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OPTIMIZATION METHOD OF BEVEL GEAR RELIABILITY BASED ON GENETIC ALGORITHM AND DISCRETE ELEMENT

METODA OPTIMALIZACJI NIEZAWODNOŚCI PRZEKŁADNI STOŻKOWEJ Z ZASTOSOWANIEM ALGORYTMU GENETYCZNEGO I ELEMENTÓW DYSKRETYCH

Gear transmission is the most basic transmission component in mechanical transmission system. Many scholars have done a lot of research on gear reliability. When the variation coefficient is used to calculate and optimize the reliability of bevel gear, in order to calculate the reliability of bevel gear, it is often assumed that the gear works under constant torque, that is, the coefficient of variation (COV) is zero, but this is not the case in practice. In this paper, a gear reliability method based on discrete element simulation is proposed. The purpose of this method is to simulate the actual working conditions of gears, calculate more accurate coefficient of variation in the real world, and improve the accuracy of gear reliability design. Firstly, the real working conditions of the bevel gear transmission are simulated by discrete element method (DEM), and in the transmission system, the tangential force COV of the bevel gear is proved to be equal to the torque COV of the crusher central shaft. Secondly, the multi-objective function model of the gear transmission system is established based on the double tooth roll crusher (DTRC). The optimal volume and reliability of the bevel gear transmission are taken as the objective function, and the teeth number, module and face width factor of basic parameters of gear are optimized by genetic algorithm (GA). Finally, the accuracy of the optimization results is verified by Monte Carlo method. The main purpose of the manuscript is to analyse the effect of actual conditions (DEM simulation) on the optimization results. The results show that the COV of nominal tangential load of bevel gear is about 0.65 under actual working conditions, so in order to guarantee the same reliability, total volume need to be increased by 34.4%. This method is similar to the selection of gear safety factor. In practical production, the selection of safety factor is often based on experience. This paper provides a new method to optimize the reliability of bevel gear, combining with DEM simulation, which provides theoretical guidance for optimal design of bevel gear.

Keywords: *bevel gear, reliability, discrete element method, monte carlo simulation, double tooth roll crusher.*

Przekładnia zębata to podstawowy element mechanicznego układu napędowego. niezawodność przekładni stożkowej jest przedmiotem wielu badań. Przy obliczeniach i optymalizacji niezawodności przekładni stożkowej z wykorzystaniem współczynnika zmienności, często przyjmuje się, że przekładnia pracuje w warunkach stałego momentu obrotowego, t.j. że współczynnik zmienności wynosi 1. Sytuacja taka jednak nie występuje w praktyce. W niniejszej pracy zaproponowano metodę optymalizacji niezawodności przekładni opartą na symulacji metodą elementów dyskretnych. Celem tej metody jest zasymulowanie rzeczywistych warunków pracy przekładni, dokładniejsze obliczenie rzeczywistego współczynnika zmienności oraz poprawa dokładności projektowania niezawodności przekładni. W pierwszej kolejności, na przykładzie kruszarki podwójnej, wyznaczono model działania układu przekładni stożkowej wykorzystujący wielokryterialną funkcję celu. Optymalną objętość i niezawodność przekładni stożkowej przyjęto jako funkcje celu. Następnie, za pomocą metody elementów dyskretnych, symulowano rzeczywiste warunki pracy przekładni. Wyznaczono moment obrotowy przekładni stożkowej i współczynnik zmienności siły wypadkowej, a podstawowe parametry koła zębatego: liczbę zębów, moduł zęba i współczynnik szerokości zębów, zoptymalizowano za pomocą algorytmu genetycznego. Trafność wyników optymalizacji weryfikowano metodą Monte Carlo. Wyniki pokazują, że badana metoda może skutecznie poprawiać niezawodność przekładni stożkowej.

Słowa kluczowe: *przekładnia stożkowa, niezawodność, metoda elementów dyskretnych, symulacja Monte Carlo, kruszarka podwójna.*

1. Introduction

Gear transmission is one of the most common transmission in mechanical systems, and is widely used in various precision mechanical transmission components, such as machine tools, vehicles, etc. In mechanical systems, the basic requirement of gear mechanism is accurate and smooth transmission of motion, considering the requirements of manufacturing costs and convenience, in the premise of ensuring the requirements of gear transmission. The optimal design of the volume or weight of the wheel drive system has become an important subject for many scholars [2, 3, 4, 7, 42, 43].

With the rapid development of computer technology and various optimization algorithms, there have been a lot of research results on

the optimal design of gear system. The methods and objects of optimal design have been continuously enriched [22, 38, 49]. Mendi [25] used GA to optimize the module of spur gear and then the optimal size of gear box shaft and rolling bearing is obtained. Savsani [34] used the particle swarm optimization and simulated annealing algorithm to optimize the design of multistage spur gear. Sa'id Golabi [13] established the objective function and constraint conditions of the optimum design of the volume / weight of the gearbox, compiled the optimization program using MATLAB, and verified the practicability of the design result by comparison with the known gearbox.

Nomenclature

Z_M	midpoint factor	$Y_{\delta_{relT}}$	relative notch sensitivity factor
Z_H	node region factor	Y_{RrelT}	relative surface factor
Z_E	elasticity factor $\left(\sqrt{N/mm^2}\right)$	Y_X	size factor
Z_{LS}	stress correction factor	σ_{Hlim}	experimental surface contact fatigue strength (N/mm ²)
Z_β	helix angle factor	σ_{HP}	allowable contact stress (N/mm ²)
Z_K	bevel gear factor	Z_{NT}	life factor
K	calculating factor	Z_L	lubricant factor
K_A	application factor	Z_V	velocity factor
K_V	dynamic factor	Z_R	surface factor
$K_{H\beta}$	face load factor	Z_W	work harden factor
$K_{H\alpha}$	transverse load factor	Z_X	size factor
F_t	nominal tangential load (N)	m_{nm}	normal module (mm)
d_m	reference diameter (mm)	$K_{F\beta}$	face load factor
l_{bm}	length of contact line in the middle of gear (mm)	$K_{F\alpha}$	transverse load factor
μ	gear ratio	Y_{FS}	form factor
σ_{Flim}	nominal stress number from reference test gears (N/mm ²)	Y_ϵ	rim thickness factor
σ_{FP}	allowable bending stress (N/mm ²)	Y_K	bevel gear factor
Y_{NT}	life factor	Y_{LS}	stress correction factor
		b	face width (mm)

