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A methodology of identification and metaidentification research on the example of Day Ahead Market System

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Abstract. The paper contains selected research results in the field of identification and metaidentification of the Day Ahead Market system of TGE S.A. Due to the proposed new approach to identification, a methodology for conducting research has been developed, which requires eight stages. Then, both the tasks and research objectives as well as the form of research occurring at all stages of research in order to meet the distinguished specific objectives and the general purpose of the research were shown in detail. Then an example of both identification and metaidentification of Day Ahead Market systems was shown. The obtained models and metamodels confirm the need and possibility of conducting this type of research at TGE S.A.

Keywords. Day Ahead Market System, identification, MATLAB and Simulink environment, methodology, metaidentification

1. Introduction

In recent years, there has been a clear increase in the interest of various entities operating on the electricity market in concepts and methods related to identification modeling, metamodeling and broadly understood possibility of delegating business processes in order to create models of systems so important at present in the era of digitalization of services [1, 7]. The basic premise of using identification modeling methods is the desire to unambiguously describe, understand, analyze and evaluate specific processes of the organization, and the final result is the creation of a system model, and inpossible conditions even a metamodel of the system. This provides support for planning, implementation of decision-making processes, and as a result, effective adaptation of the entire organization to the changing challenges of the market and its participants. This is particularly evident in the context of the current changes on the Polish Power Exchange (PPE), which is a subsystem of the TGE S.A. The growing interest in identification modeling of the PPE system is related to dynamically changing international conditions, including structural changes caused initially by SARS COVID-19, and then by the energy crisis caused by international conflicts over energy resources [19, 21]. The structural changes taking place on PPE concern the among others of the functioning and management of its subsystem, which is among others the Day Ahead Market (DAM) system, and in it the subsystem of generating the weighted average volume of electricity price (ee) based on the volume ee in each hour of the day [5, 14, 22].

Recent years, and especially 2021, have been years of rising costs of purchasing electricity on the wholesale market. The value of contracts concluded on the Polish Power Exchange confirms that in 2021 the price of energy – both with delivery for 2022 and for the following year – increased sharply. In November 2020, the price of energy in futures contracts oscillated at the level of 242 PLN/MWh, to reach the level of 470 PLN/MWh in November 2021. The second cost factor affecting energy prices in Poland was and still is the cost of purchasing CO_2 emission allowances, because the Polish power industry in 80% is based on electricity (ee) from coal. At the same time, the costs of simplifying in the period from May 2019 to November 2021 increased from PLN 100 to PLN 310 per ton. High wholesale electricity prices and the costs of purchasing CO 2 emission rights were the main reasons for the increase in retail electricity prices at the level of the final consumer. The Energy Policy adopted by the Government of the Republic of Poland in February 2021 Polish until 2040[5] and the National Recovery Plan (KPO) submitted

[5] The document was prepared on the basis of art. 15a sec. 1 of the Energy Law Act (Journal of Laws of 2020, item 833, as amended) and in accordance with the Act on the principles of conducting development policy (Journal of Laws of 2019, item 1295, as amended).

to the European Commission, announce numerous modernization and restructuring activities that will have an impact on the current operation and financial costs incurred, among others, by energy companies. The changes that the Polish energy sector is already undergoing, as exemplified by the intensive development of renewable energy sources, are a challenge for all market participants, including participants of the DAM system.[6]

Particularly important from the point of view of the development and management of PPE is the situation on the Day Ahead Market. Rapid changes taking place on the DAM mean that all its participants, i.e. suppliers as well as customers and prosumers, as well as electricity trading intermediaries, are looking for solutions that will allow them to understand the specifics of the changes taking place and thus prepare for the conclusion of specific transactions. One of the ways to meet these challenges is undoubtedly to use domain-specific models and simulate the development of projected volume-weighted average electricity prices. At the same time, this is one of the main reasons for the increased interest of the aforementioned energy market stakeholders in the possibilities of using DAM system models as a tool supporting the effective and effective implementation of decision-making processes.

In addition, there is a need to develop ever more perfect models and metamodels of systems as a new form of modelling the DAM system, which results not only from the expectations of market participants, but also from the challenges faced by the increasingly unstable global energy market of today's increasingly electronic economies and information society [1]. In addition, the role of modeling is growing due to the progressive automation, robotization and the emergence of flexible production systems developing in the development of nanotechnology and intelligent systems manifested in the integration of an electronic management system with a flexible power system, including an information- communication systems, IT process management systems, power automation system and smart grid and smart metering systems, etc. [8-9, 11, 15, 24-26].

A critical review of the literature on the subject showed [11] that the current research focuses overwhelmingly on the development of ee price forecasting models. However, there is a new

[6] On 01.06.2022, the National Recovery and Resilience Plan was approved by the European Commission and on 17.06.2022 by the Council of the EU.

research trend regarding DAM system modeling based on various methods, including: identification modeling (identification),neural modeling, fuzzy modeling, and others [3-5, 7, 13, 15, 17-18, 20]. In the light of the literature studies carried out, it can be concluded that there are no appropriate identification models of the DAM system and there are no results of research on the identification metamodeling of systems leading to the obtaining of metamodels in general, including in particular the receipt of DAM metamodels, maybe apart from the work resulting from own research. or studies in which the author of this publication participated [11].

In the course of the research on the identification of the DAM system, various types of limitations occurred, resulting, inter alia, from the lack of publicly available data on quotations on world markets, such as quotations on DAM, which could be used in comparative studies, as well as limitationsdue to the methods of computerization of DAM systems as a management system [2]. Thus, from the point of view of software engineering, the identified research problems are related to the need to obtain models and metamodels of the Day Ahead Market system, which leads to the definition of the general purpose of research in terms of IT management systems. in the form of the process of identification and metaidentification of the DAM system functioning as a management system at TGE S.A.

2. Identification of the Day Ahead Market System

2.1. Day Ahead Market as a management system

The Day Ahead Market was the first market launched on TGE S.A. since its registration. This market has been functioning as a stand-alone system since 30 June 2000 and is a physical spot market for electricity. The main purpose of its operation is to create electricity prices for other contracts concluded on the wholesale electricity market in Poland. In addition, the Day Ahead Market allows m.in market participants to pre-balance their contract positions, and energy companies to indirectly value their value (in particular for generators) by valuing the electricity they produce.

On DAM there are 24 separate hourlyquotes on which the relevant transactions are executed. The fact that the DAM system is a physical market for electricity means that as a result of the concluded transactions there is a physical flow and delivery of electricity to the customer, which is why it is intended in particular for: generators, sellers, non-tariff electricity consumers, wholesale companies. electricity trading and Brokerage houses and Department Stores Brokerage Houses. The volume of electricity trading on TGE S.A. in January 2022 amounted to 12 685 538 MWh, which means an increase by 1.7% compared to January 2021. The weighted average volume of delivered and sold ee price on DAM in January 2022 amounted to PLN 666.90 / MWh (a decrease by PLN 163.08 / MWh compared to the previous month) [21].

2.2. The essence of system identification

As a result of identification modelling, appropriate system models are obtained on the basis of the collected numerical data depending on the method used. In control theory and systems engineering, as a result of the identification process, different classes of models [16, 23-24] are obtained, including the models used in this study as Multi Input Single Output (MISO) models. Due to m.in the need for the type of identification process using the appropriate method, there are many important types of models [4, 7, 12, 17-18, 26-29]. These include a dynamic linear model, which can be determined by a discrete form output signal $y(t)$ (Fig. 1):

$$y(t) = G(z^{-1})\, u(t) + H(z^{-1})\, \varepsilon(t), \tag{1}$$

where:

$\boldsymbol{G(z^{-1})}$ – characteristics of the control track,

$\boldsymbol{H(z^{-1})}$ – characteristics of the disturbance path,

$\boldsymbol{u(t)}$ – input signal,

$\boldsymbol{\varepsilon(t)}$ – interference (noise),

$\boldsymbol{z}$ – time shift operator,

$\boldsymbol{t}$ – short time as an independent variable [days].

