Control and Cybernetics

vol. **46** (2017) No. 4

Robust soft variable structure control of perturbed singular systems with constrained input*

by

M. S. Asadinia[1] **and Tahereh Binazadeh**[2*]

[1] Ph.D. Student, Department of Electrical and Electronic Engineering, Shiraz University of Technology, Modares Blvd., Shiraz, Iran
m.asadinia@sutech.ac.ir

[2] Associate Professor, Department of Electrical and Electronic Engineering, Shiraz University of Technology, Modares Blvd., Shiraz, Iran
binazadeh@sutech.ac.ir

*corresponding author

Abstract: This paper investigates the robust soft variable structure (RSVS) control technique for perturbed singular systems with constrained input control. The aim of the RSVS control law, in addition to achieving desirable control performance for the constrained input, is the robust stability of the closed-loop system in the presence of perturbation. In this paper, the RSVS control for perturbed singular systems is designed for two cases. First, it is assumed that the perturbation term vanishes at the origin. In this case, the proposed RSVS controller leads to asymptotic stabilization of the perturbed singular system. In the second case, the perturbed singular systems with non-vanishing perturbation are considered and the robustness of RSVS is also investigated. In this situation, the proposed controller guarantees practical stability of the perturbed singular system. Finally, computer simulations are provided for two examples to verify the theoretical results.

Keywords: singular systems; robust stabilization, vanishing perturbation; non-vanishing perturbation

1. Introduction

Singular systems appear in the analysis of some practical systems and processes, such as, for instance, power systems (Ayasun, Nwankpa and Kwatny, 2005), aerospace engineering (Liu and Wen, 1997), processes related to petroleum (Gani and Cameron, 1992), robot manipulation models (Krishnan and McClamroch, 1994), mechanical systems (You and Chen, 1993), as well as economic systems (Luenberger and Arbel, 1977). Singular systems are being referred to by

*Submitted: November 2016; Accepted: February 2018.

different names like descriptor systems, generalized, algebraic-differential, semi-state and implicit, which are compositions of algebraic and dynamic equations (Dai, 1989). These systems, which have relatively more general structure in comparison with non-singular systems, are also more sophisticated and special theories have been developed for dealing with them, matching appropriately their complexity. In short, they should be solved by different methods.

During the past two decades, numerous researchers have been studying singular systems (Asadinia and Binazadeh, 2017; Dai, 1989; Yang et al., 2018, Jafari, Binazadeh, 2017). Some considerations include the issues of solvability, stability, controllability and observability conditions for singular systems (Ishihara and Terra, 2002; Men, Zhang, Li, Yang and Chen, 2006; Wu, Duan and Zhou, 2008; Duan, Wu and Zhang, 2012; Zhang and Yu, 2016). The robust stability and robust stabilization for uncertain singular systems have been analyzed by Asadinia and Binazadeh (2016), Zhang and Yang (2003), Xu, Van Dooren, Stefan and Lam (2002), Lu, Su, Xue and Chu (2008), Xu and Lam (2006). Also, optimal control of singular systems has been discussed in Balasubramaniam, Abdul Samath and Kumaresan (2007), Lin and Yang (1988), Shu and Zhu (2017).

On the other hand, a great deal of research has been reported in relation to the analysis and design methods for nonlinear singular systems. For example: the stability of nonlinear singular systems was analyzed in Yang, Zhang and Zhou (2013), Yang, Sun, Zhang and Ma (2013), Yang, Zhang, Lin and Zhou (2006), and nonlinear control techniques, such as sliding mode, feedback linearization and variable structure control have been designed for these systems (see Wu and Zheng, 2009; Lin, 2012; Liu and Wen, 1988; Guo and Gao, 2007; Wu and Ho, 2010; Xiaoping and Celikovsky, 1997). One of the advanced nonlinear control strategies, which has been applied for singular systems is soft variable structure control (consult Liu, Zhang and Gao, 2012; Liu, Kao, Gu and Karimi, 2015).

Soft variable structure control (SVSC) is a kind of variable structure control lacking sliding mode. It offers numerous benefits, such as achieving high regulation rates, shortening of settling times and little system chattering. In SVSC, controller parameter values or structures are continuously varying (Adamy and Flemming, 2004). In Liu, Zhang and Gao (2012) and Liu, Kao, Gu and Karimi (2015), the SVSC has been proposed for the nominal singular system and therefore the analysis of robust stabilization for the applied control law has not been presented.