With the rapid development of science and technology, people have higher requirements for various machines, products and parts. For gear transmission, it is necessary not only to have enough precision and strength, but also to ensure that it can complete its specified function, namely reliability design under the specified working conditions and time [18, 45, 46]. In recent years, scholars have conducted in-depth research on reliability design, and achieved considerable results. Savage [35] established the reliability model of planetary gear system based on Weibull distribution of each unit. Thompson [41] presented a generalized optimal design formula with multi-objective, which can calculate the fatigue strength of multi-stage gear. Li. [21] established the relationship between the wind turbine system and the factors and mechanisms affecting the failure based on the global tree, the success tree and the main logic diagram. Zhang [47] established the optimization design of reliability of large ball mill gear transmission based on the Bayesian analysis algorithm of Kriging model and verified the reliability calculation results by Monte Carlo method. Huang [15] proposed a method to determine the accuracy and reliability of gear motion based on truncated random variables, and discussed the practical application value of the model in detail. Gallego-Calderon and Nejad [11, 30] studied the reliability of gear transmission system in wind turbine. Zhou [48] analysed the dynamic reliability of planetary gear drive system of shearer and verified the results by Monte Carlo simulation method.

In recent decades, a large number of studies have been carried out on the failure probability of structural systems [8, 10, 23, 31, 36,

39]. Many methods have been developed to identify major failure modes, such as (A) “probabilistic” methods, including branch and bound methods [16, 19, 29, 40] and simulation-based techniques [9, 12, 24, 27, 32]; (B) “deterministic” methods, such as incremental loading method [20, 26, 28] b-unzipper method [31], mathematical programming based method [5] or heuristic technique [37, 44]. Kim [17] proposed an effective method to identify dominant failure modes in random variable space, and then analysed the system reliability and calculated the failure probability of the system, and identified the dominant failure mode in the decreasing order of its contribution to the system failure probability. Savage [33] presented a reliability model for single input pinion or equal size double input pinion reducer. Hao [14] mainly studied the influence of different failure modes of bevel gear transmission under incomplete probabilistic information on the modeling of interdependent structures.

In this paper, the probability method is used to calculate the gear reliability model based on the COV, and a new method is proposed to optimize the bevel gear parameters in combination with DEM and GA to improve the gear reliability effectively. This paper is mainly divided into three parts. In the Section 1, the bevel gear working condition is introduced, and the reliability model of single-stage transmission bevel gear is built based on reliability theory. In the Section 2, the model of bevel gear reliability is optimized by GA, and the optimization results is verified by the Monte Carlo method. The Section 3 is a summary of the full text.

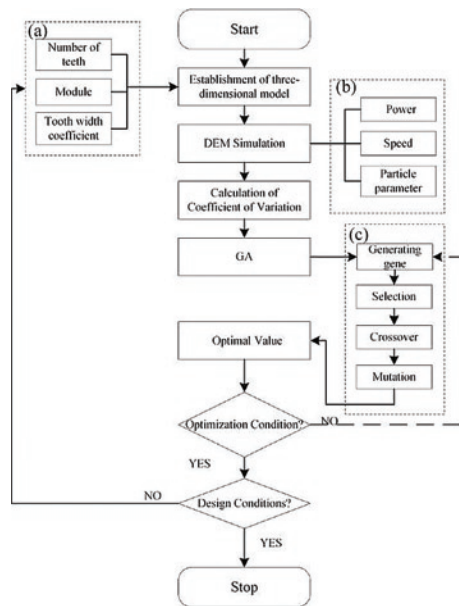


Fig. 1. Analysis flow chart

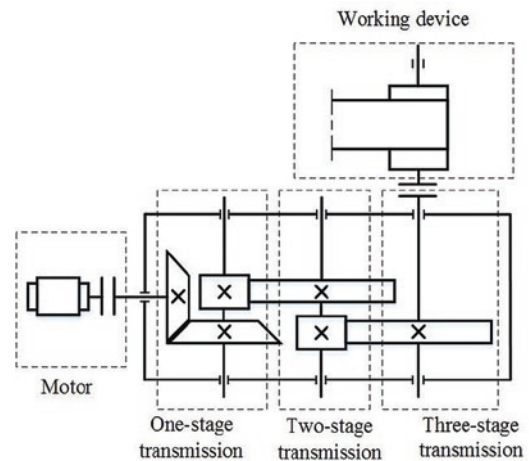


Fig. 2. Reducer of DTRC structure

Table 1. Material parameters in DEM

Material	Steel	Grit stone
Density (kg / m ³)	7850	2500
Poisson ratio	0.3	0.28
Modulus of shearing (Pa)	7.9×10 ¹⁰	2.7×10 ⁸
Recovery coefficient	0.5	0.2
Static friction coefficient	0.3	0.4
Kinetic friction coefficient	0.01	0.01

The discrete element model of DTRC mainly includes: material condensation model, prototype operation model and particle factory, material parameter setting Table 1.

The equivalent form of simulated particles in DEM is usually irregular particles, as shown in Figure 3 (a). The theoretical model of particles is assumed to be spherical in order to facilitate the establishment of equivalent model. The radius of equivalent model is 300mm, which is formed by the condensation of small particles (as shown in the Figure 3 (b)) with radius 40mm. The condensation radius is set to 40.5 mm.

