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Numerical Investigations of Finite Amplitude Waves Interaction in Water

A. Baranowska

Naval Academy, ul. Smidowicza 71, 81-919 Gdynia, POLAND
e-mail: abar@amw.gdynia.pl

The aim of this paper is numerical analysis of the finite waves interaction in water. The mathematical model is built on the basis of Khokhlov-Zabolotskaya-Kuznetsov (KZK) equation. The problem is considered as an axial symmetric one. The finite-difference method has been used to solve the KZK equation. The paper presents the mathematical model and some results of numerical investigations of finite amplitude waves interaction. One shows the waveform and spectrum of the wave for fixed distances from the source, and the pressure amplitude changes as a function of distance of different harmonic components.

1. Introduction

The finite amplitude waves during their propagation in the same direction in water interact. The appearance of different frequency waves is the result of this interaction. One can observe, for instance, sum frequency wave and difference frequency one. The most important in practical application is existence of the difference frequency wave. The generation of this wave has practical application at investigation and construction of the parametric acoustic arrays.

The finite waves interaction problem has been described in the literature for many years. This problem is considered in experimental investigations (for example [4]) and theoretical one [1, 3, 6]. It is possible to find the works about practical use of parametric arrays [5], too.

The aim of this paper is the numerical analysis of the finite amplitude waves interaction in water. This problem can be described using KZK equation, which describes the changes in acoustic pressure in nonlinear and dissipative medium along the sound beam. The paper presents the results of numerical investigations of finite waves interaction in water. These investigations were carried out using own computer program.

2. Mathematical model

The mathematical model of the finite amplitude waves interaction was worked out assuming that two finite amplitude waves produced by a circular piston propagate in water. The source is placed on the y0z plane and waves propagate in the x axis direction. It means that x axis corresponds to the beam axis (Fig. 1).

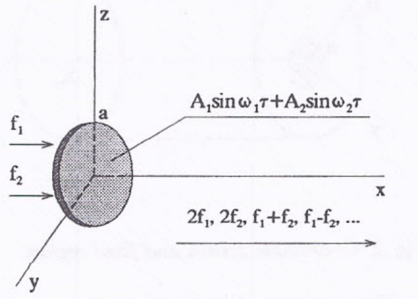

Fig. 1. The geometry of the problem.

Assuming the axial symmetry of the source it is comfortably to solve the problem in cylindrical coordinates (x,r), where $r=(y^2+z^2)^{1/2}$.

The waves distribution on the piston is defined by

$$p'(x=0,r,\tau) = = A_1(r)\sin\omega_1\tau + A_2(r)\sin\omega_2\tau \tag{1}$$

for $r \le a$ (a - piston radius). Moreover $p'(x=0,r,\tau)=0$ for $r>a$. In formula (1) parameters A_1 and A_2 are primary wave amplitudes, angular frequency are defined by $\omega_1=2\pi f_1$ and $\omega_2=2\pi f_2$ respectively.

The mathematical model is built on the basis on KZK equation:

$$\frac{\partial}{\partial\tau}\left(\frac{\partial p'}{\partial x} - \frac{\epsilon}{\rho_o c_o^3} p' \frac{\partial p'}{\partial\tau} - \frac{b}{2\rho_o c_o^3}\frac{\partial^2 p'}{\partial\tau^2}\right) = = \frac{c_o}{2}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p'}{\partial r}\right) \tag{2}$$

where $p'=p-p_o$ is an acoustic pressure, $\tau=t-x/c_o$ - time in the coordinate system fixed in the zero phase of the propagating wave, ρ_o - medium density at rest, c_o - speed of sound, b - dissipation coefficient of the medium, ϵ - nonlinearity parameter.

The finite amplitude waves interaction problem is solved in fixed region and fixed time interval. The solution is look for inside a hypothetical cylinder with radius R_1, for $\tau\in[0,T_1]$ (Fig. 2).

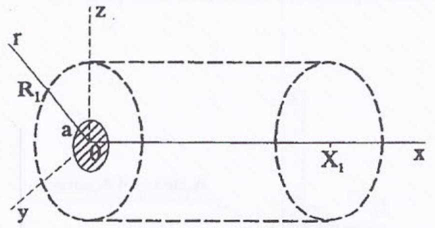

Fig. 2. Coordinate system and fixed region.

The pressure changes along the sound beam are obtained after computer calculations. To solve Eq. (2) numerically function $p'(x,r,\tau)$ is discretized in both space and time. The net is defined in following form:

$$x_n=n\Delta x,\quad r_k=k\Delta r,\quad \tau_m=m\Delta\tau$$
$$\Delta x=\frac{X_1}{N_x},\quad \Delta r=\frac{R_1}{N_r},\quad \Delta\tau=\frac{T_1}{N_\tau} \tag{3}$$

where n designate the *n*th step in the x direction, k the *k*th step in the r direction and m the *m*th step time. The finite-difference method was used to solve the problem numerically therefore the Eq. (2) and the boundary conditions were approximated by difference schemes.