However, let us add in this context, that in practice it is not realistic to assume that mathematical model is an exact description of a physical system. At best, a model can be an approximation of a real system. The difference between the mathematical model and the true system can be modeled through perturbation terms. The perturbation terms could result from modeling errors, aging, or uncertainties and disturbances, which exist in any realistic problem (Marquez, 2013). Therefore, in order to guarantee the robust stabilization of the perturbed systems, explicit consideration of the perturbation terms is necessary.

In this paper, a class of singular systems under vanishing and non-vanishing perturbations is considered and the robust version of soft variable structure control (called RSVS) is extended for these systems. To the best of authors' knowledge, this subject has not been studied in the literature. In the case when perturbation terms are vanishing, it is demonstrated that the RSVS controller leads to asymptotical stability of the closed-loop perturbed system. However, designing of controller for perturbed singular systems under non-vanishing perturbation results only in what is called practical stability. In this situation, the best one can expect is that the state variables be ultimately bounded by small bound.

The scheme of content of this paper is as follows. The singular systems' definition and some preliminaries are given in Section 2. Problem statement and aims of the paper are presented in Section 3. The main results, concerning the design of the RSVS controller for singular systems under vanishing perturbations, are discussed in Section 4. The robustness of soft variable structure control for singular systems in the presence of non-vanishing perturbations is investigated in Section 5. Additionally, two examples are provided to show the effectiveness of the proposed controller in Section 6. Finally, conclusions are given in Section 7.

2. Preliminaries

Consider the following singular system:

$$E\dot{x}(t) = Ax(t) + bu(t) \tag{1}$$

where $x \in \mathrm{R}^n$ is the state vector, $u \in \mathrm{R}$ is the input vector, $A, E \in \mathrm{R}^{n\times n}$ are constant matrices, $b \in \mathrm{R}^n$ is a constant vector, and $rank(E) = r < n$.

Definition 1 *(Xu and Lam, 2006):*

1. *The pair (E, A) is said to be regular, if $\det(sE - A)$ is not identically zero for any $s \in C$, where C is regarded as the complex plane.*
2. *The pair (E, A) is called impulse-free, if $\det(sE - A) = rank(E)$.*
3. *The pair (E, A) is said stable, if all the roots of $\det(sE - A) = 0$ have negative real parts.*
4. *The pair (E, A) is said admissible, if it is regular, impulse-free and stable.*

Definition 2 *(Dai, 1989): The singular system (1) is called controllable if, for any $t_1 > 0$, $x(0) \in R^n$ and $w \in R^n$, there exists a control input $u(t)$ such that $x(t_1) = w$.*

Lemma 1 *(Dai, 1989) The singular system (1) is completely controllable if and only if for any $s \in C$, it holds that:*

$$rank \begin{bmatrix} sE - A & b \end{bmatrix} = n$$

and

$$rank \begin{bmatrix} E & b \end{bmatrix} = n.$$

Lemma 2 *(Zhang and Yang, 2003) The singular system (1) is admissible if and only if there exists a solution $R \in R^n$ to the Lyapunov equation:*

$$A^T R + R^T A = -Q \tag{2}$$

satisfying

$$E^T R = R^T E \geq 0 \tag{3}$$

for any positive-definite Q.

3. Problem definition

Consider a perturbed singular system as follows:

$$E\dot{x}(t) = Ax(t) + bu(t) + d(x,t) \tag{4}$$

where $x \in \mathrm{R}^n$is the state vector, $u \in \mathrm{R}$ is the input vector and $d(x,t) : D \times [0,\infty) \to \mathrm{R}^n$ is the perturbation vector, which is piecewise continuous in t and locally Lipschitz in x on $D \subseteq \mathrm{R}^n$, where D is a domain that contains the origin. The perturbation vector $d(x,t)$ might result from the modeling errors or uncertainties and disturbances, which exist in any practical system. In a topical situation, the nonlinear function $d(x,t)$ is not known, but some information about its upper bound is available.

