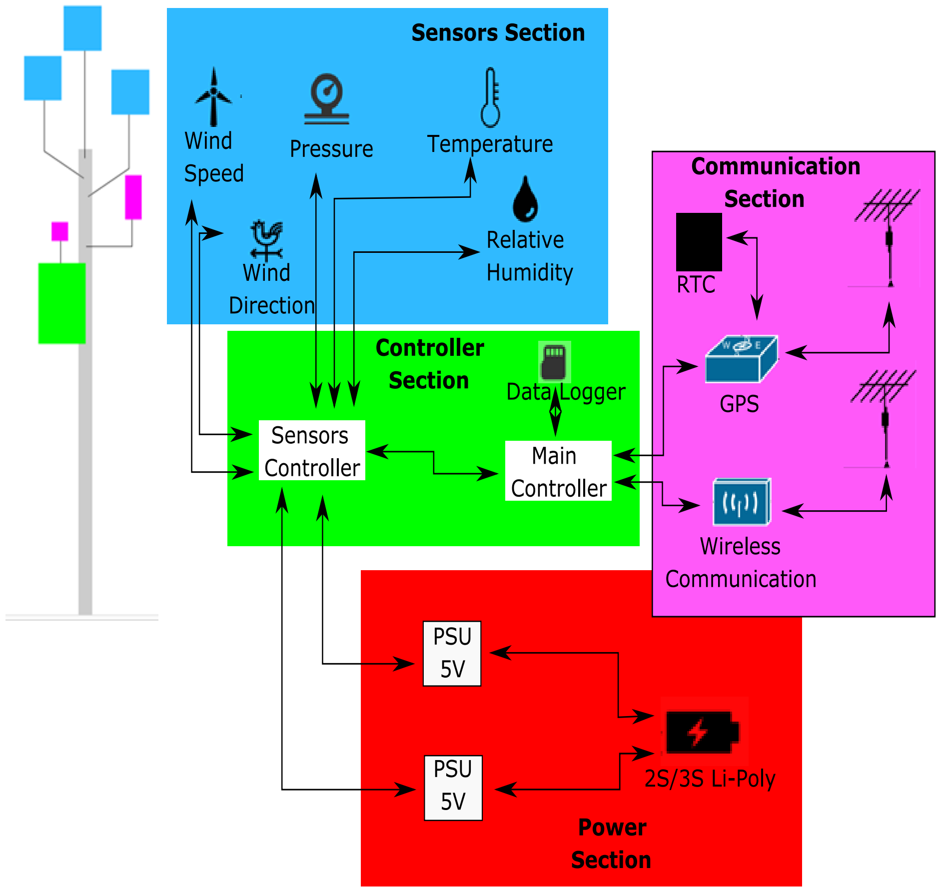

Fig. 7. Layout of the sensor and instruments on a sensor tower

The recorded shell radial velocity trends and atmospheric data during the live fire tests will allow identification of the aerodynamic coefficients of the test munitions by applying the algorithm (see Fig. 3).

6. CONCLUSION

The increasing detection and tracing accuracy of sensors (in radars and optical electronic arrays) applied in fire control systems demands improved ballistic computing. This demand could be satisfied by applying a shell trajectory model with enough accuracy and computing speed.

The greatest obstacle in the development of a shell trajectory model is the determination of the aerodynamic characteristics of a shell. The indirect methods of identification of the aerodynamic characteristics suggested in this paper could greatly facilitate the development of the model.

The validity of the method based on firing table data was demonstrated in this work. For the other method (identification by recorded live fire test data), a suitable live fire test concept is proposed. The test station for the other method was designed to remove as many unknowns present in the applied shell trajectory model as possible to reduce their potential negative impact on the identification of aerodynamic characteristics of an artillery shell.

Experimental verification of the method by live fire testing will be the focus of future work.

FUNDING

The paper is based upon work supported by the Polish National Centre for Research and Development under Contract No DOBR/0046/R/ID1/2012/3.

REFERENCES

[1] STANAG 4144 *Procedures to determine the fire control inputs for use in direct fire control systems*, 12 August 2005.

[2] STANAG 4119 *Adoption of a standard cannon artillery firing table format*, 5 February 2007.

[3] Shanks D., T. Walton. 1957. *A New General Formula for Representing the Drag on a Missile Over the Entire Range of Mach Number*. NAVORD Report 3634.

[4] Pope R.L. 1978. *The analysis of trajectory and solar aspect angle records of shell flights. Theory and computer programs*. Department of Defence, Defence Science and Technology Organistion, Weapons Systems Research Laboratory, South Australia.

[5] Baranowski Leszek. 2011. *Modelowanie, identyfikacja i badania numeryczne dynamiki lotu obiektów balistycznych na potrzeby systemów kierowania ogniem artylerii naziemnej*. Warszawa: Wojskowa Akademia Techniczna.

[6] STANAG 4355 *The Modified Point Mass and Five Degrees of Freedom Trajectory Models*, 2009.

[7] Baranowski Leszek, Błażej Gadomski, Przemysław Majewski, Jacek Szymonik. 2016. "Comparison of explicit and implicit forms of the modified point mass trajectory model". *Journal of Theoretical and Applied Mechanics* 54 (4) : 1183-1195.

[8] Baranowski Leszek, Błażej Gadomski, Przemysław Majewski, Jacek Szymonik. 2016. "Explicit "ballistic M-model": a refinement of the implicit "modified point mass trajectory model"". *Bulletin of the Polish Academy of Sciences: Technical Sciences* 64 (1) : 81-89.

[9] Coleman F. Thomas, Yuying Li. 1994. "On the convergence of reflective Newton methods for large-scale nonlinear minimization subject to bounds". *Mathematical Programming* 67 (1-3) : 189-224.

[10] Coleman F. Thomas, Yuying Li. 1996. "An interior, trust region approach for nonlinear minimization subject to bounds". *SIAM Journal on Optimization* 6 (2) : 418-445.

[11] Baranowski Leszek, Wojciech Furmanek. 2013. „Problem walidacji modelu trajektorii lotu 35mm pocisku TP-T w warunkach normalnych". *Problemy Techniki Uzbrojenia* 125 : 35-44.

Koncepcja badań poligonowych na potrzeby identyfikacji współczynników aerodynamicznych 35 mm pocisku przeciwlotniczego

Leszek BARANOWSKI[1], Błażej GADOMSKI[2], Przemysław MAJEWSKI[2], Jacek SZYMONIK[2]

[1]*Wojskowa Akademia Techniczna im. Jarosława Dąbrowskiego*
ul. gen. Sylwestra Kaliskiego 2, 00-908 Warszawa
[2]*PIT-RADWAR S.A., ul. Poligonowa 30, 04-051 Warszawa*

Streszczenie. W artykule zaprezentowano ogólny algorytm identyfikacji współczynników aerodynamicznych na podstawie danych zawartych w tabelach strzelniczych oraz na podstawie zarejestrowanych podczas strzelań poligonowych danych o parametrach lotu pocisku. Przedstawiony algorytm oraz koncepcja badań poligonowych pozwalają pozyskać model matematyczny ruchu pocisku w postaci zmodyfikowanego modelu ruchu punktu materialnego. Model taki może być wykorzystany np. w systemie kierowania ogniem.
Słowa kluczowe: balistyka, tabele strzelnicze, identyfikacja, aerodynamika