3. Numerical investigations

The numerical investigations were made assuming that waves were propagated by circular piston with radius a=25 mm in nondissipative medium (b=0) where speed of sound c_o=1490 m/s, medium density ρ_o=998 kg/m^3. Moreover nonlinearity parameter ϵ=3.5.

The correct choose of numerical parameters (step sizes, size of space) is very important in numerical investigations [2].

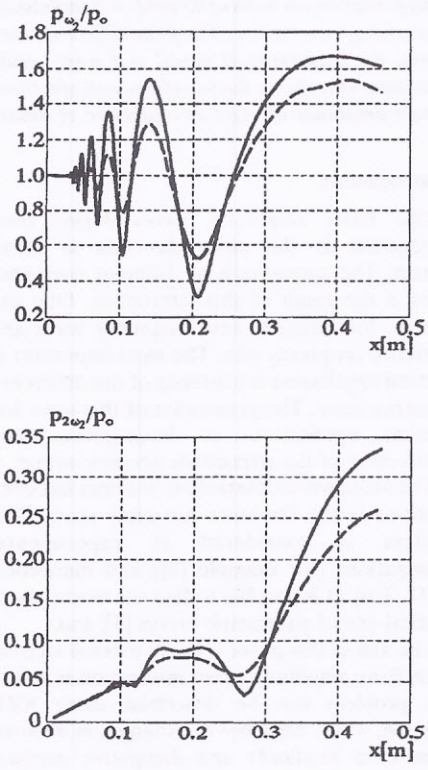

Fig. 3. Normalized on-axis pressure amplitude for f_2 and $2f_2$ frequency wave as a function of distance from the source for different values of step size Δx.

Figure 3 shows the on-axis pressure amplitude changes for the f_2 and $2f_2$ frequency wave

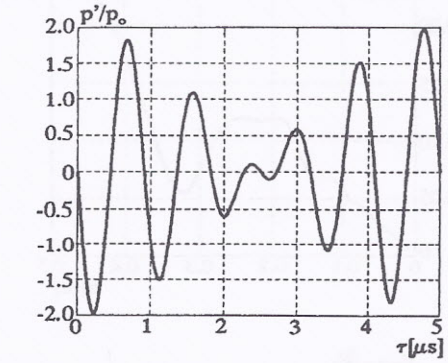

Fig. 4. The waveform as a function of time on the source.

respestively. In this example the distribution on the source $p'/p_o = -\sin\omega_1 - \sin\omega_2$, $f_1 = 1.2$ MHz, $f_2 = 1$ MHz, $p_o = 150$ kPa (Fig. 4). The pressure amplitudes were calculate for two different values of step sizes Δx. The dashed line presents the amplitude changes calculated when $\Delta x = 5.8 \cdot 10^{-5}$ m and the solid one presents the result for $\Delta x = 2.9 \cdot 10^{-5}$ m. Of course, lowering of step sizes make that the accuracy of numerical calculations increase.

The harmonic analysis is very often used to investigate wave distortion. Pressure changes along the sound beam (in fixed space points) are obtained during numerical calculations. The knowledge of these pressure changes allow to calculate the spectrum changes. To find spectrum of the wave obtained during numerical calculations the fast Fourier transform (FFT) is used.

