

# A planning model for the chemical integrated system under uncertainty by grey programming approach

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A model to optimize the planning of the chemical integrated system comprised by multi-devices and multi-products has been proposed in this paper. With the objective to make more profits, the traditional model for optimizing production planning has been proposed. The price of chemicals, the market demand, and the production capacity have been considered as mutative variables, then an improved model in which some parameters are not constant has been developed and a new method to solve the grey linear programming has been proposed. In the grey programming model, the value of credibility can be suggested by the decision-makers, and the results of the production planning calculated by the model can help them to achieve their desired target. An actual case has been studied by the proposed methodology, and the proposed methodology can be popularized to other cases.

**Keywords:** planning, chemical integrated system, grey programming.

## INTRODUCTION

Rational use of resources and energy, and environmental protection are two important issues in the 21<sup>st</sup> century, which results in the recognition of the Circular Economy (CE) by more and more people. Circular Economy emphasizes achieving the harmony of economic system and ecosystem with small resources and environmental costs and realizing the optimum economic benefits with high resource utilization efficiency and small environmental impact<sup>1-3</sup>. Sporadic chemical plants are not conducive to the CE concept due to their multiple disadvantages, i.e. high overall investment, high processing costs, small flexibility of management and production<sup>4</sup>, accordingly, integrated chemical projects have recently received more and more attention in China and some other countries because of their higher utilization of resources and energy as well as less emission of wastes than those of the sporadic plants<sup>5</sup>, consequently, the production planning of integrated systems has become a research hotspot<sup>6-9</sup>.

With the objective function of economic goal, a linear programming model was built to optimize production planning of the multi-refineries based on regional resources limitations<sup>10-11</sup>. A quantitative approach for designing the responsive supply chain under demand uncertainty was presented, and a multi-period mixed integer nonlinear programming (MINLP) model was developed for the bi-criterion optimization of economics and responsiveness<sup>12</sup>. Another MINLP model whose objective function expresses minimizing the total costs consisting of inventory and production costs was developed, and a branch-and-price algorithm for solving the optimization was proposed<sup>13</sup>. An integrated neuro-fuzzy and MILP approach was used to design supply network, the demand was forecasted by neuro-fuzzy method, and the optimal product flow between factories, warehouses and distributors are calculated<sup>14</sup>.

The proposed approaches by predecessors are too complicated to solve or too simple to acquire the accurate solution. The traditional linear programming model is

easy, but there are too many approximations and hypotheses in the model to acquire the accurate solution. The papers published before usually provide a model with many parameters, e.g. the price of the production, the price of the raw materials, the production capacity, the market demand etc. The assumption that these parameters are constants is not accurate because the price of the raw materials and the production capacity are mutative with time.

Grey programming has the ability to deal with poor, incomplete, or uncertainty problems of systems, and it has been widely used in many aspects such as economics, agriculture, medicine, geography, industry, etc<sup>15</sup>.

The “grey information” (parameter belongs to an interval) is used instead of “white information” (parameter is a constant). Grey programming is different from interval programming. The main difference of grey programming and interval programming is the concept of grey parameter and interval parameter. Grey parameter is a parameter which belongs to an interval, but interval parameter is a set in the form of an interval, Li and Liu have discussed the difference between grey numbers and interval numbers completely<sup>16</sup>.

Fuzzy programming has been applied to optimize the planning of the integrated systems in many studies, which mentioned that some variables are not constants<sup>17-22</sup>. Fuzzy programming can also deal with uncertainty problems, but membership functions are needed in fuzzy programming, meanwhile it can not reflect the interval which the mutative parameter belongs to directly. That is the reason why the grey programming has been selected for the planning of chemical integrated system instead of fuzzy programming in this paper.

In the grey programming, some variables are assigned intervals instead of precise values, in the problems such as planning of chemical integrated system, a designed planning under an uncertainty environment with some grey variables is of vital importance for the stakeholders

to make a right decision on the planning of chemical integrated systems.