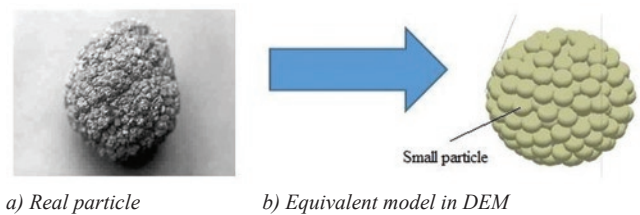


Fig. 3. Particle model transformation

The optimal design of the bevel gear transmission system could be carried out according to the established optimization model based on DEM. The whole process of the method is shown in Figure 1. Firstly the 3D model of the DTRC is established according to the actual size, as shown in Figure 1 (a); Then the whole model is imported into EDM simulation in Figure 1 (b) according to the actual conditions of crusher (Table 2), and the torque COV of the crusher shaft torque is calculated in order to deduce the torque COV of bevel gear; Finally, based on GA, the optimization design of bevel gear is carried out in Figure 1 (c).

2. Modeling of bevel gear reliability system

2.1. Gear working condition analysis

In this paper, the double tooth roll crusher (2PGC1040×3610) is taken as an example to simulate and calculate the torque change of the central shaft of the DTRC under a certain working condition with the discrete element method (DEM). Through the mean and variance of the central shaft torque, the torque COV of the bevel gear in the transmission process is deduced.

In the reducer of the DTRC, torque change of the DTRC central shaft under a certain working condition directly affect the force situation of the bevel gear in the transmission system. According to the load of transmission system, as shown in Figure 2, a series transmission in this paper is adopted, so the load of the crusher central shaft is transferred to the bevel gear in a certain proportion. Through calculation of COV, the torque COV of the bevel gear is equal to the torque COV of the crusher central shaft and meanwhile is equal to the nominal tangential load (F_t) COV of the bevel gear.

The basic theory of DEM is that the discontinuities of the system are discretized into elements, and each element can interact with each other and satisfy Newton's law of motion when the equations of motion of the system are satisfied. The static or dynamic relaxation iteration method is used to solve the force and motion of each unit in each time step, and then the macroscopic motion law of the whole system is obtained. Based on discrete element, the flow model of material particles in machine can be established, and the information of particle velocity, flow rate, force and wear with machine can be obtained [1,6].

Table 2. Working parameters of bevel gears

Name	Value	Name	Value
Power of motor(P)	710kW	Life expectancy(hours)	8.76×10 ⁴
Motor speed(n)	175rad/s	Lubricant	Mine Gear Oil 100
Crossed axis angle	90°	Mode of production	Medium-duty
Accuracy of manufacture	6	Bevel gear ratio	3
Main gear	42CrMo(41~45HRC) Tempered	Pinion	20CrMnMo(56~62HRC) Tempered

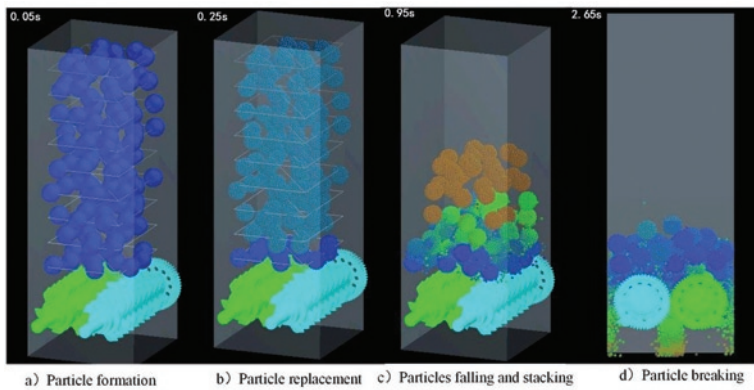


Fig. 4. Simulation process in DEM

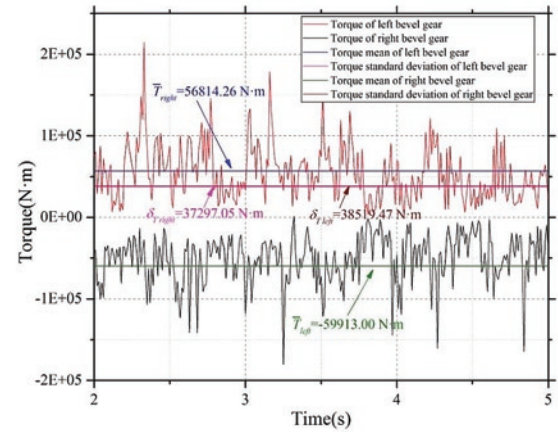


Fig. 5. Torque output curve of DTRC in DEM

Table 2 shows the working parameters of the bevel gear deceleration part of the DTRC. The rotational speed of the motor is 175 rad/s, after deceleration of the reducer (Figure 2), the rotational speed is

Table 3. Coefficient of variation of parameters in bevel gear transmission system [14]