Expressing as quotients of polynomials, one obtains character models:$G(z^{-1})\ i\ H(z^{-1})z^{-1}$

$$A(z^{-1})\, y(t) = \frac{B(z^{-1})}{F(z^{-1})}\, u(t) + \frac{C(z^{-1})}{D(z^{-1})}\, \varepsilon(t), \tag{2}$$

where:

$A(z^{-1}) = 1 + a_1 z^{-1}+...+a_{na} z^{-na}$,

$B(z^{-1}) = 1 + b_1 z^{-1}+...+b_{na} z^{-nb}$,

$C(z^{-1}) = 1 + c_1 z^{-1}+...+c_{nc} z^{-nc}$,

$D(z^{-1}) = 1 + d_1 z^{-1} + ... + d_{nd} z^{-nd}$,

$F(z^{-1}) = 1 + f_1 z^{-1} + ... + f_{nf} z^{-nf}$,

z-1 – time delay by one unit, e.g. y(t) $^{z-1}$ = y(t-1),

na – the degree of the polynomial A(z),

nb – degree of polynomial B(z),

nc – degree of the polynomial C(z),

nd – the degree of the polynomial D(z),

nf – the degree of the polynomial F(z).

From the point of view of computational formalism, it is assumed that system identification is a process that uses measurement data to create a system model. This type of models also includes parametric models obtained as a result of identification, including ARX methods, used for many years, especially in technical and economic sciences, including the management and control of the DAM system.

Different methods of obtaining parametric models depending on the test object are used, such as m.in: AR, ARX, ARMAX, etc. This is the case, for example, if in model (2):.

$$C(z^{-1}) = D(z^{-1}) = F(z^{-1}) = 1 \quad (3)$$

is obtained:

$$y(n) = \frac{B(z^{-1})}{A(z^{-1})} u(n) + \frac{1}{A(z^{-1})} \varepsilon(n). \quad (4)$$

i.e. the ARX (AutoRegressive with Exogenous Input) model – Fig. 2.

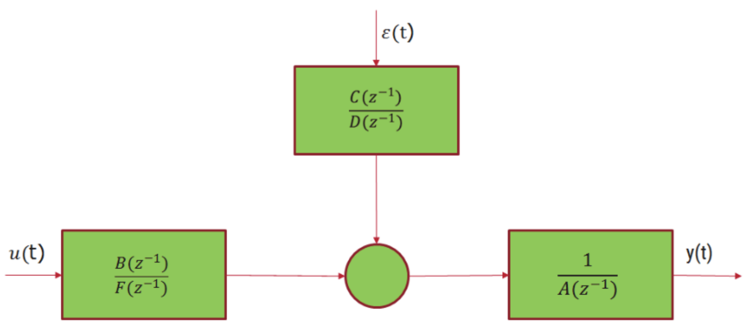

Figure 1. Block diagram of the parametric model in general form. Note: in-text markings. Source: [20].

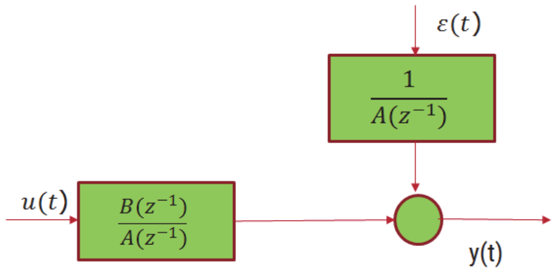

Figure 2. Flowchart of the ARX parametric model. Ozreferences in the text. Source: [20, 24].

Therefore, in the identification research conducted so far, prognostic, planning or programming models were obtained as a result of the use of modeling methods known in the literature of this type of systems as the Day Ahead Market system at TGE S.A., which were then used to search for new system states, i.e. specific output quantities depending on the relevant input quantities [11].

2.3. System identification modeling

Identification modeling of the DAM TGE S.A. system will be carried out using the ARX parametric model. The system situation of DAM is shown in terms of systems engineering in Figure 3. The DAM system (and thus the model) is associated with the closer environment (customers and suppliers of ee) by two types of couplings, i.e. energy coupling and financial coupling [7, 14, 16, 24]. Energy coupling includes the delivery and sale of ee on the stock exchange, and the financial coupling is related to the payment for the delivered ee (cost incurred) and the revenues obtained depending on the average price obtained and the volume of ee sold.

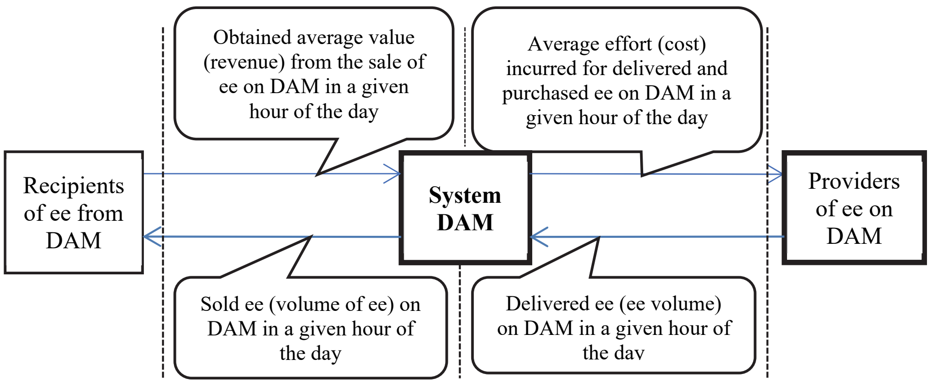

Figure 3. Placement of the DAM system in terms of systems engineering in a closer environment (suppliers and ee recipients). Source: own study based on [16, 24].

As a result of the identification modeling and identification metamodeling, a model and metamodel of the DAM system were obtained. The research experiment used an example of numbers concerning the DAM system of TGE S.A. for figures recorded on DAM from the period of operation of TGE S.A. with a period of 184 days and progress of one month (half-yearly periods), in particular on data from 1.01.2016 to 31.12.2019 eye, selected results of which are included in this publication. This made it possible to obtain a catalogue of parametric discrete ARX models, which are then converted to continuous mand those to continuous models in statespace. This resulted in discrete and continuous metamodels, as well as metamodels in statespace.

As a result of the DAM system test, data on the actualego of the u system, which operates on TGE S.A., were obtained. By conducting identification on it, a model of the DAM system is obtained, and by carrying out metaidentification, a metamodel of the DAM system is obtained. The DAM metamodel can be further used to design the DAM system model. In a formalized way, the identification of the DAM system is called the problem of finding a mapping of the matrix of measurement data recorded on the DAM system in the form [24, 30]:

$$Z^N = [\text{output, input}] \tag{5}$$

in the vector of model parameters [θ] written in the form of **th** matrix **of theta** format, with N being the number of ordered observations of consecutive values of input variables and subsequent values of output variables. It should be emphasized here that there are no results of research on the identification and metaidentification of the Day Ahead Market system at TGE S.A. conducted in order to obtain models and metamodels as replacement schemes of real systems [11]. An extremely important research problem is therefore the definition of the research object itself, i.e. the DAM system as a management system functioning on the Polish Power Exchange S.A. It is assumed in terms of control theory and systems that theidentification process is carried out using a set of values of the input and output quantities of the DAM system, i.e. respectively: the volume of ee delivered and sold on DAM (input quantities) and the averagevolume-related ee price obtained in this respect in individual hours of the day (output size). As a result of the identification, the ARX model catalog is obtained, i.e. also the model parameters, which are the coefficients of polynomials B(z) associated with the input quantities and the coefficients of polynomials A(z) associated with the output quantities.

3. Methodology of identification and metaidentification tests of the Day Ahead Market system

3.1. Description of the research process

Due to the assumed main objective of the research and its specific objectives and the complexity of identification studies, the designed research process consists of eight stages of activities. Their course and synthetic description are shown in Figure 4. The research process designed for the purpose of achieving the general objective and the specific objectives of the conducted research assumes the implementation of the following activities within eight separate stages, i.e.:

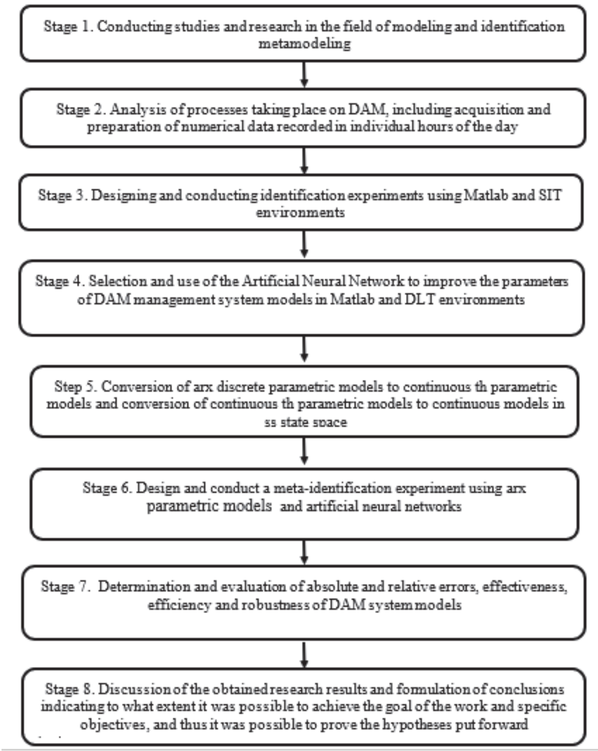

Figure 4. Scheme of the research process. Source: own study.