In this paper, a grey programming model for the production planning of chemical integrated system is firstly established, and then a new method to solve the grey programming model is proposed, which is proved to be that the new way could solve the grey programming efficiently.

**MATHEMATIC MODEL**

With the purpose to make more profits, the objective function has been set by Eq.(1)

$$Profit = Sales\ income - material\ costs - processing\ costs - transportation\ costs \quad (1)$$

The profit contains four parts, sales income, materials costs, processing costs and transportation. The four terms can be calculated by Eq.(2), Eq.(3), Eq.(4), Eq(5), respectively.

$$Sales\ income = \sum_i P_i F_i \quad (2)$$

$$material\ costs = \sum_j P_j F_j \quad (3)$$

$$processing\ costs = \sum_k P_k F_k \quad (4)$$

$$transportation\ costs = \sum_p (C_p + S_{mn}^p k_{mn}^p) \quad (5)$$

It is apparent that Eq.(1) can be transformed into Eq.(6) .

$$max\ P = \sum_i P_i F_i - \sum_j P_j F_j - \sum_k P_k F_k - \sum_p (C_p + S_{mn}^p k_{mn}^p) \quad (6)$$

The constraints of the model have been shown as follows:

1. The total material balance

$$\sum_{i=1}^n \sum_{j=1}^m F_{i \rightarrow k}^j + \sum_u F_{in \rightarrow k}^u - \sum_{s=1}^q \sum_{t=1}^p F_{s \rightarrow k}^t - \sum_v F_{k \rightarrow out}^v = 0 \quad (7)$$

The total material balance involves four terms, the first term represents materials provided by other plants, the second term expresses materials provide by the outside market, the third term represents materials provided to other plants by this plant, and the fourth term denotes materials for sales.

2. The material source balance in a device

$$F_k^j = F_{l \rightarrow k}^j + F_{in \rightarrow k}^j \quad (8)$$

Eq.(8) indicates that one material in one special device is comprised from materials produced by the other devices and bought from the market.

3. The distribution of the purchased materials

$$F_{in}^j = \sum_k F_{in \rightarrow k}^j \quad (9)$$

This constraint represents that the purchased materials are assigned to several devices as raw materials.

4. The constraint of yield in each device

$$F_p^k = \left[ \min \left( \frac{F_k^j}{v_j} \right) \right] v_p \xi_k^j, \quad i = 1, 2, \dots, w \quad (10)$$

This constraint presented in Eq.(10) means that the way to calculate the output of each product in one de-

vice. The output should be calculated according to the raw material that is insufficient in the chemical reaction.

5. Production capacity of each device

$$\sum_{j=1}^w F_k^j \leq F_{k,upper} \quad (11)$$

The constraint (10) indicates that handling capacity of each device can not exceed its maximum production capacity.

6. Supply of raw material constraint

$$F_k^j \leq F_{in,j} + \sum_i F_{i \rightarrow k}^j \quad (12)$$

The supply of raw material constraint (see Eq.(12)) represents that the supply of one kind of raw material in one device can not outstrip the total supply provided by all the other devices and the market.

7. Market demand constraint

$$F_{k \rightarrow out}^p \leq F_p^{market\ demand} \quad (13)$$

This constraint (Eq.(13)) indicates that the production for sales can not exceed the demand of the market.

8. Distribution of product constraint

$$F_p = \sum_{s=1}^q F_{k \rightarrow s}^p + F_{k \rightarrow out}^p \quad (14)$$

Eq.(14) expresses how to distribute the product, part of the products are provided to other devices as raw materials, and the residual parts are for sales.

9. Nonnegative constraints

$$X \geq 0 \quad (15)$$

All the parameters represent their practical significance in the model, and it should be emphasized that all the parameters in the model are nonnegative.