Variables	Mean	Std	COV	Unit	Variables	Mean	Std	COV	Unit
F_t	5.85×10^4	3.8×10^4	0.6495726	N·m	Z_X	1	0.033	0.033	-
K_A	1	0.033	0.033	-	Z_L	0.9658	0.0318714	0.033	-
K_V	1.034147	0.034127	0.033	-	Z_V	0.968213	0.031951	0.033	-
$K_{H\beta}$	1.65	0.05445	0.033	-	Z_R	0.936408	0.030901	0.033	-
$K_{F\beta}$	1.65	0.05445	0.033	-	Z_W	1	0.033	0.033	-
$K_{H\alpha}$	1	0.033	0.033	-	Y_{Fa1}	2.243285	0.074028	0.033	-
$K_{F\alpha}$	1	0.033	0.033	-	Y_{Fa2}	2.249664	0.074238	0.033	-
l_{bm}	27.138	0.13569	0.005	mm	Y_{sa1}	1.880157	0.06204	0.033	-
b	28	0.14	0.005	mm	Y_{sa2}	1.889086	0.06234	0.033	-
d_{V1}	50.886	0.25433	0.005	mm	Y_e	0.717334	0.0035867	0.005	-
m_{mn}	2.54827	0.01274	0.005	mm	Y_K	1.000244	0.005	0.005	-
Z_{M-B}	0.9874	0.00494	0.005	-	σ_{Flim}	380	76	0.2	N / mm ²
Z_H	2.49457	0.01247	0.005	-	Y_{ST}	2.0	0.066	0.033	-
Z_E	189.8117	9.4905	0.05	(N / mm ²) ^{1/2}	Y_{NT1}	0.912	0.030096	0.033	-
F_t	5.85×10^4	3.8×10^4	0.6495726	N·m	Y_{NT2}	0.933	0.0300789	0.033	-
Z_{LS}	1	0.033	0.033	-	Y_{relT1}	1.004782	0.033158	0.033	-
Z_β	1	0.033	0.033	-	Y_{relT2}	1.003269	0.033108	0.033	-
Z_K	0.8	0.004	0.005	-	Y_{RelT1}	1.024202	0.033809	0.033	-
σ_{Hlim}	1370	164.4	0.12	N / mm ²	Y_{RelT2}	1.024202	0.033809	0.033	-
Z_{NT}	1.07	0.03531	0.033	-	Y_{X1}	1	0.033	0.033	-

reduced to 2.77 rad/s, the total gear ratio is 63.18, and the input power of motor is 710 kW. The bevel gear works at the one-stage transmission with a gear ratio of 3, a rotational speed of 175 rad/s for the input shaft, and a rotational speed of 58.3 rad/s for the output shaft. The material and processing parameters are shown in the table.

Figure 4 shows the change of particle position at different time in DEM simulation. The working speed of DTRC shaft is 2.77 rad/s. It can be seen that the particle does not contact the crusher when the simulation time reaches 1 s, so the torque of the crusher central shaft is zero. After 1 s, the crusher starts to work. The torque change curve of the central shaft under the steady working condition of 2-5s crusher is exported, as shown in Figure 5.

Figure 5 is torque output curve of DTRC in DEM, the output torque of the left and right tooth rollers can be seen to be basically equal. The load distribution information is extracted and the mean value and standard deviation are calculated as follows:

$$\begin{cases} \bar{T}_e = \frac{\bar{T}_{right} + \bar{T}_{left}}{2} = 58363.63N \cdot m \\ \delta T_e = \frac{\delta T_{right} + \delta T_{left}}{2} = 37908.26N \cdot m \end{cases} \quad (1)$$

For accuracy of calculation, the mean and standard deviation (Std) are taken as the mean values of the left and right axes.

$$COV(F_t) = COV(\bar{T}) = \frac{\delta T_e}{\bar{T}_e} \quad (2)$$

In Table 3, except that the force F_t is simulated by DEM, the other gear parameters are assumed to follow the normal distribution [14], so the COV of the parameters is calculated, and the reliability calculation process is shown in the appendix of this paper.

2.2. Gear modeling analysis

The theory of system reliability bound, such as Cornell bounds [4] and Ditlevsen bounds [7], is often used to estimate the reliability of the system. It is also called the narrow reliability bound. By evaluating the joint failure probability of each pair of failure modes, a narrower estimate of the system failure probability is obtained. For a series system with m failure modes, a narrow bound of system reliability estimation is given:

$$P_{f_1} + \sum_{i=2}^m \max(P_{f_i} - \sum_{j=1}^{i-1} P_{f_{ij}}, 0) \leq P_f \leq \sum_{i=1}^m P_{f_i} - \sum_{i=2}^m \max(P_{f_{ij}}) \quad (3)$$

Where, P_{f_1} is the failure probability of the first failure mode and $P_{f_{ij}}$ is the joint failure probability of each pair of failure modes. Considering the convenience of the analysis, we only consider the primary bevel gear drive, which is a series system.

2.3. Determination of Objective Functions

According to the definition of robust optimal design involving reliability sensitivity, the robust design optimization model of bevel gear transmission system based on reliability is established:

$$\min f(X) = \sum_{k=1}^M w_k f_k(X) \quad (4)$$

$$s.t. R[g_i(X) > 0] \leq R_i, i = 1, \dots, N \quad (5)$$

$$X^L \leq X \leq X^U, X \in X^{ndv} \quad (6)$$

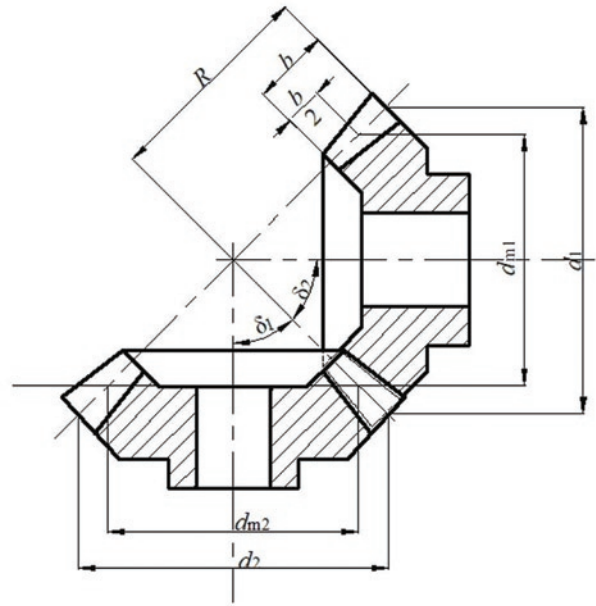


Fig. 6. Bevel gear transmission geometric parameters

Where, X is the design variables, $X = [m_{nm}, z_1, \varphi_R]^T$, X^L and X^U is the lower and upper bounds of the design variables, respectively. M , N and ndv represent the number of sub-objective functions, reliability functions and design variables, respectively. ω_k is the weight of each sub-function which shows the importance of each sub-function in calculating reliability. The formulas are as follows:

$$\omega_k = [f_1(X^{*k} - f_1(X^{*1}))] / \{ [f_1(X^{*k}) - f_1(X^{*1})] + [f_2(X^{*(k-1)}) - f_2(X^{*2})] + \dots + [f_k(X^{*1}) - f_k(X^{*k})] \} \quad (7)$$

where, the weighting factor should satisfy the following conditions:

$$\sum_{k=1}^n \omega_k = 1 \quad (\omega_k \geq 1) \quad (8)$$

The reliability function is the objective function f_1 , and the bevel gear transmission is series system. $R_1(X)$, $R_2(X)$, $R_3(X)$, $R_4(X)$ represent the reliability of contact strength of the gear tooth surface, bending strength of the gear tooth root, contact strength of pinion tooth surface and bending strength of pinion tooth root, respectively. In order to ensure the reliability of gear transmission, the structure of transmission will be destroyed if any gear is damaged. Therefore, the reliability objective function of bevel gear transmission can be designed as follows:

$$f_1(X) = f(X) = 1 - R_1(X)R_2(X)R_3(X)R_4(X) \quad (9)$$

where, calculation formula of $R_1(X)$, $R_2(X)$, $R_4(X)$ is in the appendix.

Meanwhile, the volume of bevel gear group is also the objective function f_2 , the formula is:

$$f_2(\mathbf{X}) = (f_p(\mathbf{X}) + f_g(\mathbf{X})) / V_{\max} \quad (10)$$

where, $f_p(\mathbf{X})$, $f_g(\mathbf{X})$ represent the volume of the pinion and the main gear, respectively.

$$f_p(\mathbf{X}) = \frac{\pi u d_1^2}{12 \times 10^9} \left(1 - \left(\frac{d_1 - b/u}{d_1} \right) \right) \quad (11)$$

$$f_g(\mathbf{X}) = \frac{\pi d_2^3}{12 \times u \times 10^9} \left(1 - \left(\frac{d_2 - bu}{d_2} \right) \right) \quad (12)$$

$$b = \varphi_R m_n \quad (13)$$

$$d_1 = z_1 m_n \quad (14)$$

$$d_2 = d_1 u \quad (15)$$

where, u is bevel gear ratio.

Therefore, the formula (1)-(13) is simultaneous, then the objective function is as follows:

$$\min f(\mathbf{X}) = \omega_1 f_1(\mathbf{X}) + \omega_2 f_2(\mathbf{X}) \quad (16)$$

$$f_1(\mathbf{X}) = 1 - R_1(\mathbf{X})R_2(\mathbf{X})R_3(\mathbf{X})R_4(\mathbf{X}) \quad (17)$$

$$f_2(\mathbf{X}) = \frac{\pi u (z_1 m_n)^2}{12 \times 10^9} \left(1 - \left(\frac{z_1 - \varphi_R / u}{z_1} \right) + u - u \left(\frac{z_1 - \varphi_R}{z_1} \right)^2 \right) / V_{\max} \quad (18)$$

$$\text{s.t. } R[g_i(\mathbf{X}) > 0] \leq R_i \quad (19)$$

$$(R_1 \cap R_2 \cap R_3 \cap R_4) \geq 0.9 \quad (20)$$

$$g_i(\mathbf{X}) \geq 0 \quad (21)$$

$$\begin{cases} 1 \leq m_e \leq 50 \\ z_{\min} \leq z_1 \leq 44 \\ 1/4 \leq \varphi_R \leq 1/3 \end{cases} \quad (22)$$

where, $i = 1, 2$, z_{\min} is the minimum number of pinion teeth, determined by the type of bevel gear in Table 4. $z_{\min} = 13 \sim 15$.

Table 4. Selection of gear types and minimum module

Type of Gear	Coniflex gear	Skew bevel gear	Zerol bevel gear	Gleason spiral bevel gear
Power range (kW)	<50	<500	<3700	>3700
Minimum teeth number	13	13	15	12

3. Case Analysis

3.1. Analysis of results

Figure 7-11 shows the optimization results of DEM and GA. From those figures, it can be seen that the optimization result is good. As shown in Figure 7, the optimization result of GA tends to be stable after 70 iterations by using the objective function of weight $\omega_1 = 0.3$ and $\omega_2 = 0.7$, that is, the optimal value may be reached. It can be seen that the total volume and failure rate of gears will affect the objective function, and the objective function decreases first and then rise. That indicates that the reliability of gear system has a great influence on the objective function before 25 generations, while after 25 generations, the total volume of gear system has great influence on the objective function.

Figure 8 shows number of teeth and module curve (DEM and GA) and Figure 9 shows face width factor curve (DEM and GA). Combined with Figure 7-9, GA is used to optimize bevel gear parameters (teeth number, module and face width factor), which change trend is approximate, finally after 80 iterations tend to balance. Variable approximation is $\mathbf{X}_{op} = (m_n, z_1, \varphi_R)^T \approx (11, 32.23, 0.264)^T$, optimal value $f(\mathbf{X})_{\min} = 0.5661$.

Figure 10 shows the failure rate and reliability curve (DEM and GA), after 30 iterations, the reliability gradually tends to 1, while Gear failure rate gradually approach 0. Figure 11 shows the change of contact stress on tooth surface and bending stress on tooth root during iteration. According to experience, the pinion is most easily worn out during transmission. The allowable contact stress of pinion is 783.57 MPa and the bending stress of pinion tooth root is 466.95 MPa. As the Figure 11 shows, after 30 generations, the contact strength and bending strength of gear are lower than the allowable value, and the curve tends to be stable.

Table 5 shows the differences of optimization results using DEM and without DEM. In mode 1, the result is not using DEM and the COV of nominal tangential load (COV_{F_t}) is assumed to be 0; in mode 2, the COV of nominal tangential load (COV_{F_t}) is obtained from DEM. The results of mode 1 and mode 2 is all optimized by GA.