1) **Stage 1.** For the purpose of conducting studies and research in the field of identification modeling and metamodeling (shorter: identification) of the DAM TGE S.A. system. a comparative study of modeling with a structure was carried out: modeled system/object/system/process/etc., type of modeling, including identification/forecasting methods used, data used for modeling(input/output), forecasting horizon, purpose of research (matching the model to the system, i.e. to real data), measures used, forecasting results obtained (m.in types of errors and their values), authors/title of the book/article,

comments, including on the computing environment from which the test results are published in the paper [11].

2) **Stage 2.** In order to carry out the experiments, it was necessary to analyze the processes taking place on the DAM, as well as to obtain numerical data recorded in individual hours of the day, i.e. regarding the volume of electricity delivered and sold on DAM and the volume-weighted average price of electricity obtained in individual hours of the day, and preliminary procedures should be carried out on the data, including normalization of numerical data. , which was included in the paper [10].

3) **Stage 3.** In order to obtain a catalog of DAM system models, identification experiments had to be designed and carried out using numerical data recorded on DAM in the MATLAB and System Identification Toolbox environments.

4) **Stage 4.** Due to the need to improve the parameters of the DAM system model, it was necessary to select the architecture and method of learning the Artificial Neural Network in the MATLAB and Deep Learning Toolbox environments and to conduct a neural learning experiment, from this stage of research the results were published in the paper [10].

5) **Stage 5.** Obtaining information about the degree of internal organization of the management system, which is the DAM system and the level of control occurring in it, was associated with the conversion of parametric discrete ARX models to continuous th parametric models and the conversion of continuous th parametric models into continuous models in the space of ss states.

6) **Stage 6.** In order to obtain a metamodel of the DAM system, it was necessary to design and conduct a metaidentification experiment using the parameters of the DAM system model catalog in the MATLABA and System Identification Toolbox environments, as well as the Deep Learning Toolbox.

7) **Stage 7.** Assessment of the quality of the model in relation to the DAM system required the determination of absolute and relative errors, as well as the effectiveness of the DAM system models, the efficiency and robustness of the system and the DAM system models, as well as the examination of the sensitivity of the DAM system model, which was published m.in, in the paper [26].

8) **Stage 8**. A discussion of the obtained research results was held and conclusions were formulated indicating to what extent the goal of the work and specific objectives were achieved, as well as what are the further directions of research.

For the purpose of the research, 24 research tasks were accepted, for which research questions were formulated, which are summarised in Table 1. In the research process, four catalogues of data on the Day Ahead Market system of TGE S.A. were generally used, including the following data (Stage 2) [22]:

– from 1 January 2015 to 30 June 2015 with an identification period of 184 days, on the basis of which 24 hourly parametric models were obtained, respectively: discrete and continuous and 24 continuous models in statespace, i.e. a total of 72 models that were used in Stage 3 and Stage 5,

Table 1. Research objectives and tasks. Source: own study.

Scientific objectives of the dissertation	Research task	Research questions	Form of research
Specific objective No 1: Location of the Day Ahead Market system in terms of systems engineering and management and quality sciences against the background of research issues related to the Electricity Market system	**Research task No. 1:** Description of the Electricity Market system in Poland	What is the state and structure of the Electricity Market system in Poland? What are the requirements for implementing a REE system? What are the models of the REE system? What are the electricity markets from the point of view of trading? What are the characteristics of trading ee on electricity markets?	Literature research
	Research task No. 2: Recognition of the Polish Power Exchange as a subsystem on the Electricity Market	What are the characteristics of TGE S.A.? What is the trading status of ee on TGE S.A.?	Literature research/ individual consultations
	Research task No. 3: Characteristics of the Day Ahead Market system as a management system	What is the purpose and tasks of DAM? What kind of transactions are concluded on DAM? How do DAM quotes take place? What are the DAM participants?	Literature research/ individual consultations
Specific objective No 2: Overview of identification modelling methods in a systemic approach	**Research task No. 4:** Description of the essence of identification	What is the identification process? What are the identification phases? What is the place of identification modeling against the background of system modeling?	Literature research
	Research task No. 5: Overview of the possibilities of obtaining system models	What model classes can be obtained as a result of identification? What is the mathematical description of the PARAMETRIC model of ARX?	Literature research
	Research task No. 6: Review of methods used in modeling TGE S.A. systems	What models are used in the functioning of commodity exchanges?	Literature research
	Research task No. 7: Understanding DAM Identification Modeling	What is the DAM system situation? What is the process of modeling and metamodeling an DAM system? What is the definition of DAM as a management system?	Qualitative research
	Research task No. 8: Conduct a critical literature review to obtain sufficiently accurate DAM models	What is the state and directions of development of DAM system modeling?	Literature research

Specific objective No 3: Development of a methodology for identification and metaidentification tests of the DAM system	**Research task No. 9:** Description of the stages of the research process	What stages of the research process should be distinguished? What data catalogues should be used in the research process? How should the process of identification and metaidentification be addressed?	Qualitative research
	Research task No. 10: DAM system approach	What is the essence of the DAM system approach? How do I get the DAM system model?	Qualitative research
	Research task No. 11: Evaluation of the model and system in terms of efficiency, effectiveness and robustness	What is the effectiveness of the model? What is the efficiency and robustness of the system and model?	Qualitative research
Specific objective No 4: Identification of the DAM system	**Research task No. 12:** Formulation of assumptions regarding the design of the process of identification and metaidentification of the DAM system	What assumptions have been made about the DAM model? What assumptions have been made regarding the identification of the DAM system? What assumptions have been made about the metaidentification of the DAM system?	Empirical research (design)
	Research task No. 13: Identification of the DAM system using data from 1.01.2015 to 30.06.2015	What types of mathematical models are obtained as a result of DAM system identification? What is the interpretation of the parameters of the obtained models?	Empirical research (identification and evaluation)
	Research task No. 14: Identification of the DAM system using data from the period from 1.01.2013 to 30.04.2016	What types of mathematical models are obtained as a result of DAM system identification? What model catalogue is obtained for metaidentification? What is the interpretation of the parameters of the obtained models? What is the level of the management system and the degree of internal organization of the DAM system?	Empirical research (identification and evaluation)
	Research task No. 15: Identification of the DAM system using data from the period from 1.01.2016 to 31.12.2019	What types of mathematical models are obtained as a result of DAM system identification?	Empirical research (identification)
Specific objective No 5: Conduct metaidentifica tion DAM system	**Research task No. 16:** Metaidentification of the DAM system using 35 DAM system models obtained on the basis of data from 1.01.2013 to 30.04.2016.	What types of mathematical models are obtained as a result of the metaidentification of the DAM system? Is it possible to obtain metamodels of the DAM system as a result of designing and teaching ANNs?	Empirical research (metaidentificat ion and ANN learning)
Specific objective No 6: Building simulation models and then conducting simulation studies using them, comparative studies and sensitivity testing	**Research task No. 17:** Design and construction of a simulation model	Is it possible to design and build a simulation model of an DAM system?	Empirical research (simulation studies)
	Research task No. 18: Conducting simulation tests	Is it possible to conduct simulation studies using a simulation model for any values of input signals (volume of delivered and sold ee)?	Empirical research (simulation)

	Research task No. 19: Conducting comparative studies and sensitivity tests	Is it possible to conduct comparative studies using a simulation model for any values of input signals (volume of delivered and sold ee)? Is it possible to conduct a sensitivity test?	Empirical research
	Research task No. 20: Conducting tests of the efficiency, effectiveness and robustness of the system and the model of the DAM system	What is the efficiency, effectiveness and robustness of the DAM system and model?	Empirical research (studies of the efficiency, effectiveness and robustness of the system and model)
Specific objective No 7: Interpretation of parameters and testing the quality of models	**Research task No. 21:** Interpretation of model parameters	What is the interpretation of parametric model parameters? What is the interpretation of the parameters of the state variable models?	Empirical research (examination of model parameters)
	Research task No. 22: Model quality studies	What is the study of the effectiveness of the model and the system? What is the test of the effectiveness of the model to the system? What is the robustness test of the model and the system?	Empirical research (efficiency, effectiveness and robustness depending on the volume of ee and the weighted average volume of the price of ee)
Specific objective No 8: Formulation of final conclusions and further directions of research	**Research task No. 23:** Preparation of final conclusions	What conclusions can be drawn indicating the degree of achievement of the general objective and the specific objectives of the work? How were the hypotheses verified?	Qualitative research
	Research task No. 24: Development of further research directions	What further research directions have been developed?	Qualitative research