**GREY PROGRAMMING MODEL**

Grey numbers represent the numbers that are not crisp values but incomplete information. The linear programming that concerns grey numbers has been called grey linear programming. The grey linear programming (GLP) proposed by Professor Deng Jvlong when he was doing research on uncertain system in 1980s, had been applied in many engineering fields because it can solve the uncertain decision problem efficiently<sup>23-29</sup>. Both the objective function and the constraints have different forms of uncertainty that behave in the form of grey information<sup>30</sup>. The grey model, with some of grey parameters (none constant), is more objective and precise than the traditional models<sup>31</sup> when maximizing (minimizing) the objective function under certain constraints.

The traditional linear programming (TLP) can not be used to solve the grey linear programming (GLP), because some of the parameters are not crisp values, therefore, developing a methodology for solving the GLP is of vital importance.

**Definition 1** Assuming  $x = (x_1, x_2, \dots, x_n)$  are decision variables,  $\otimes_C, \otimes_A, \otimes_a, \otimes_B, \otimes_b$  are all grey parameters sets,

$$\begin{aligned} &Max (Min) \otimes_C X \\ &st. \begin{cases} \otimes_A X \leq \otimes_a \\ \otimes_B X = \otimes_b \end{cases} \end{aligned} \quad (16)$$

The model shown in Eq.(16) is Grey Linear Programming.

In this paper, the price of the product, the price of the raw materials, the production capacity and the market demand have been recognized as grey numbers. Then, the model for the planning of chemical integrated system in grey programming methodology can be established, as shown in Eq.(17), Eq.(18), Eq.(19), Eq.(20), Eq.(21), Eq.(22), Eq.(23), Eq.(24), Eq.(25) and Eq.(26).

The objective function:

$$\max \otimes(P) = \sum_i \otimes(P_i)F_i - \sum_j \otimes(P_j)F_j - \sum_k \otimes(P_k)F_k - \sum_p (C_p + \otimes(k_{mm}^p)S_{mm}^p) \quad (17)$$

The constraints of the model:

$$\sum_{i=1}^n \sum_{j=1}^m F_{i \rightarrow k}^j + \sum_u F_{in \rightarrow k}^u - \sum_{s=1}^q \sum_{t=1}^p F_{s \rightarrow k}^t - \sum_v F_{k \rightarrow out}^v = 0 \quad (18)$$

$$F_k^j = F_{l \rightarrow k}^j + F_{in \rightarrow k}^j \quad (19)$$

$$F_{in}^j = \sum_k F_{in \rightarrow k}^j \quad (20)$$

$$F_k^{production(p)} = \left[ \min \left( \frac{F_k^j}{v_j} \right) \right] v_p \xi_k^j, \quad i = 1, 2, \dots, w \quad (21)$$

$$\sum_{j=1}^w F_k^j \leq \otimes(F_{k,upper}) \quad (22)$$

$$F_k^j \leq F_{in,j} + \sum_i F_{i \rightarrow k}^j \quad (23)$$

$$F_{k \rightarrow out}^p \leq \otimes(F_p^{market\ demand}) \quad (24)$$

$$F_p = \sum_{s=1}^q F_{k \rightarrow s}^p + F_{k \rightarrow out}^p \quad (25)$$

$$X \geq 0 \quad (26)$$

**METHOD FOR SOLVING GREY PROGRAMMING**

The usual approach to solving grey linear programming is to transform the grey linear programming into traditional linear programming. In order to solve the grey linear programming accurately, a methodology for solving the grey linear programming problem as shown in Eq.(27), is urgently needed.

$$f(X) = \otimes_c X$$

$$s.t. \begin{cases} AX \leq \otimes_a \\ BX = b \\ X \geq 0 \end{cases} \quad (27)$$

$\otimes_a$  and  $\otimes_c$  are grey vector, assuming  $\otimes_c^i = [\bar{\otimes}_c^i, \hat{\otimes}_c^i]$ ,