Comparing mode 1 with mode 2, it can be seen that when mode 1 reaches the best point $(m_n, z_1, \varphi_R)^T \approx (11, 28.57, 0.298)^T$ and mode 2 reaches the best point $(m_n, z_1, \varphi_R)^T \approx (11, 32.23, 0.264)^T$, the interesting phenomenon is occur that there is a small change in module relative to mode 1, which is always 11mm. The teeth number increased from 28.57 to 32.23, while the face width factor decreased from 0.298 to 0.264. The reliability changes little and the gear volume increases by 34.4%, which shows that the mode 2 design increases the cost, but the mode 2 design is more suitable for the actual situation. The method is same as the safety factor in gear design. Because of the different working conditions of gear, the selection of safety factor is often based on experience. This method provides theoretical guidance for the selection of gear safety factor.

In the mode 2, the optimization result based on the DEM increase the number of gear tooth than the mode 1, resulting in the increase of the volume, but the reliability change is relatively small.

The result shows that when the variation coefficient method is used to optimize the gear reliability, the gear working condition also has a great influence on the reliability optimization.

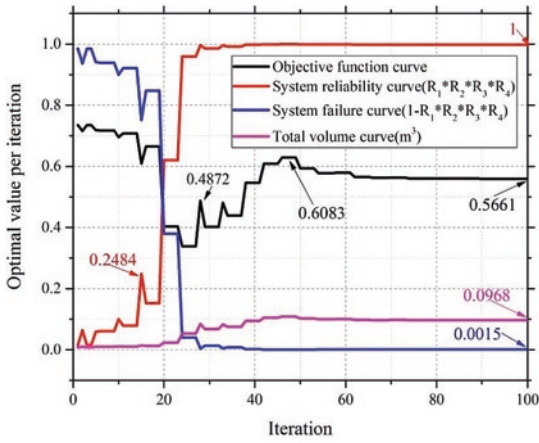


Fig. 7. Gear parameters curve

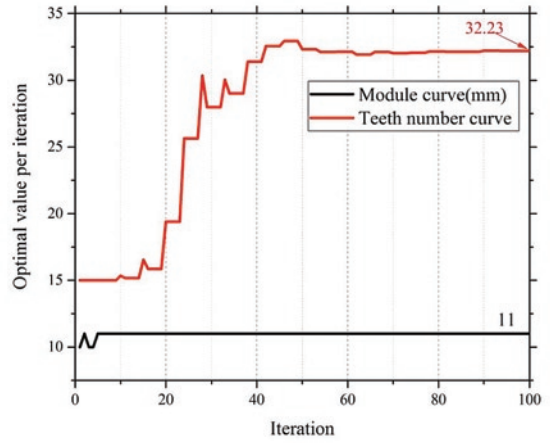


Fig. 8. Number of teeth and module curve

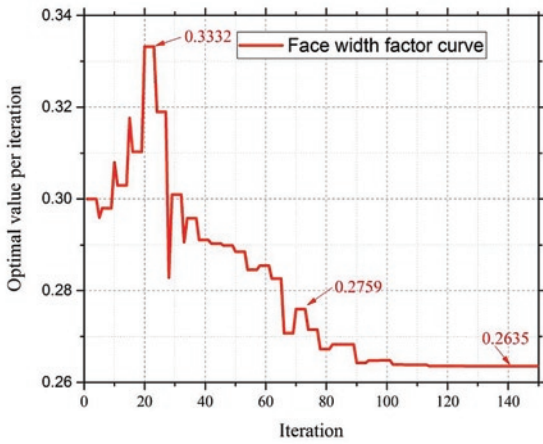


Fig. 9. Face width factor curve

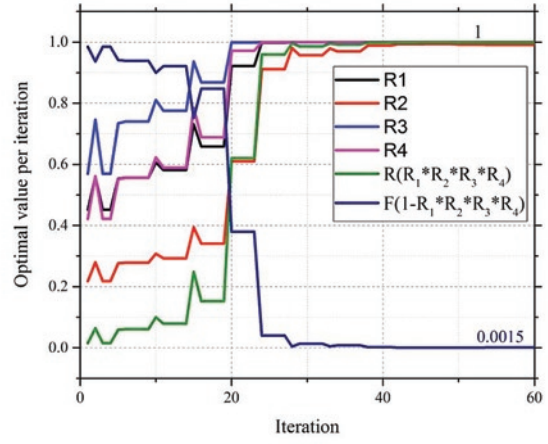


Fig. 10. Reliability and failure rate curve

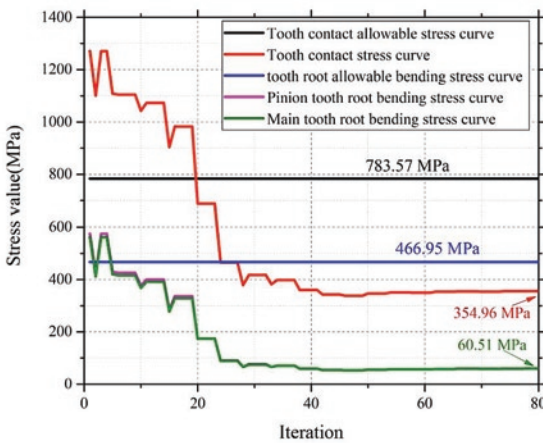


Fig. 11. Stress iterative curve

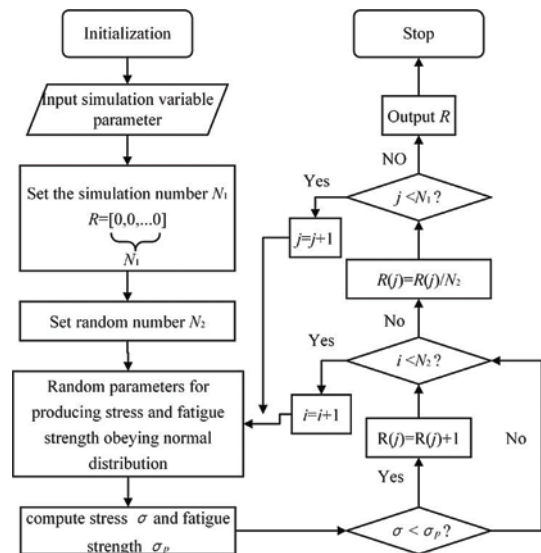


Fig. 12. Reliability calculation using Monte Carlo method

Table 5. Differences of optimization results using DEM and without DEM

Design mode	m_e	z_1	φ	R_1	R_2	R_3	R_4	System Reliability	Volume (m^3)	F_{ob}
1 Reliability optimization design ($COV_{F_t} = 0$)	11	28.57	0.298	0.9999	0.9903	1	1	0.9903	0.0721	0.4227
Teeth number upward rounding ($COV_{F_t} = 0$)	11	29	0.298	0.9999	0.9925	1	1	0.9925	0.0754	0.4403
2 Reliability optimization design (DEM) ($COV_{F_t} = 0.6495726$)	11	32.23	0.264	0.9998	0.9900	1	1	0.9900	0.0969	0.5661
Teeth number upward rounding (DEM) ($COV_{F_t} = 0.6495726$)	11	33	0.264	0.9999	0.9931	1	1	0.9931	0.104	0.6050

Note: $F_{ob} = \omega_1 \times Reliability + \omega_2 \times \frac{Volume}{V_{max}}, V_{max} = 0.052m^3, \omega_1 = 0.3, \omega_2 = 0.7$

Table 6. Results Comparison of Monte Carlo (MC) and theoretical value (TV)

Reliability	Teeth number upward rounding ($COV_T = 0$)		Teeth number upward rounding (DEM) $COV_T = 0.6495726$	
		MC	TV	MC
R_1	0.9999	0.998201	0.9999	0.999896
R_2	0.9925	0.992051	0.9931	0.994681
R_3	1	1	1	1
R_4	1	0.992494	1	0.999439

3.2. Verification of reliability with Monte Carlo method

Monte Carlo method is also called statistical test method, stochastic simulation method, or random sampling technique or statistical test method.

When the Monte Carlo method is used to calculate the reliability of gear transmission, the parameters related to the geometric dimensions of the gears are regarded as constants, while the other parameters are considered to obey normal distribution.

As shown in Figure 12, this paper uses the Monte Carlo method to calculate the gear transmission reliability flow chart, the basic process is:

- Step 1. Input the parameters of simulation variables that need to verify the results;