- from 1 January 2013 to 30 April 2016 with an identification period of 184 days, on the basis of which 24 catalogues of hourly parametric models were obtained, with each catalogue containing 35 models set in the accepted rolling periods with a progress of one month, i.e. a total of 840 discrete and continuous models and 840 continuous models in statespace, which have been used in Step 3 and Stages 5 to 6,
- from 1 January 2016 to 31.12.2019 with an identification period of 365 days, on the basis of which in each of the four years, i.e. for the following years: 2016, 2017, 2018, 2019, 24 discrete and continuous parametric models were obtained, followed by 24 continuous models in state space, i.e. a total of 288 models that were used in Stage 3 and Stage 5, and for 2019 in the scope of parametric corrections using PPS in Stage 4 and in the study of quality, efficiency, effectiveness and robustness in Stage 7,

– from 1 January 2020 to 31.12.2020 with an identification period of 365 days, on the basis of which a simulation model was built to test the sensitivity of 24 discrete and continuous parametric models, followed by 24 continuous models in state space, i.e. a total of 72 models were built for sensitivity testing, which were used in Stage 7.

The process approach to identification (Stage 3) and metaidentification (Stage 6)of the DAM system is illustrated in Figure 5.

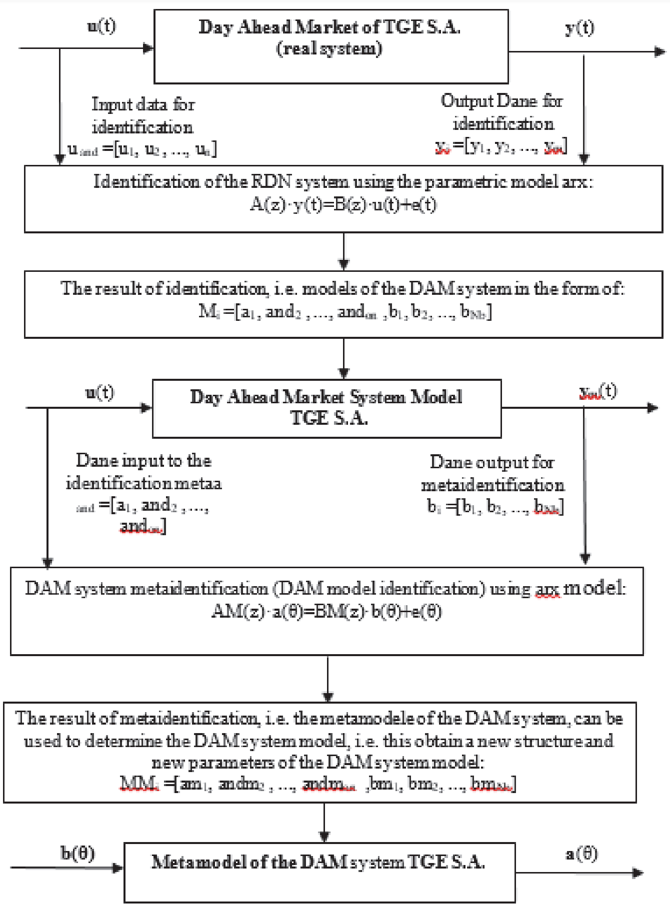

Figure 5. Diagram of the identification and metaidentification process of the DAM TGE S.A. system
Source: own study based on [24].

As a result of the identification, the output is a catalog of parametric arx models, whose parameters are used for metaidentification, as a result of which a metamodel of the DAM system is obtained. Identification of the DAM system and assessment of the quality of the obtained models is associated with the need to approach the DAM system and define the concept of efficiency, effectiveness and robustness of the model in system categories.

4. Identification of the Day Ahead Market system

The identification of the DAM system was carried out for annual data collected from TGE S.A. from the period from 1.01.2016 to 31.12.2019. As a result of identification, DAM system models were obtained for individual years respectively: 2016, 2017, 2018 and 2019. Each year, 24 MISO parametric models of discrete and continuous types were obtained, followed by 24 continuous models in state space. For detailed analysis, 4 hourly models were used, i.e. for hours: 6, 12, 18 and 24. Thus, for the 6th hour of 2019, a discrete parametric model was obtained in the form [10-12, 26]:

$$A(z) \cdot y(t) = B_1(z) \cdot u_1(t) + B_2(z) \cdot u_2(t) + \cdots + B_{24}(z) \cdot u_{24}(t) + e(t), \tag{6}$$

where:

$$A(z) = 1 - 0.3777 \cdot z^{-1} + 0.1551 \cdot z^{-2} - 0.1395 \cdot z^{-3} - 0.1729 \cdot z^{-4} - 0.07633 \cdot z^{-5} - 0.0725 \cdot z^{-6},$$

$$B_1(z) = -0.01233 \cdot z^{-1} - 0.021 \cdot z^{-2} + 0.002389 \cdot z^{-3} - 0.003882 \cdot z^{-4} + 0.0452 \cdot z^{-5} - 0.0008132 \cdot z^{-6} - 0.01498 \cdot z^{-7} - 0.01678 \cdot z^{-8} - 0.002096 \cdot z^{-9} + 0.0196 \cdot z^{-10},$$

$$B_2(z) = 0.02169 \cdot z^{-1} + 0.02292 \cdot z^{-2} - 0.005696 \cdot z^{-3} + 0.01106 \cdot z^{-4} - 0.08478 \cdot z^{-5} - 0.002471 \cdot z^{-6} + 0.02813 \cdot z^{-7} + 0.04245 \cdot z^{-8} + 0.02165 \cdot z^{-9} - 0.03317 \cdot z^{-10},$$

……

$$B_{24}(z) = -0.01645 \cdot z^{-1} - 0.000098 \cdot z^{-2} + 0.01285 \cdot z^{-3} + 0.009628 \cdot z^{-4} + 0.008773 \cdot z^{-5} - 0.000536 \cdot z^{-6} + 0.001467 \cdot z^{-7} - 0.001157 \cdot z^{-8} + 0.003415 \cdot z^{-9} + 0.01136 \cdot z^{-10},$$

When converting a discrete parametric model to a continuous parametric model using, for example, a form function[7] [6]:

th6101h62019=d2c(arx6101h62019,'tustin') (7)

[7] In the command of the form th6101h62019=d2c(arx6101h62019,'tustin'): th6101h62019 - name of the continuous parametric model, where: th6101 refers to the degrees of polynomials in the parametric discrete model arx6101h62019 (na=6, nb=10, nk=1), h6 - means model for 6:00 am, 2019 – model for the whole year 2019, d2c() - function converting the model from discrete to continuous form, 'tutsin' - integration method.

the following continuous parametric model for the 6 o'clock of 2019 was obtained:

$$y(t) = [((B_1(s)/F_1(s)) \cdot u_1(t) + ((B_2(s)/F_2(s)) \cdot u_2(t) + \dots + ((B_{24}(s)/F_{24}(s)) \cdot u_{24}(t) + [C(s)/D(s)] \cdot e(t), \quad (8)$$

where:

$$B_1(s) = -0.02731 \cdot s^{10} - 0.2861 \cdot s^9 + 2.816 \cdot s^8 - 9.212 \cdot s^7 + 49.11 \cdot s^6 - 103.4 \cdot s^5 + 198.3 \cdot s^4 - 254.7 \cdot s^3 - 42.29 \cdot s^2 - 81.72 \cdot s - 3.196,$$

$$B_2(s) = 0.03977 \cdot s^{10} + 0.501 \cdot s^9 - 6.2 \cdot s^8 + 18.33 \cdot s^7 - 90.71 \cdot s^6 + 193 \cdot s^5 - 306.8 \cdot s^4 + 339.7 \cdot s^3 + 169 \cdot s^2 + 36.33 \cdot s + 14.83,$$

....