$\otimes_a^i = [\bar{\otimes}_a^i, \hat{\otimes}_a^i]$  and the “white” value can be calculated by the drift relations of grey coefficient, as shown in Eq.(28) and Eq.(29).

$$\otimes_c^i = \bar{\otimes}_c^i + \alpha_c (\hat{\otimes}_c^i - \bar{\otimes}_c^i) \quad (28)$$

$$\otimes_a^i = \bar{\otimes}_a^i + \alpha_a (\hat{\otimes}_a^i - \bar{\otimes}_a^i) \quad (29)$$

where  $\alpha_c \in [0,1]$ ,  $\alpha_a \in [0,1]$

It is apparent that if  $\alpha$  takes value between 0 and 1, the “white” values  $\otimes_c^i$ ,  $\otimes_a^i$  will fluctuate between the upper bound and the lower bound.

If  $\alpha_c = 1$ ,  $\alpha_a = 1$ , the maximum value ( $f_{max}$ ) of the objective function can be obtained, namely, as shown in Eq.30.

$$f(X) = \hat{\otimes}_c X$$

$$s.t. \begin{cases} AX \leq \hat{\otimes}_a \\ BX = b \\ X \geq 0 \end{cases} \quad (30)$$

If  $\alpha_c = 0$ ,  $\alpha_a = 0$ , the minimum value ( $f_{min}$ ) of the objective function can be obtained, namely, as shown in Eq.31.

$$f(X) = \bar{\otimes}_c X$$

$$s.t. \begin{cases} AX \leq \bar{\otimes}_a \\ BX = b \\ X \geq 0 \end{cases} \quad (31)$$

It is apparent that the value of  $\otimes_c^i$ ,  $\otimes_a^i$  may not reach the lower bound or the upper bound simultaneously, in other words,  $\alpha_c$ ,  $\alpha_a$  can't take value of 0 or 1 simultaneously. When  $\alpha_c$ ,  $\alpha_a$  are two arbitrary values in the interval [0,1], the optimal value of the objective function is assumed to be  $f_a$ , therefore

$$f_{min} < f_a < f_{max} \quad (32)$$

**Definition 2.** Credibility ( $\mu_a$ ) is defined in Eq.(33)

$$\mu_a = \frac{f_a - f_{min}}{f_{max} - f_{min}} \quad (0 \leq \mu_a \leq 1) \quad (33)$$

The credibility reflects the desired target and satisfaction of the decision-makers. If the credibility is known, the desired target can be also calculated. The objective of maximizing the grey linear programming (see Eq.27) is to find out the proper decision variables  $X$  to fulfill the desired target  $f_a$ . The process of maximizing the grey linear programming (see Eq.27) can be transformed to traditional linear programming, as shown in Eq.34.

$$Min \sum_i \alpha_c^i + \sum_j \alpha_a^j$$

$$s.t. \begin{cases} f_a - [\bar{\otimes}_c + \alpha_c (\hat{\otimes}_c - \bar{\otimes}_c)] X \leq 0 \\ AX \leq [\bar{\otimes}_a + \alpha_a (\hat{\otimes}_a - \bar{\otimes}_a)] \\ BX = b \\ X \geq 0 \end{cases} \quad (34)$$

The objective function and constraints indicate their special meanings respectively. The first constraint expresses that the values of  $\alpha_c$ ,  $\alpha_a$ , and the decision variables  $X$  can make the value of the objective function reach the desired target. The meanings of other constraints are the same as the constraints in traditional linear programming. The objective function is to minimize the resource costs as much as possible.