In this paper, the robust soft variable structure control (RSVS) for the perturbed singular system (4) is designed, satisfying the following constraint:

$$|u| \leq u_{\max}, \tag{5}$$

where $u_{\max} > 0$ is the maximum allowable amplitude of input, due to actuator saturation. In this paper, the procedure of robust controller designing is investigated for two cases. First, it is assumed that the perturbation term $d(x,t)$ vanishes at the origin (i.e. $d(0,t) = 0 \ \ \forall t \geq 0$). In this situation, the perturbed singular system (4) has an equilibrium point at the origin. Therefore, the RSVS controller, which will be proposed, guarantees the asymptotical stability of the closed-loop perturbed singular system. In the second case, it is assumed that $d(0,t) \neq 0$ and an RSVS controller is designed for the singular system with non-vanishing perturbation. In this case, the perturbed singular system (4) has no equilibrium point. Therefore, in this case, it cannot be expected that $x(t) \to 0$ as $t \to \infty$. The best thing that can be expected is that $\| x(t)\|$ be ultimately bounded by an appropriately small bound, which corresponds to the concept of practical stability. Thus, the designed RSVS controller leads to practical stabilization of the perturbed singular system (4).

4. RSVS control design for singular systems with vanishing perturbations

In this section, the singular system (4) is considered under the assumption of vanishing perturbation. The RSVS control law, which guarantees the asymptotical stability of the closed-loop perturbed singular system is presented in the following theorem.

THEOREM 1 *Consider the perturbed singular system (4). Suppose the perturbation term $d(x,t)$ satisfies:*

$$\|d(x,t)\| \leq \gamma \|x\|, \quad d(0,t) = 0, \ \forall t \geq 0, \forall x \in D \tag{6}$$

where γ is a known positive constant. Then, the following RSVS control law

$$u(t) = -\left(g + p(t)h\right)^T x(t) \tag{7}$$

with the following differential equation

$$\dot{p}(t) = \frac{x^T(t)\hat{R}^T b h^T x(t) - p(t)S(x,p)}{r} \tag{8}$$

guarantees asymptotical stabilization of the perturbed singular system (4) if the matrix satisfies the following condition:

$$\lambda_{\max}(\hat{R}) < \frac{\lambda_{\min}(\hat{Q})}{2\gamma} \tag{9}$$

where the vector $g \in R^n$ is chosen in such a way that the pair $(\hat{A} = A - bg^T, E)$ is admissible, also the vector $h \in R^n$ is selected so as to ensure better dynamic quality. The parameter $p(t)$ is determined by Eq. (9). Moreover, r is a positive constant and $S(x,p)$ is a positive function. Additionally, $\hat{R} \& \hat{Q} \in R^{n\times n}$ are positive-definite matrices, which satisfy the singular Lyapunov equation

$$\hat{A}^T\hat{R} + \hat{R}\hat{A} = -\hat{Q}$$

with constraint $E^T\hat{R} = \hat{R}^T E \geq 0$, and $\lambda_{\min}$ and $\lambda_{\max}$ denote the minimum and maximum eigenvalues, respectively.

PROOF: By substituting control law (7) into (4) and considering (8), total state equations of the perturbed singular system become as follows

$$\begin{cases} E\dot{x}(t) = (\hat{A} - pbh^T)x(t) + d(x,t) \\ \dot{p}(t) = \frac{x^T(t)\hat{R}^T b h^T x(t) - pS(x,p)}{r} \end{cases} \tag{10}$$

The perturbed singular system (10) has a unique equilibrium point in the origin, $\left[x^T, p\right] = [0, 0]$. Consider the following function as a Lyapunov function candidate:

$$V(x,p) = x^T E^T \hat{R} x + rp^2 \tag{11}$$

By Lemma 2 and since the pair $(\hat{A}, E)$ is admissible, there exists a positive-definite matrix $\hat{R}$ for an arbitrarily positive-definite matrix $\hat{Q}$, such that

$$\hat{A}^T\hat{R} + \hat{R}^T\hat{A} = -\hat{Q} \tag{12}$$

$$E^T\hat{R} = \hat{R}^T E \geq 0. \tag{13}$$

The derivation of $V(x,p)$ along the trajectories of (10) is given by:

$$\begin{aligned}\dot{V} &= \dot{x}^T E^T \hat{R}x + x^T E^T \hat{R}\dot{x} + 2rp\dot{p} = \left((\hat{A} - pbh^T)x + d(x,t)\right)^T \hat{R}x \\ &+x^T\hat{R}^T\left((\hat{A} - pbh^T)x + d(x,t)\right) + 2rp\frac{x^T\hat{R}^T bh^T x - pS(x,p)}{r} \\ &= x^T(\hat{A}^T\hat{R} + \hat{R}^T\hat{A})x + 2x^T\hat{R}^T d(x,t) - 2p^2 S(x,p).\end{aligned}$$