$$B_{24}(s) = 0.006079 \cdot s^{10} + 0.03525 \cdot s^9 + 2.183 \cdot s^8 - 2.081 \cdot s^7 + 32.8 \cdot s^6 - 58.96 \cdot s^5 + 74.968 \cdot s^4 - 211.2 \cdot s^3 + 19.52 \cdot s^2 - 68.22 \cdot s + 19.93,$$

$$C(s) = s^6 + 12 \cdot s^5 + 60 \cdot s^4 + 160 \cdot s^3 + 240 \cdot s^2 + 192 \cdot s + 64,$$

$$D(s) = s^6 + 11.04 \cdot s^5 + 41.99 \cdot s^4 + 107.2 \cdot s^3 + 128.6 \cdot s^2 + 125.3 \cdot s + 13.46,$$

$$F_1(s) = s^{10} + 19.04 \cdot s^9 + 154.3 \cdot s^8 + 740 \cdot s^7 + 2363 \cdot s^6 + 5246 \cdot s^5 + 8203 \cdot s^4 + 8943 \cdot s^3 + 6389 \cdot s^2 + 2435 \cdot s + 215.4,$$

$$F_2(s) = s^{10} + 19.04 \cdot s^9 + 154.3 \cdot s^8 + 740 \cdot s^7 + 2363 \cdot s^6 + 5246 \cdot s^5 + 8203 \cdot s^4 + 8943 \cdot s^3 + 6389 \cdot s^2 + 2435 \cdot s + 215.4,$$

....

$$F_{24}(s) = s^{10} + 19.04 \cdot s^9 + 154.3 \cdot s^8 + 740 \cdot s^7 + 2363 \cdot s^6 + 5246 \cdot s^5 + 8203 \cdot s^4 + 8943 \cdot s^3 + 6389 \cdot s^2 + 2435 \cdot s + 215.4.$$

As a result of converting a parametric continuous model to a model in continuous state space using functions such as the form[8] [6]:

[A B C D KDAM X0DAM]=th2ss(th6101h62019) (9)

the following model was obtained for the 6th hour of 2019:

[8] In the command of the form [A B C D KDAM X0DAM]=th2ss(th6101h62019): th6101h62019 - name th2ss() - function converting the model from parametric continuous model to continuous model in state space; A, B, C, D, KDAM, X0DAM – appropriate model matrices in the state space, other notations as until further notice[3].

$$\frac{dx_1}{dt} = -19{,}0379\, x_1 - 9{,}6436\, x_2 - 5{,}7815\, x_3 - 4{,}6152\, x_4 - 2{,}5617\, x_5 - 2{,}0026\, x_6 - 1{,}0917 x_7 - 0{,}7799\, x_8 - 0{,}5945\, x_9 - 0{,}2103\, x_{10} + u_1,$$

$$\frac{dx_2}{dt} = 16\, x_1,$$

$$\frac{dx_3}{dt} = 8\, x_2,$$

$$\frac{dx_4}{dt} = 4\, x_3,$$

$$\frac{dx_5}{dt} = 4\, x_4,$$

$$\frac{dx_6}{dt} = 2\, x_5,$$

$$\frac{dx_7}{dt} = 2\, x_6,$$

$$\frac{dx_8}{dt} = x_7,$$

$$\frac{dx_9}{dt} = 0.5\, x_8,$$

$$\frac{dx_{10}}{dt} = 0.25\, x_9,$$

$$\frac{dx_{11}}{dt} = -19{,}0379\, x_{11} - 9{,}6436\, x_{12} - 5{,}7815\, x_{13} - 4{,}6152\, x_{14} - 2{,}5617\, x_{15} - 2{,}0026\, x_{16} - 1{,}0917 x_{17} - 0{,}7799\, x_{18} - 0{,}5945\, x_{19} - 0{,}2103\, x_{20} + u_2,$$

$$\frac{dx_{12}}{dt} = 16\, x_{11},$$

$$\frac{dx_{13}}{dt} = 8\, x_{12},$$

$$\frac{dx_{14}}{dt} = 4\, x_{13},$$

$$\frac{dx_{15}}{dt} = 4\, x_{14},$$

$$\frac{dx_{16}}{dt} = 2\, x_{15},$$

$$\frac{dx_{17}}{dt} = 2\, x_{16},$$

$$\frac{dx_{18}}{dt} = x_{17},$$

$$\frac{dx_{19}}{dt} = 0.5\ x_{18},$$

$$\frac{dx_{20}}{dt} = 0.25\ x_{19},$$

.... (10)

$$\frac{dx_{231}}{dt} = -19{,}0379\ x_{231} - 9{,}6436\ x_{232} - 5{,}7815\ x_{233} - 4{,}6152\ x_{234} - 2{,}5617\ x_{235} - 2{,}0026\ x_{236} - 1{,}0917 x_{237} - 0{,}7799\ x_{238} - 0{,}5945\ x_{239} - 0{,}2103\ x_{240} + 0.5 u_{24},$$

$$\frac{dx_{232}}{dt} = 16\ x_{231},$$

$$\frac{dx_{233}}{dt} = 8\ x_{232},$$

$$\frac{dx_{234}}{dt} = 4\ x_{233},$$

$$\frac{dx_{235}}{dt} = 4\ x_{234},$$

$$\frac{dx_{236}}{dt} = 2\ x_{235},$$

$$\frac{dx_{237}}{dt} = 2\ x_{236},$$

$$\frac{dx_{238}}{dt} = x_{237},$$

$$\frac{dx_{239}}{dt} = 0.5\ x_{238},$$

$$\frac{dx_{240}}{dt} = 0.25\ x_{239},$$

$$\frac{dx_{241}}{dt} = -19{,}0379\ x_{241} - 9{,}6436\ x_{242} - 5{,}7815\ x_{243} - 4{,}6152\ x_{244} - 2{,}5617\ x_{245} - 2{,}0026\ x_{246}$$

$$\frac{dx_{242}}{dt} = 16\ x_{241},$$

$$\frac{dx_{243}}{dt} = 8\ x_{242},$$

$$\frac{dx_{244}}{dt} = 4\ x_{243},$$

$$\frac{dx_{245}}{dt} = 4\,x_{244},$$

$$\frac{dx_{246}}{dt} = 2\,x_{245},$$

and for the6th year of 2019, the output equation:

$$y_6(t) = 0{,}2338\,x_1(t) + 0{,}4394\,x_2(t) + 0{,}08591\,x_3(t) + \cdots + 0{,}3949\,x_{246}(t) - +0{,}0273\,u_1(t) + 0{,}0397\,u_2(t) + \cdots + 0{,}0061\,u_{24}(t), \quad (11)$$

where:

x_1(t), x_2(t),.... , x_{246}(t) – the relevant state variables,

u_1(t), u_2(t),.... , u_{24}(t) – the corresponding input variables.

In the same way, discrete parametric models, continuous parametric models and continuous models in statespace for hours: 12, 18, 24 were obtained.

5. Metaidentification

In the literature on the subject of DAM system research, so far no appropriate metaidentification models have been developed, including parametric metamodels and metamodels in state space. Such metamodels can be obtained, m.in, using control and systems theory [16, 24], and in particular using the metaidentification of the DAM system or using artificial neural networks (ANN) [10].

The process of metaidentification of systems in general, including the DAM system, is a complex process, requiring prior obtaining a sufficient number of system models by identification, the parameters of which are used to build a database for the metaidentification process [24]. In order to carry out the process of metaidentification of the DAM system, a numerical example based on data recorded on DAM was used in the approximate period from 0 January 1, 2013 to 30 April 2016. during the contractual period of 184 days (six months), and subsequent models can be obtained from consecutive figures recorded on the DAM taken with the progress of one month. This made it possible to obtain a catalogue of 35 discrete arx parametric models of the DAM system for each hour of the day (rolling models), assuming that the volume of electricity (ee) in each hour of the day are inputs to the system, and the volume-weighted volume-weighted price figures are outputs from the system [24].

Then, using the parameters of this type of 35 parametric discrete arx models, a secondary identification (model identification) was carried out, which was called metaidentification, because it results in models of system models (metamodels) [24]. Thus,the identification of the DAM system was first carried out for all 24 input quantities and subsequent output quantities, thus obtaining 24 discrete arx parametric models for one measurement sample, that is, for 184 days. This experiment was repeated 35 times, and therefore a total of 35 x 24 = 840 parametric models of discrete arx were obtained. These models provided data for metaidentization in the form of coefficients of polynomials B(z) related to input quantities (ee volume) and in the form of coefficients of polynomials A(z) associated with output quantities (volume-weighted average price ee). Metaidentification was also taught to the Perceptron ANN [12, 26].