This method can be popularized to solve all the grey linear programming. The grey linear programming (see Eq.16) can be transformed to traditional linear programming, as shown in Eq.35.

$$\begin{aligned}
 & \text{Min} \sum \alpha_c + \sum \alpha_a + \sum \alpha_a + \sum \alpha_b + \sum \alpha_b \\
 & \begin{cases} f_a - (\bar{\alpha}_c + \alpha_c (\hat{\alpha}_c - \bar{\alpha}_c)) X \leq 0, \text{if maximize the objective function} \\ f_a - (\bar{\alpha}_c + \alpha_c (\hat{\alpha}_c - \bar{\alpha}_c)) X \geq 0, \text{if minimize the objective function} \end{cases} \\
 \text{st.} & \begin{cases} (\bar{\alpha}_a + \alpha_a (\hat{\alpha}_a - \bar{\alpha}_a)) X \leq (\bar{\alpha}_a + \alpha_a (\hat{\alpha}_a - \bar{\alpha}_a)) \\ (\bar{\alpha}_b + \alpha_b (\hat{\alpha}_b - \bar{\alpha}_b)) X = (\bar{\alpha}_b + \alpha_b (\hat{\alpha}_b - \bar{\alpha}_b)) \end{cases} \quad (35)
 \end{aligned}$$

$$\begin{aligned}
 & \text{Max } \otimes_C X \\
 \text{st.} & \begin{cases} \otimes_A X \leq \otimes_a \\ \otimes_B X = \otimes_b \end{cases} \quad (36)
 \end{aligned}$$

When  $\alpha_c, \alpha_a$  and  $\alpha_b$  equal 0,  $\alpha_A, \alpha_B$  equal 1, the worst maximum  $f_{min}$  can be obtained, and when  $\alpha_c, \alpha_a$  and  $\alpha_b$  equal 1,  $\alpha_A, \alpha_B$  equal 0, the best maximum  $f_{max}$  can be obtained in the process of maximizing grey linear programming, as shown in Eq.36.

The procedure for maximizing grey linear programming model has been shown as follows,

**Step 1:** Finding out the worst maximum  $f_{min}$  and the best maximum  $f_{max}$ .

**Step 2:** Making a decision on credibility  $\mu_\alpha$ , calculating the desired target  $f_a$ .

**Step 3:** Transforming the grey linear programming into traditional linear programming

**Step 4:** Solving the traditional linear programming.

In order to verify the proposed methodology for solving the grey linear programming, an example from reference<sup>32</sup> has been studied by the proposed methodology.

**Example 1.** The grey linear programming

Grey objective function (see Eq.(37)).

$$\max f = \otimes_1 x_1 + \otimes_2 x_2 \quad (37)$$

Grey constraints

$$\begin{aligned}
 & \otimes_{11} x_1 + \otimes_{12} x_2 \leq 360 \\
 & 3x_1 + 10x_2 \leq 300 \\
 & 4x_1 + 5x_2 \leq 198 \\
 & x_1 \in [0,100], x_2 \in [0,100] \\
 & \otimes_1 \in [1,7], \otimes_2 \in [4,12] \\
 & \otimes_{11} \in [1,21], \otimes_2 \in [4,10]
 \end{aligned} \quad (38)$$

The grey linear programming can be transformed into the traditional linear programming, as shown in Eq.39 and Eq.40, respectively.

$$\min \alpha_1 + \alpha_2 + \alpha_{11} + \alpha_{12} \quad (39)$$

$$\begin{aligned}
 & f_a - (1+6\alpha_1)x_1 + (4+8\alpha_2)x_2 \leq 0 \\
 & (1+20\alpha_{11})x_1 + (4+6\alpha_{12})x_2 \leq 360 \\
 & 3x_1 + 10x_2 \leq 300 \\
 & 4x_1 + 5x_2 \leq 198 \\
 & x_1 \in [0,100], x_2 \in [0,100] \\
 & 0 \leq \alpha_1 \leq 1 \\
 & 0 \leq \alpha_2 \leq 1 \\
 & 0 \leq \alpha_{11} \leq 1 \\
 & 0 \leq \alpha_{12} \leq 1
 \end{aligned} \quad (40)$$

The worst and best optimums can be obtained using the simplex method.

$$f_{min} = 120.0000, f_{max} = 371.3333$$

The proposed credibility  $\mu_\alpha$  equals 0.7, then the desired target can be obtained  $f_a = 295.9333$ , and the computing results have been shown in Table 1.