- Step 2. Set the number of simulations N_1 and make the N_1 dimension line vector $R = [0, 0, \dots, 0]$;
- Step 3. Generating N_2 random variables of normal distribution;
- Step 4. Calculate the stress σ and fatigue strength σ_p ;
- Step 5. Comparing the magnitude of stress and fatigue strength σ_p , if $\sigma < \sigma_p$, it accumulates to the reliable database, otherwise it goes directly to the next step;
- Step 6. If the total number of random numbers reaches N_2 , go to the next step, otherwise, return to Step4;
- Step 7. Calculate the reliability of j th order $R(j) = R(j)/N_2$;

- Step 8. If the number of simulations reaches N_1 , enter the reliability R of all the simulations and end the program. Otherwise, $j = j + 1$, return to Step3.

Suppose the simulation number N_1 is 1 and the number of random numbers N_2 is one million. In Table 6, comparison of reliability between Monte Carlo and theoretical calculations (Section 3.1) can be obtained.

In Table 6, the simulation results of reliability obtained by Monte Carlo method and theoretical value are very similar, so it can be considered that the calculation of results by normal distribution is correct.

4. Summary

In this paper, the multi-objective function model of bevel gear is established, and the parameters of bevel gear are optimized by GA and DEM. The feasibility of the method is proved theoretically. In the design of gear reliability, the calculation of reliability by COV often assumes that the moment of transmission is subjected to constant load, but the actual situation is not always so. The innovation of this paper is to use DEM. Combined with DEM of real working conditions, the torque COV of gear transmission is calculated, which makes the designed gear more in line with the actual situation. The results of this paper verify the feasibility of the method by Monte Carlo method and provide reference for the design and optimization of gears.

The results show that it is closer to gear design when the optimum design of gear under actual working conditions are considered, and this method is similar to the selection of gear safety factor. In practical production, the selection of safety factor is often based on experience.

This paper provides a new method to optimize the reliability of bevel gear, combining with DEM simulation, which provides theoretical guidance for optimal design of bevel gear.

There are also many defects in this paper. After all, the DEM is only theoretical simulation, and there is still a lot of gap with the actual situation. At the same time, the GA is used in the process of global optimization, because of the defects of the algorithm. It is possible to fall into a local optimal situation in the process of optimization. Of

course, with the continuous improvement of computer performance and research of algorithm, this method will have a lot of room for improvement.

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Appendix:

1. Calculation of Reliability of Bevel Gear Transmission system

According to the IOS standard, reliability design is to treat the design variables such as load, strength and their influencing factors as random variables, and apply the reliability theory and method to make the designed products meet the expected reliability requirements. In this paper, the reliability design of bevel gear transmission system mainly includes: the reliability of tooth surface contact strength of the pinion and the gear, the reliability of tooth surface contact strength of the pinion and the gear are expressed as R_1, R_2, R_3, R_4 , respectively.