The experiment adopted the arx identification method [12, 26] and the numerical data obtained by identifying the DAM system at 24 input quantities and one output quantity (MISO models). As a result of the identification carried out using the arx method, 35 parametric discrete models were obtained, such as for the first period marked as p1 for data recorded on DAM from 1.01.2013 to 30.06.2013 for all hours of the day in terms of the volume delivered and sold ee and for 1 hour for the volume-weighted average price of ee (contractual 184 days) in the form of the presented relationship (6). The obtained DAM system model for the p1 period has an accuracy in relation to the DAM system in the amount of 87.8%. Examples of the values of the coefficients of the polynomial A(z) and the polynomialof type B(z) are presented in the papers [12, 26].

Using the generated coefficients in 35 models for each separate hour of the day (24 polynomials B(z), i.e. 4 coefficients in each of the 24 models), a total of 96 input quantities were obtained. Then, from the obtained ranges of variation of the relevant coefficients used in identification, 560 pairs of input-outputvalues were generated in a random way.

Subsequently, metaidentification was carried out and four separate metamodels were obtained for each coefficient of the polynomial $AM_i(z)$, such as for hours 0–1 and the coefficient at operator $^{z-1}$, in the form of a discrete arx748 model:

$$AM_1(z)\, a_1(\theta) = BM_{1,1}(z)\, b_1(\theta) + BM_{1,2}(z)\, b_2(\theta) +... + BM_{24,96}(z)\, b_{96}(\theta) +e(\theta), \quad (12)$$

where:

$AM_1(z) = 1\text{-}0.9557\ ^{z-1} + 0.166\ ^{z-2} + 0.04223z\text{-}3\ \text{-}0.4683\ ^{z-4} + +0.7337\ ^{z-5} - 0.3706\ ^{z-6} - 0.2331\ ^{z-7}$,

$BM_{1,1}(z) = 0{,}00001\ ^{z-8} + 0{,}00001\ ^{z-9} + 0{,}00001\ ^{z-10} + +0{,}00001\ ^{z-11}$,

$BM_{1.2}(z) = 0.00001z\text{-}8 + 0.00001z\text{-}9 + 0.00001z\text{-}10 + +0.00001z\text{-}11$,

....

$BM_{24.96}(z) = -2.88z\text{-}8 +3.923z\text{-}9 - 0.8226z\text{-}10\text{-}1.227\ ^{z-11}$,

na – degree of polynomial $AM_1(z)$, na=7,

nb – degree of polynomials $BM_j(z)$, nb=4,

nk – output delay in relation to input, nk=8,

θ – long time [month].

The accuracy of the metamodel to the data obtained for 35 models was 59.67% – Fig. 6.

Using the generated parameters of 35 DAM system models, the ANN of the DAM system metamodel was also taught on the example of 0–1 hour. The course of the learning curve is shown in Figure 7. The Perceptron Bilayer Type Artificial Neural Network as a neural metamodel of the MIMO type DAM system (input coefficients u_1, output coefficients y_1) for hours 0-1 had four input quantities (coefficients b_i) and four output quantities (coefficients a_j).

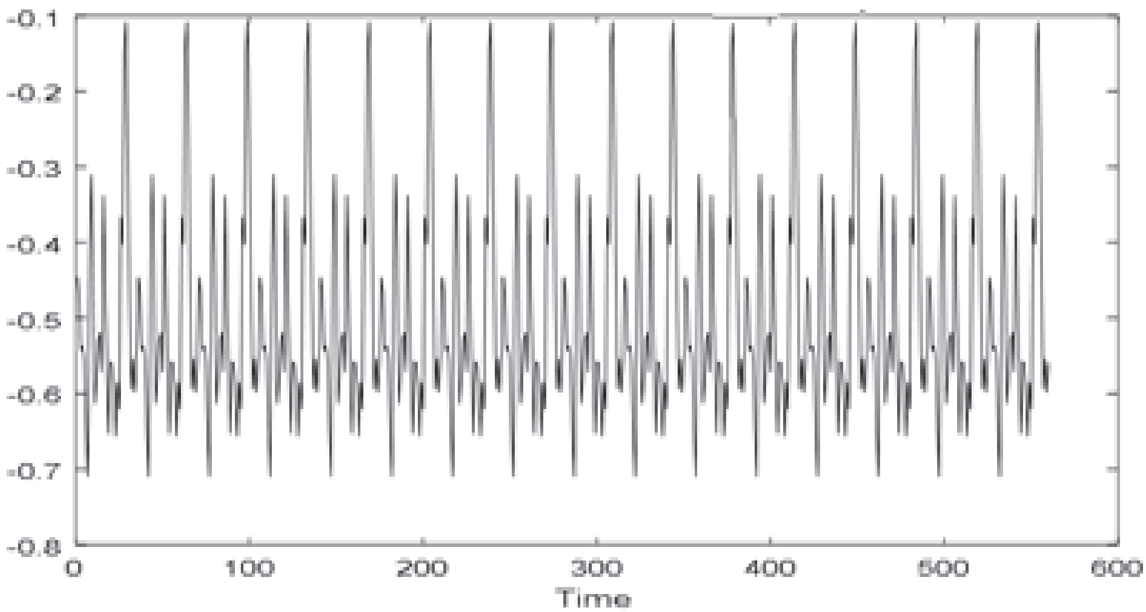

Figure 6. The waveform of the value of the coefficient a_{11} of the polynomial $AM_1(z)$ in the metamodel of the DAM system and the DAM system model for 560 input-output samples concerning the period p1 in the period 0-1 hours, i.e. from the period from 1.01 to 30.06.2013. Determinations: Time, x-axis – long time concerning the p1 period (subsequent models of the DAM system). Source: own development in the MATLAB environment using SIT [6, 30].

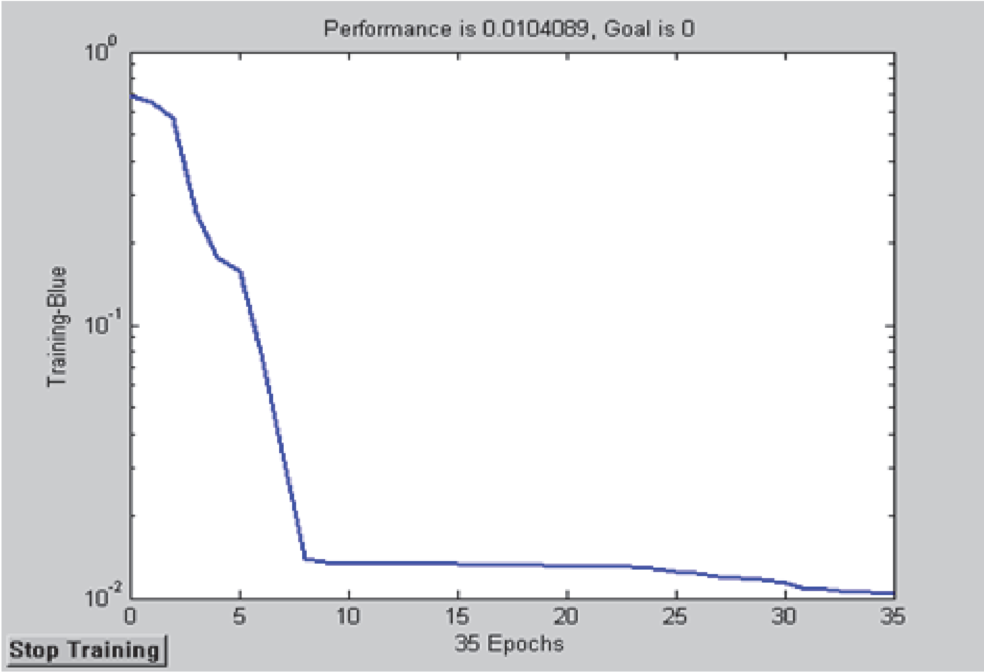

Figure 7. The course of the MSE learning error for the considered metamodel of the DAM system, that is, for the period p1. Designations: X-axis – epochs corresponding to subsequent models of the DAM system, y-axis – MSE error value. Source: own elaboration using MATLAB and Simulink [6].

The input quantities to the metamodel of the DAM system were the four coefficients of the polynomial $BM_1(z)$, standing at subsequent operators z^{-i}, and the output quantities were the four coefficients of the polynomial $AM_1(z)$, standing similarly with subsequent operators z^{-i}. In the example under consideration, there were 35 training pairs.