**Table 1.** The computing results when  $\mu_\alpha = 0.7$

$x_1$	$x_2$	$\alpha_1$	$\alpha_2$	$\alpha_{11}$	$\alpha_{12}$
49.5000	0.0000	0.8297	0	0	0

In order to compare the developed methodology with the Genocop Algorithm, the results using the two methods have shown in Table 2 when the optimums (desired targets) have been fixed as  $f_a = 305.8$ .

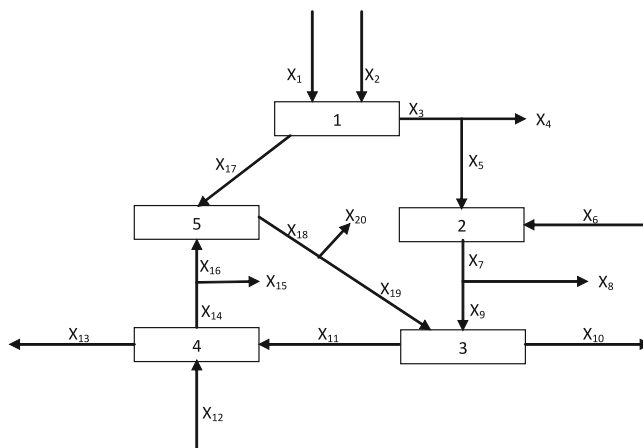
It is apparent that only the first uptake coefficient  $\alpha_1$  is a little bigger than the coefficient calculated by Genocop Algorithm, and all the other uptake coefficients are smaller than that calculated by Genocop Algorithm. It indicates that the proposed method is better than Genocop Algorithm, because the smaller the uptake coefficient is, the more remnant resources there will be. The best method for programming is to achieve the aim with the least costs. Therefore, the developed methodology is a promising way to solve the grey programming.

### CASE STUDY

In order to illustrate the model, an illustrative case has been studied, and due to the secrecy of the company, the specific information about this case has been hidden in this paper. The network of this chemical integrated system has been shown in Fig. 1 and the main information of the chemical integrated system has been shown as follows:

1. Raw materials from the market: X1, X2, X6, X12.
2. Products for sales completely: X10, X13.
3. Products partly for sales and partly for internal-used: X3, X7, X14, X18.
4. Products for internal-used completely: X11, X17.
5. Relationship between the main stream and the sub-streams:

$$X3 = X4 + X5,$$



**Figure 1.** The flow diagram of the integrated system

**Table 2.** Comparison of the results calculated by the proposed method and Genocop Algorithm

The proposed method							Genocop Algorithm		
$f_a$	$x_1$	$x_2$	$\alpha_1$	$\alpha_2$	$\alpha_{11}$	$\alpha_{12}$	$x_1$	$x_2$	$\alpha_i (i = 1,2,3,4)$
305.8	49.5	0	0.86	0	0	0	7.70	27.69	0.7

$$\begin{aligned} X7 &= X8+X9, \\ X14 &= X15+X16, \\ X18 &= X19+X20. \end{aligned}$$

The price of the product, the price of the raw materials, the market demand of the products, the base costs and unit costs of transportation between the devices, the unit processing costs, and the stoichiometric ratios of the materials in each device have been shown in Table 3, Table 4, Table 5, Table 6, Table 7 and Table 8, respectively.

The grey model for production planning is used to solve the problem. The best maximal profit  $f_{\max} = 3011.133 \times 10^4$  ¥ (Yuan) and the worst maximal profit  $f_{\min} = 1219.232 \times 10^4$  ¥ (Yuan). According to the experience of the decision-makers, they propose credibility  $\mu_\alpha = 0.6678$ , so the decision-makers will be satisfactory to the desired target  $f_\alpha = 2300 \times 10^4$  ¥ (Yuan). The computing results are shown in Table 9.