Considering (12), one has:

$$\dot{V} = -x^T\hat{Q}x + 2x^T\hat{R}^T d(x,t) - 2p^2 S(x,p). \tag{14}$$

Since $S(x,p) > 0$, $\hat{R} > 0$ and $\|d(x,t)\| \leq \gamma\,\|x\|$, we obtain:

$$\begin{aligned}&\dot{V} \leq -\lambda_{\min}(\hat{Q})\,\|x\|^2 + 2\lambda_{\max}(\hat{R})\,\|x\|\,\|d(x,t)\| \\ &\leq -\lambda_{\min}(\hat{Q})\,\|x\|_2^2 + 2\gamma\,\lambda_{\max}(\hat{R})\,\|x\|^2 = -(\lambda_{\min}(\hat{Q}) - 2\gamma\,\lambda_{\max}(\hat{R}))\,\|x\|^2.\end{aligned} \tag{15}$$

Hence, the origin is a robustly asymptotically stable point of the closed-loop system (10) if $(\lambda_{\min}(\hat{Q}) - 2\gamma\,\lambda_{\max}(\hat{R})) > 0$, or, in another way of stating this:

$$\lambda_{\max}(\hat{R}) < \frac{\lambda_{\min}(\hat{Q})}{2\gamma}. \tag{16}$$

The matrix $\hat{R}$ is given by computing the singular Lyapunov equation (12) in such a way as to satisfy both conditions (12) and (13). The solution $\hat{R}$ may be found by linear matrix inequality (LMI) method. □

Remark 1 *The RSVS control law (7) guarantees asymptotical stability of the perturbed singular system (10) for any $S(x,p) > 0$. However, this control law should satisfy the condition of constrained input (5). Thus, the function $S(x,p)$ should be designed so as to ensure compliance with condition (5).*

From condition (5), one has:

$$-u_{\max} \leq -(g + ph)^T x \leq u_{\max}. \tag{17}$$

From the above inequality, one can compute the admissible range of the selection parameter p as follows:

$$\begin{cases} \frac{-g^T x - u_{\max}}{h^T x} \leq p \leq \frac{-g^T x + u_{\max}}{h^T x}, & h^T x > 0 \\ \frac{-g^T x + u_{\max}}{h^T x} \leq p \leq \frac{-g^T x - u_{\max}}{h^T x}, & h^T x < 0 \end{cases}. \tag{18}$$

The selection parameter p will be infinite, when $x \to 0$. Hence, the additional restriction on $p(t)$ is assumed as follows

$$-p_1 \leq p(t) \leq p_2 \tag{19}$$

where p_1 and p_2 are positive constants. Combining (18) and (19) leads to:

$$\alpha(x(t)) \leq p(t) \leq \beta(x(t)) \tag{20}$$

where $\alpha(x)$ and $\beta(x)$ are functions of the state variables and are defined by following formulas (Adamy and Flemming, 2004):

$$\alpha(x) = \begin{cases} \frac{-g^T x + u_{\max}}{h^T x}, & h^T x \leq \frac{g^T x - u_{\max}}{p_1} \\ -p_1, & \frac{g^T x_1 - u_{\max}}{p_1} < h^T x < \frac{g^T x + u_{\max}}{p_1} \\ \frac{-g^T x - u_{\max}}{h^T x}, & h^T x \geq \frac{g^T x + u_{\max}}{p_1} \end{cases} \tag{21}$$

$$\beta(x) = \begin{cases} \frac{-g^T x - u_{\max}}{h^T x}, & h^T x \leq \frac{-g^T x - u_{\max}}{p_2} \\ p_2, & \frac{-g^T x - u_{\max}}{p_2} < h^T x < \frac{-g^T x + u_{\max}}{p_2} \\ \frac{-g^T x + u_{\max}}{h^T x}, & h^T x \geq \frac{-g^T x + u_{\max}}{p_2} \end{cases} . \tag{22}$$