ANN learned the metamodel of the DAM system after 8 epochs, and the MSE error dropped byd value 0.7555 to the value of 0. 255 $^{10-1}$, so by an order of magnitude. It was assumed that the ANN had one hidden layer of neurons with four neurons. Both in the hidden layer and in the output layer, the tansig() function was adopted as a function of neuronal activation. The generated neural metamodel is shown in Figures 8–10.

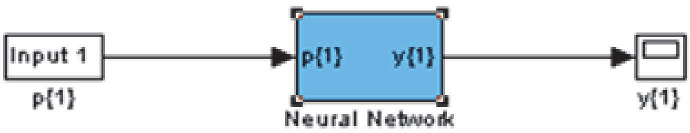

Figure 8. Neural metamodel of the DAM system in the MATLABA and Simulink environment. Designations: Input 1 (p{1}) – input vector to ANN, Neural Network – Artificial Neural Network, y{1} – output vector from ANN. Source: own development in MATLAB and Simulink using NNT [6].

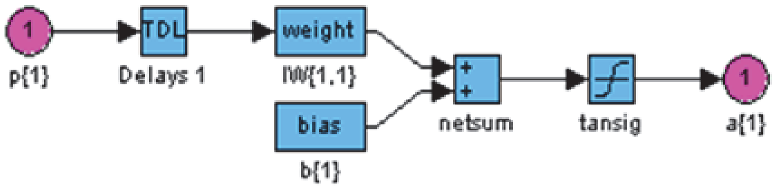

Figure 9. Model of the first layer of neurons in ANN as a metamodel of the DAM system

Designations: weight – weight matrix of the form IW{1,1}, bias – vector of biases of the form b{1}, netsum – summation block, Delays 1 – delay of input signals by one unit of time, tansig – neuron activation function, p{1} – matrix of input quantities to the first layer of neurons, a{1} – matrix of output quantities from the first layer of neurons. Source: own development in MATLAB and Simulink environment using NNT [6].

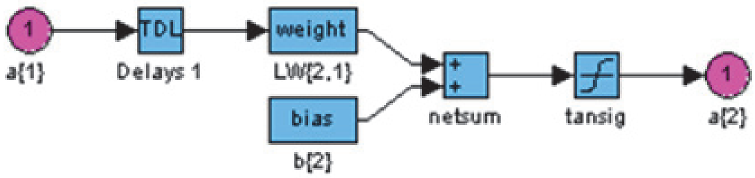

Figure 10. Model of the second layer of neurons in ANN as a metamodel of the DAM system. Designations analogous to Figure 4.16. Source: own study in matlab and Simulink environment using NNT [6].

For the purpose of comparative studies of the behavior of the neural metamodel in relation to the DAM system model, a flowchart was constructed as in Figure 11. An exampleof the obtained waveforms of discrepanciesand between the output signals from the metamodel and the DAM system model is shown in Figures 12-15.

High metamodel accuracy was achieved for the DAM model, measured by the discrepancy in the output quantities from the metamodels with respect to the parameter data of the respective DAM model. It can be noted that depending on the rolling model, discrepancies in the output signals between the metamodel and the data of the corresponding DAM system model basically change in the range from – 0.2 to 0.2.

To sum up, in order to identify the DAM system on the basis of figures listed on the Day Ahead Market of the Polish Power Exchange in the period from 1 January 2013 to 30 April 2016, 24 input variables were distinguished in relation to the total volume of electricity delivered and sold of all transactions carried out at the trading session in individual hours of the day (input variables) [MWh] and single volumes were distinguished. output related to the price obtained by the weighted average volume obtained from the supply and sale of electricity from all transactions on the trading session for a given hour of the day [PLN/MWh].

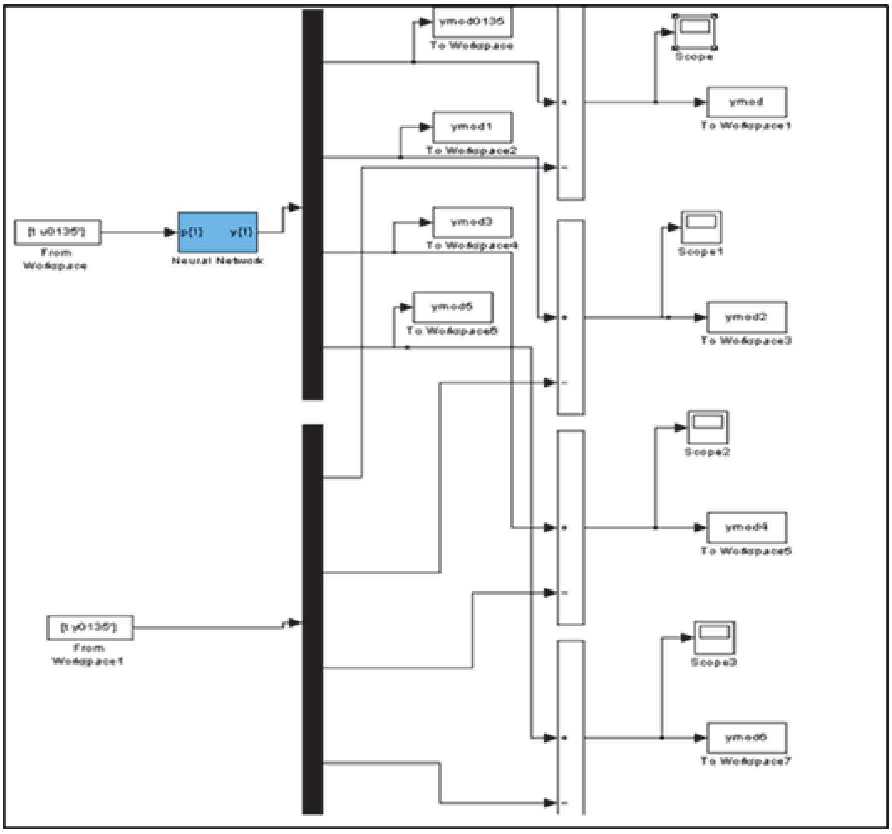

Figure 11. Example of a flowchart in Simulink for comparative studies of a neural metamodel in ANN with an DAM system model. Source: own development in MATLAB and Simulink environment using NNT [6].

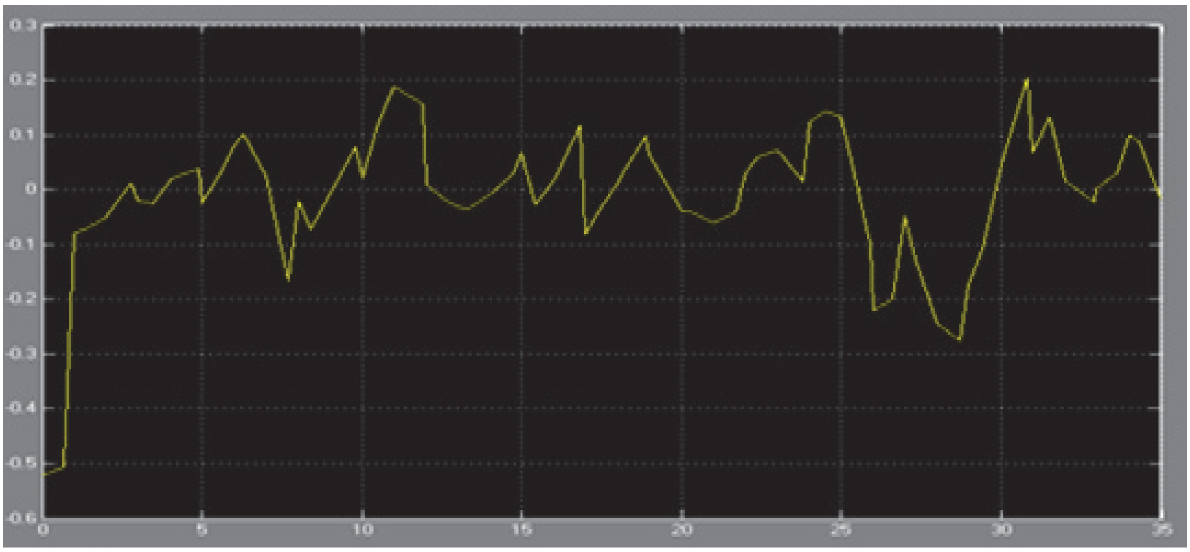

Figure 12. The course of discrepancies between the output signal on the first neuron from the second layer of metamodel neurons and the analogous output signal from the DAM system model. Designations: y-axis – divergence, x-axis – epochs as a long time. Source: Aboutown work in matlab and Simulink environments using NNT [6].