1.1. Reliability calculation of tooth surface contact strength (R_1, R_2)

In the calculation of tooth surface contact fatigue strength, destructive pitting is the limit state of tooth surface work. The strength condition of the tooth surface without fatigue pitting is that the contact stress of the tooth surface joint is not greater than the contact allowable stress. Its reliability calculation formula is as follows:

$$R = P\{\sigma_H < \sigma_{HP}\} \quad (23)$$

1.1.1. Bevel gear tooth surface contact stress

Computational formula:

$$\sigma_H = Z_M Z_H Z_E Z_{LS} Z_\beta Z_K \sqrt{\frac{K K_A K_V K_{H\beta} K_{H\alpha} F_t}{d_m l_{bm}}} \times \frac{\sqrt{u^2 + 1}}{u} \quad (24)$$

Formula of COV:

$$\sigma_H = Z_M Z_H Z_E Z_{LS} Z_\beta Z_K \sqrt{\frac{K K_A K_V K_{H\beta} K_{H\alpha} F_t}{d_m l_{bm}}} \times \frac{\sqrt{u^2 + 1}}{u} \quad (25)$$

where, $\bar{Z}_E, \bar{F}_t, \bar{K}, \bar{K}_{H\beta}, \bar{K}_{H\alpha}$ are the mean values of the corresponding parameters.

$$V_{\sigma_H} = [V_{Z_E}^2 + \frac{1}{4}(V_K^2 + V_{F_t}^2 + V_{K_V}^2 + V_{K_{H\beta}}^2 + V_{K_{H\alpha}}^2 + V_{K_V} V_{K_{H\alpha}} + V_{K_V} V_{K_{H\beta}} + V_{K_{H\alpha}} V_{K_{H\beta}})]^{0.5} \quad (26)$$

$$S_{\sigma_H} = \overline{\sigma_H} V_{\sigma_H} \quad (27)$$

$$R = P\{\sigma_F < \sigma_{FP}\} \quad (36)$$

The average tangential force on the bevel gear face is:

$$\bar{F}_t = \frac{2000T}{d_{m1}} \quad (28)$$

1.1.2. Contact fatigue strength of bevel gears

Formula of tooth surface contact fatigue strength:

$$\sigma_{HP} = \sigma_{Hlim} Z_{NT} Z_L Z_V Z_R Z_W Z_X \quad (29)$$

Formula of COV:

$$\bar{\sigma}_{HP} = \bar{\sigma}_{Hlim} \bar{Z}_{NT} \bar{Z}_L \bar{Z}_V \bar{Z}_R \bar{Z}_W Z_X \quad (30)$$

$$V_{\sigma_{HP}} = \left(V_{\sigma_{Hlim}}^2 + V_{Z_{NT}}^2 + V_{Z_L}^2 + V_{Z_V}^2 + V_{Z_R}^2 + V_{Z_W}^2 \right)^{0.5} \quad (31)$$

$$S_{\sigma_{HP}} = \bar{\sigma}_{HP} V_{\sigma_{HP}} \quad (32)$$

Assuming that the contact stress limit function is:

$$g_1(\mathbf{X}_1) = \sigma_H - \sigma_{HP} \quad (33)$$

The formula of the contact fatigue strength of tooth surfaces is as follows:

$$Z_R = \frac{\bar{\sigma}_{HP} - \bar{\sigma}_H}{\left(S_{\sigma_{HP}}^2 + S_{\sigma_H}^2 \right)^{0.5}} \quad (34)$$

$$R = \Phi(Z_R) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{Z_R} e^{-\frac{x^2}{2}} dx \quad (35)$$

1.2. Reliability calculation of tooth root bending strength (R_3, R_4)

In the calculation of tooth root bending strength, the fracture of tooth root is regarded as the limit state. The condition that the tooth root does not break is that the bending stress of the tooth root joint is not greater than the bending allowable stress. The reliability calculation method is as follows:

1.2.1. Root bending stress

Formula of bending stress of tooth root:

$$\sigma_F = K K_A K_V K_{F\beta} K_{F\alpha} F_t \frac{Y_{FS} Y_\epsilon Y_K Y_{LS}}{b m_{nm}} \quad (37)$$

Formula of COV:

$$\bar{\sigma}_F = \bar{K} \bar{K}_A \bar{K}_V \bar{K}_{F\beta} \bar{K}_{F\alpha} \bar{F}_t \frac{Y_{FS} Y_\epsilon Y_K Y_{LS}}{b m_{nm}} \quad (38)$$

$$V_{\sigma_F} = \left(V_{\bar{K}}^2 + V_{\bar{F}_t}^2 + V_{\bar{K}_V}^2 + V_{\bar{K}_{F\beta}}^2 + V_{\bar{K}_{F\alpha}}^2 + V_{K_{F\alpha}} V_{K_{F\beta}} + V_{K_{F\alpha}} V_{K_V} + V_{K_{F\beta}} V_{K_V} \right)^{0.5} \quad (39)$$

$$S_{\sigma_F} = \bar{\sigma}_F V_{\sigma_F} \quad (40)$$

1.2.2. Tooth root bending fatigue strength

Formula of tooth root bending fatigue strength:

$$\sigma_{FP} = \sigma_{Flim} Y_{NT} Y_{\delta rel} Y_{Rrel} Y_X \quad (41)$$

Formula of COV:

$$\bar{\sigma}_{FP} = \bar{\sigma}_{Flim} \bar{Y}_{NT} \bar{Y}_{\delta rel} \bar{Y}_{Rrel} \bar{Y}_X \quad (42)$$

$$V_{\sigma_{FP}} = \left(V_{\bar{\sigma}_{Flim}}^2 + V_{\bar{Y}_{NT}}^2 \right)^{0.5} \quad (43)$$

Assuming that the bending stress limit function is:

$$g_2(\mathbf{X}_2) = \sigma_F - \sigma_{FP} \quad (44)$$

The formula for the reliability of tooth root bending fatigue strength is:

$$Z_R = \frac{\bar{\sigma}_{FP} - \bar{\sigma}_F}{\left(S_{\sigma_{HF}}^2 + S_{\sigma_F}^2 \right)^{0.5}} \quad (45)$$

$$R = \Phi(Z_R) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{Z_R} e^{-\frac{x^2}{2}} dx \quad (46)$$

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