The optimal uptake coefficients reflect their influences on optimizing the profit, the bigger the uptake coefficient is, the more effects it takes on the optimal profit. The results indicate that the price of X10, the price of X13, the price of X12 and the supply of X12 are the main factors which are the most sensitive, influence the profit of the whole system. It is apparent that, if the price of X10, and the price of X13 are higher, the price of X12 is lower, and the supply of X12 is more adequate, more

profits the integrated system will make. The decision-makers can draw up an appropriate production plan according to the results calculated by the grey model.

The relationship between the profit and the credibility has been shown in Fig. 2. The maximum profit of the integrated system changes with the credibility which is proposed by the decision-makers, obviously, if the value of the credibility is bigger, more profits there will be

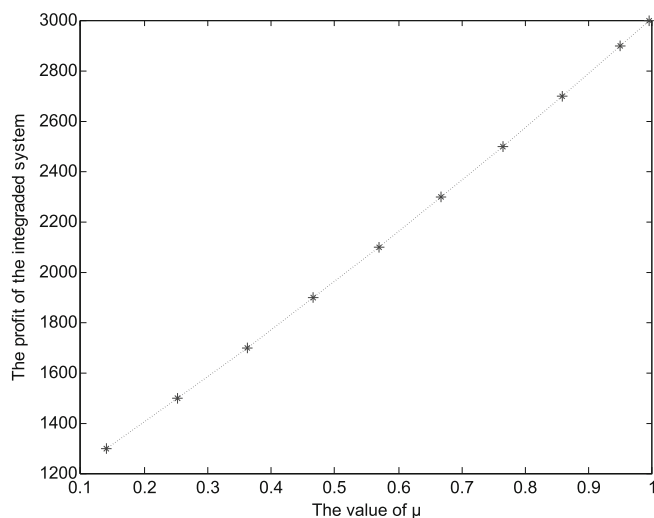


Figure 2. The maximum profit of the integrated system with the proposed credibility

Table 3. Price ranges of each product

Material	Unit	X3	X7	X10	X13	X15	X20
Price range	$10^4$ Yuan·t <sup>-1</sup>	19.5~20	11.7~12	7~7.5	14.9~16	21.8~22	8.7~9

Yuan is the monetary unit of The People's Republic of China

Table 4. Price ranges of each raw material

Material	Unit	X1	X2	X6	X12
Price range	$10^4$ Yuan·t <sup>-1</sup>	12~12.1	6~6.3	7~7.15	11~11.5

Table 5. Market demand or maximum supply of some chemicals

Chemical	Unit	X1	X2	X6	X8	X10
Demand	$10^3$ ton	1.3~1.5	0.7~0.8	1.3~1.5	0.7~0.8	1.4~1.5
Chemical	Unit	X12	X13	X15	X20	
Demand	$10^3$ ton	1.4~1.5	0.9~1.0	0.9~1.0	0.7~0.75	

Table 6. The base costs and unit costs of transportation between some devices

Path	Unit	1, 2	2, 3	3, 4	4, 5	1, 5	5, 3
Base cost	$10^4$ Yuan	200	80	75	150	40	90
Unit cost	$10^4$ Yuan·t <sup>-1</sup>	1.5	1.2	0.8	2.4	1.6	1.0

Table 7. The unit processing costs

Device	Unit	1	2	3	4	5
Unit processing costs	$10^4$ Yuan·t <sup>-1</sup>	1.5	2.2	0.8	2.0	3

Table 8. The ratio of processing materials in each device

Ratio of	X1:X2	X5:X6:X7	X9:X19:X10:X11	X11:X12:X13:X14	X16:X17:X18
Ratio	1:2	1:1:2	1:2:0.75:2.25	1:1:0.8:1.2	1:0.5:1.5