As a result of the identification, 24 models were obtained with 24 input variables (volumes from individual hours of the day) and with a single output variable (volume-weighted average prices in individual 24 hours of the day). A total of 35 periods of six-month data range were distinguished, covering contractual 184 days of quotations on DAM (rolling models), and identification was carried out with the progress of one month.

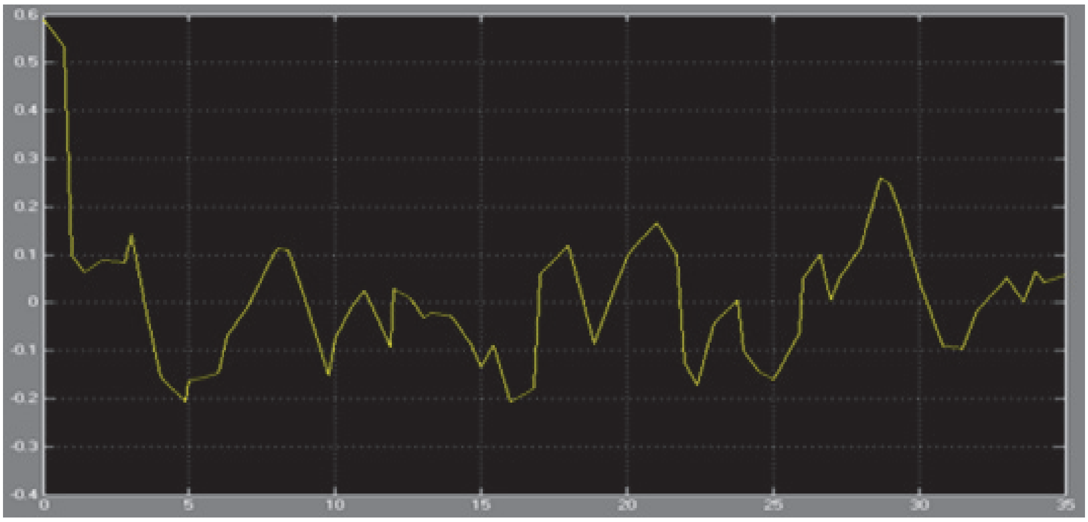

Figure 13. The course of discrepancies between the output signal on the second neuron from the second layer of metamodel neurons and the analogous output signal from the DAM system model. Designations: y-axis – discrepancy, x-axis – epoch as a long time. Source: own development in MATLAB and Simulink using NNT [6].

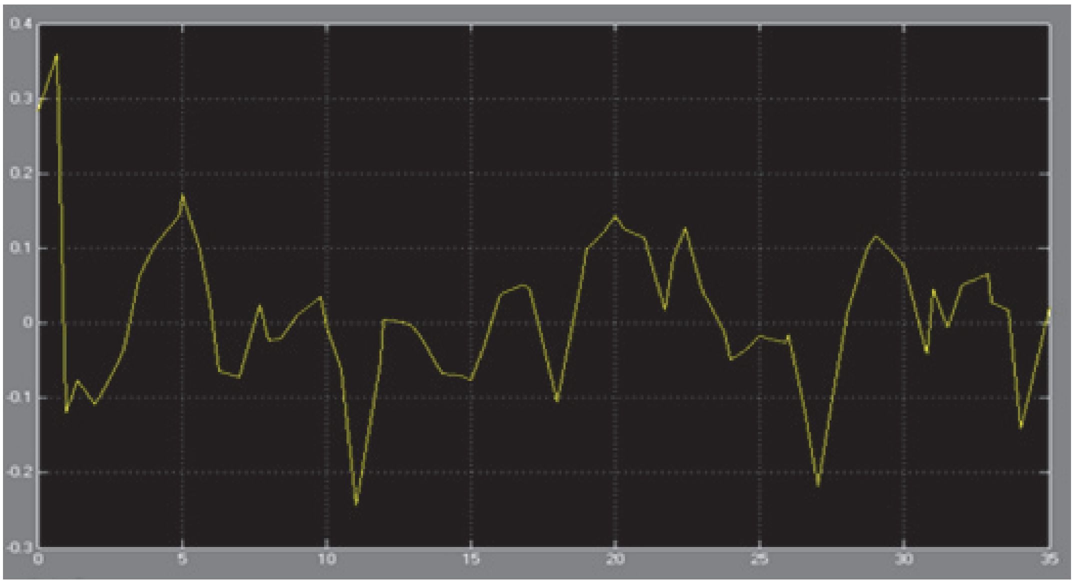

Figure 14. The course of discrepancies between the output signal on the third neuron from the second layer of metamodel neurons and the analogous output signal from the DAM system model. Designations: y-axis – divergence, x-axis – epochs as a long time. Source: own development in matlab and Simulink environment using NNT [6].

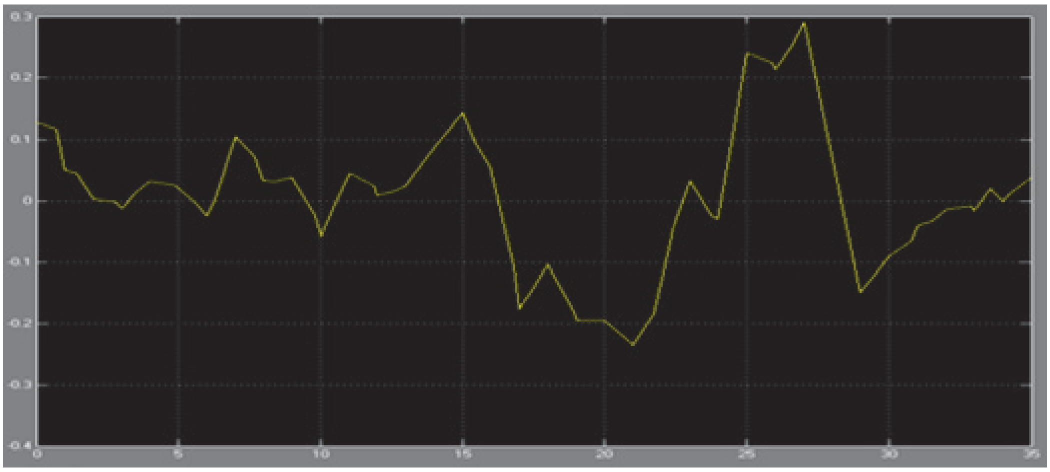

Figure 15. The course of discrepancies between the output signal on the fourth neuron from the second layer of metamodel neurons and the analogous output signal from the DAM system model. Designations: y-axis – divergence, x-axis – epochs as a long time. Source: own development in matlab and Simulink environment using NNT [6].

In this way, parametric discrete models of the linear type (arx) were first obtained, and on their basis a metamodel using the Perceptron Artificial Neural Network. In addition, the learning results are shown on the example of data obtained for hours 0-1, that is, the ANN of the DAM system metamodel, that is, the coefficients of the system model, were taught.

6. Conclusion

All participants of T GE S.A. are interested in identifying the Day Ahead Market system, i.e. both suppliers, recipients (more and more often also prosumers) as well as intermediaries in electricity trading, including TGE S.A.

One of the groups of modeling methods is identification modeling. In the results of identification u, a sufficiently large catalog of 35 parametric models in the rolling system was obtained, which made it possible to carry out metaidentization leading to parametric metamodels and neuronal metamodels of the DAM system, and it turned out that depending on the rolling model, discrepancies were obtained. in the output quantities from the metamodels in relation to the given parameters of the relevant DAM system model, for example, the a_1 parameter (which is the weight of the volume-weighted average price ee) changed in the range from -0.2 to 0.2 – values expressed without a name.

The metaidentification was carried out on the basis of figures listed on the Day Ahead Market of TGE S.A. in the period from 1 January 2013 to 30 April 2016, for 24 input variables in relation to the total volume of electricity delivered and sold of all transactions carried out at the exchange session in individual hours of the day (input variables) [MWh] and for individual output quantities related to the received the volume-weighted average price obtained from the supply and sale of electricity from all transactions on the trading session for a given hour of the day [PLN/MWh]. In this way, 24 models were obtained with 24 input variables (volumes from individual hours of the day) and with single output variables as a volume weighted average of electricity prices in with a specific 24 hours of the day.

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