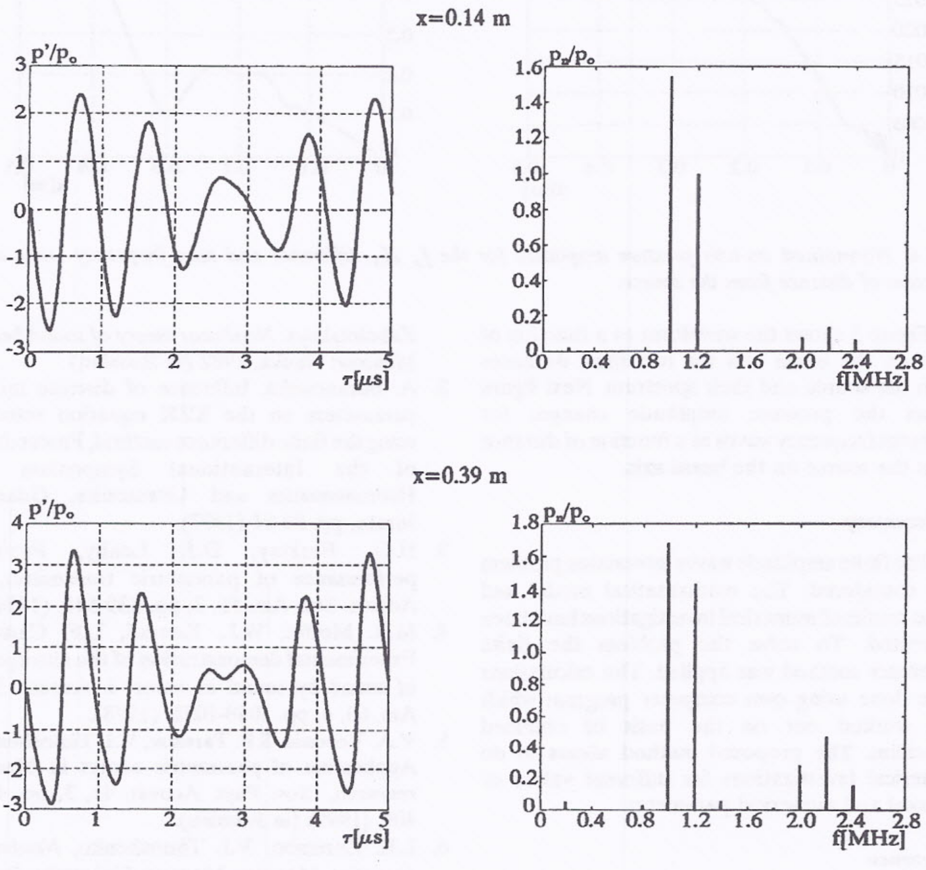

Fig. 5. The waveform as a function of time on the beam axis for fixed distances from the source and the spectrum corresponding to it.

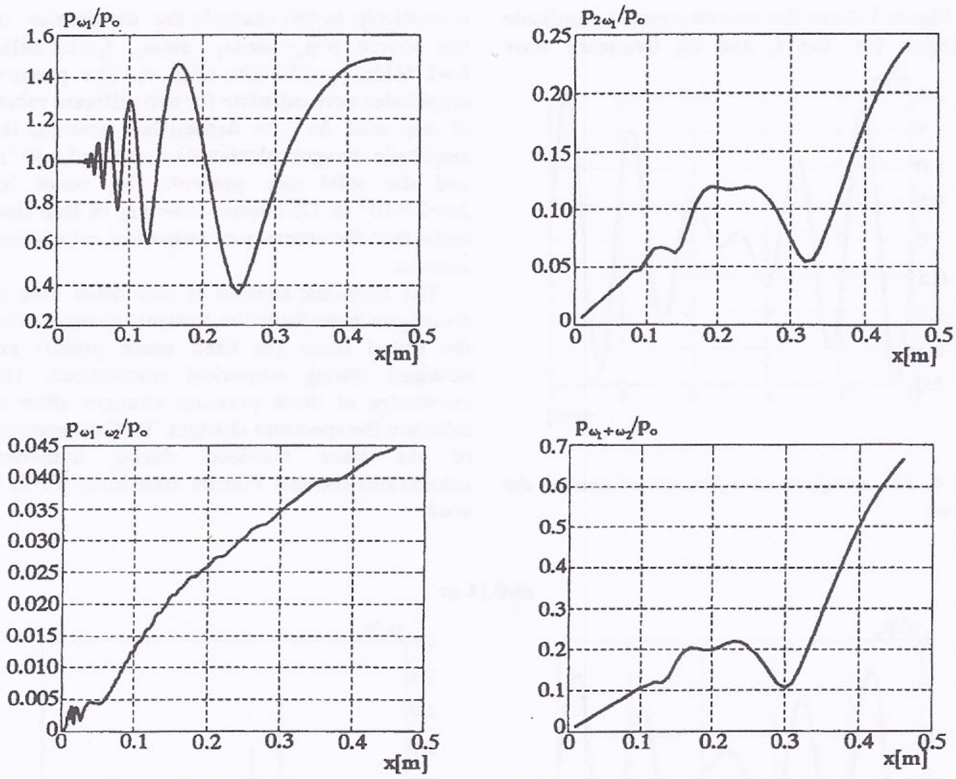

Fig. 6. Normalized on-axis pressure amplitude for the f_p, $2f_p$, difference and sum frequency wave as a function of distance from the source.

Figure 5 shows the waveform as a function of time on the beam axis for two fixed distances from the source and their spectrum. Next figure shows the pressure amplitude changes for different frequency waves as a function of distance from the source on the beam axis.

4. Summary

The finite amplitude waves interaction problem was considered. The mathematical model and some results of numerical investigations have been presented. To solve the problem the finite difference method was applied. The calculations were done using own computer program which was worked out on the basis of obtained algorithm. The proposed method allows to do numerical investigations for different values of physical and numerical parameter.

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