Table 9. Results of production for the case

X1	X2	X3	X4	X5	X6	X7
177.78	355.56	320.00	160.00	160.00	320.00	0.00
X8	X9	X10	X11	X12	X13	X14
0.00	320.00	240.00	720.00	720.00	576.00	864.00
X15	X16	X17	X18	X19	$\alpha_3$	$\alpha_4$
437.33	426.67	213.33	640.00	640.00	0.0665	1.0000
$\alpha_{10}$	$\alpha_{16}$	$\alpha_i (i = 1, 2, 5, \dots, 9, 11, \dots, 15, 17, \dots, 29)$				
1.0000	1.0000	0				

theoretically, but the risk also increases, because of the ever-changing market. The production planning which is solved by the proposed model acts as the most stable and safest way to satisfy the desired target of the decision-makers, because the prices of some products are limited at the lower bound, the maximum profit is achieved in a way which may make the actual profit bigger than the desired target with the designed production plan.

In order to fit the philosophy “don't put all the eggs into one basket”, the method to optimize the production planning by choosing the credibility is better for the decision-makers to make a strategic decision correctly and wisely. Grey programming model for the planning of chemical integrated system has the ability to deal the planning problems with uncertainty and incomplete information, it takes into account the ever-change market and some other uncertainty factors sufficiently comparing to the traditional method.

## CONCLUSIONS

This paper presents a planning model based on grey programming for the integrated chemical system under uncertainties. The price of chemicals, the market demand and the production capacity were considered as constants in the traditional model, which can not fulfill the fact such as ever-change market, therefore, a grey programming model has been developed, in which some variables are considered as non-constants. For solving the grey model accurately and conveniently, a new method for solving grey programming has been proposed.

The proposed method is an object-oriented planning model, and it can help the decision-makers to achieve their desired target. The desired target which reflects expectations of the decision-makers can be set by them according to the best and worst maximum profit, then, the grey model is used to plan the integrated system, the results can be used as references for the decision-makers to design and manage the chemical integrated system.

The developed model for the planning of the chemical integrated system in grey programming methodology can consider the uncertainty factors in the decision-making process. And the decision-makers/stakeholders are allowed to participate in the planning process by setting the desired target. The proposed methodology can provide a feasible solution for them to design a stable chemical integrated system, and it can be popularized to some other cases for the planning of the chemical integrated system.

Although the uncertainty problem has been solved by the proposed method, some further work also needs to be done in the future. The environmental and social issues have not been incorporated in the presented model. Therefore, the further work will focus on developing a multi-objective model which considers not only the economic aspects but also the environmental and social issues for the planning of the integrated systems. Meanwhile, in order to help the decision-makers to design a steady and reliable plan under uncertainties, the exact parameters could also be extended to grey parameters, namely, a multi-objective grey model is imperative for the planning of a sustainable integrated system.

## Nomenclature

$C_p$	unit cost for the transportation of chemical p
$F_i$	yield of product i
$F_j$	supply of raw material j
$F_k$	processing capacity of device k
$F_k^j$	input of raw material j in device k
$F_{i \rightarrow k}^j$	supply of chemical j from device i to device k
$F_{in \rightarrow k}^u$	supply of chemical u purchased from the market
$F_{k \rightarrow out}^v$	chemical v for sales in devices k
$F_{in}^j$	the total of chemical j purchased from the market
$F_{k,upper}$	the upper bound of processing capacity in device k
$\mu$	credibility
$F_p^{market\ demand}$	demand of chemical p in the market
$k_{mn}^p$	unit costs for the transportation of chemical p from device m to device n
$P_i$	price of product i
$P_j$	price of raw material j
$P_k$	unit processing costs in device k
$S_{mn}^p$	supply of chemical p from device m to device n
$v_j$	stoichiometric number of reactant j
$v_p$	stoichiometric number of reactant p
$\alpha$	uptake coefficient
$\otimes$	grey information

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