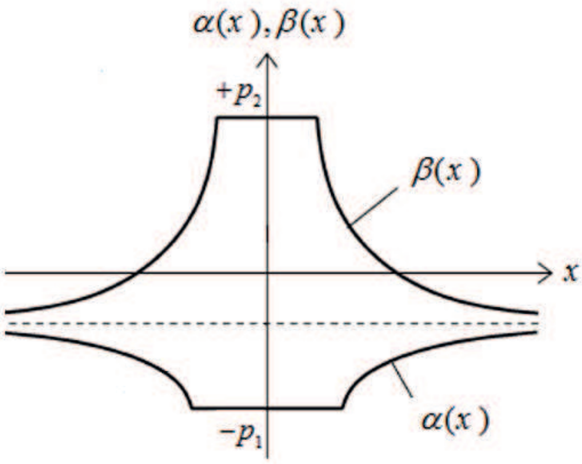

Figure 1. Functions $\alpha(x)$ and $\beta(x)$ (Adamy and Flemming, 2004)

Figure 1 shows the plots of the functions $\alpha(x)$ and $\beta(x)$ for a single variable x. A suitable choice for $S(x, p)$ is as follows:

$$S(x,p) = \begin{cases} \eta_1(1 - \frac{\alpha(x)}{p}) + \eta_2 \frac{\alpha(x)}{p}, & p \leq \alpha(x) \\ \eta_2, & \alpha(x) < p < \beta(x) \\ \eta_1(1 - \frac{\beta(x)}{p}) + \eta_2 \frac{\beta(x)}{p}, & p \geq \beta(x) \end{cases} \tag{23}$$

where $\eta_1 >> 1$, $0 < \eta_2 << 1$. Using (21) and (22), it can be shown that inequality $S(x, p) > 0$ is satisfied. Consequently, the perturbed singular system (4) is asymptotically stabilizable by the control law (7).

5. RSVS control for singular systems with non-vanishing perturbations

In this case, the more general situation, that of $d(0,t) \neq 0$, is considered, where the origin may not be an equilibrium point of the perturbed system (4). In the following, a theorem is given, which establishes the practical stability of the closed-loop system with control law (7).

Theorem 2 *Consider the perturbed singular system (4). Suppose the perturbation term $d(x,t)$ satisfies:*

$$\|d(x,t)\| \leq \delta, \quad d(0,t) \neq 0, \qquad \forall t \geq 0, \forall x \in D \tag{24}$$

where δ is a known positive constant. The robust control law (7) with differential equation (8) guarantees the robust practical stabilization of the perturbed singular system (4) with non-vanishing perturbation $d(x,t)$, in the region

$$\|x\| \geq 2\delta\lambda_{\max}(\hat{R})/\theta\lambda_{\min}(\hat{Q}) \tag{25}$$

where $0 < \theta < 1$, and matrix $\hat{R}$ is found from the singular Lyapunov equation (12) for any arbitrary positive-definite matrix $\hat{Q}$.

Proof: Use $V(x,p)$, proposed in (11), as the Lyapunov function candidate. Then, the derivative of $V(x,p)$ along the trajectories of (10) satisfies

$$\begin{aligned}\dot{V} &= \dot{x}^T E^T \hat{R}x + x^T E^T \hat{R}\dot{x} + 2rp\dot{p} = (E\dot{x})^T \hat{R}x + x^T \hat{R}^T E\dot{x} + 2rp\dot{p}\\ &= \left((\hat{A} - pbh^T)x + d(x,t)\right)^T \hat{R}x + x^T \hat{R}^T \left((\hat{A} - pbh^T)x + d(x,t)\right)\\ &\quad +2rp\left(\frac{x^T \hat{R}^T bh^T x - pS(x,p)}{r}\right).\end{aligned} \tag{26}$$

One has

$$\begin{aligned}\dot{V} &= x^T(\hat{A}^T\hat{R} + \hat{R}^T\hat{A})x - 2px^T\hat{R}^T bh^T x + 2x^T\hat{R}^T d(x,t)\\ &\quad +2px^T\hat{R}^T bh^T x - 2p^2 S(x,p)\\ &= -x^T\hat{Q}x + 2x^T\hat{R}^T d(x,t) \ - 2p^2 S(x,p)\end{aligned} \tag{27}$$

where the matrix $\hat{R}$ is obtained from solving the Lyapunov equation (12) so as to ensure satisfaction of $E^T\hat{R} = \hat{R}^T E \geq 0$, and $S(x,p)$ is defined according to formula (23). Therefore:

$$\begin{aligned}\dot{V} &\leq -\lambda_{\min}(\hat{Q})\,\|x\|^2 + 2\lambda_{\max}(\hat{R})\,\|x\|\,\|d(x,t)\|\\ &= -(1-\theta)\lambda_{\min}(\hat{Q})\,\|x\|^2 - \theta\lambda_{\min}(\hat{Q})\,\|x\|^2 + 2\lambda_{\max}(\hat{R})\,\|x\|\,\delta\end{aligned} \tag{28}$$

where $0 < \theta < 1$ is a positive constant. If the term

$$\left(-\theta\lambda_{\min}(\hat{Q})\,\|x\|^2 + 2\lambda_{\max}(\hat{R})\,\|x\|\,\delta\right)$$

is negative definite, or, in other formulation,

$$\|x\| \geq 2\delta\lambda_{\max}(\hat{R})/\theta\lambda_{\min}(\hat{Q}),$$

then:

$$\dot{V} \leq -(1-\theta)\lambda_{\min}(\hat{Q})\,\|x\|^2\,, \quad \forall\,\|x\| \geq 2\delta\lambda_{\max}(\hat{R})/\theta\lambda_{\min}(\hat{Q}). \tag{29}$$

It can be seen from (29) that the trajectories of the state variables of the closed-loop perturbed singular system (11) are bounded for all $t \geq 0$ and the robust practical stability of the singular system (4) under non-vanishing perturbations is guaranteed by the proposed control law (7). □

6. Computer simulations

In this section, two illustrative examples will be presented, meant to show the robust performance of the RSVS controller.

First Example: Consider the following perturbed singular system:

$$E\dot{x}(t) = Ax(t) + bu(t) + d(x,t) \tag{30}$$

with the following specifications:

$$E = \begin{bmatrix} 1 & 1 & 1 \\ 5 & 7 & -1 \\ 2 & 3 & -1 \end{bmatrix}, \quad A = \begin{bmatrix} -1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & -5 & 0 \end{bmatrix},$$
$$b = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad d(t,x) = \begin{bmatrix} \mu_1 x_1 \sin x_2 \\ \mu_2 x_2 \\ 0 \end{bmatrix} \tag{31}$$

where μ_1 and μ_2 are unknown constants, with $|\mu_1| \leq 1$; $|\mu_2| \leq 0.5$. The control input is confined to the range

$$|u| \leq u_{\max} = 20. \tag{32}$$

From the given information about the perturbation term, one has:

$$\begin{aligned} \|d\| &= \sqrt{\mu_1^2 x_1^2 \sin^2 x_2 + \mu_2^2 x_2^2} \\ &\leq \sqrt{x_1^2 + 0.25 x_2^2} \\ &\leq \sqrt{x_1^2 + 0.25 x_2^2} \leq \|x\|\,. \end{aligned} \tag{33}$$

Therefore, the value of the constant number γ in condition (6) is $\gamma = 1$. Also, vectors $g^T = \begin{bmatrix} 5 & 5 & 10 \end{bmatrix}$ and $h^T = \begin{bmatrix} 0 & -8 & 1 \end{bmatrix}$ have been chosen. Thus, the matrix for the Lyapunov equation (12) such that it satisfies conditions (13) and (16) has been found by LMI as

$$\hat{R} = \begin{bmatrix} 0.250 & 0.003 & -0.009 \\ 0.003 & 0.040 & -0.006 \\ -0.009 & -0.006 & 0.061 \end{bmatrix}.$$

For the above matrix $\hat{R}$, $\lambda_{\max}(\hat{R}) = 0.2391$, which satisfies condition (16). Finally, the parameters $\eta_1 = 10^6$, $\eta_2 = 10^{-2}$, $p_1 = p_2 = 100$ and $r = 1$ are chosen. Then, the robust control law will be as follows:

$$u = -5x_1 - 5(1-p)x_2 - 2(5+p)x_3$$

with the following dynamic equation:

$$\dot{p} = 2.95 \times 10^{-5}x_2^2 + 1.016 \times 10^{-4}x_3^2 + 4.1 \times 10^{-5}x_1x_2$$
$$-1.64 \times 10^{-5}x_1x_3 - 2.658 \times 10^{-4}x_2x_3 - 10^{-3}pS(x,p)$$

where

$$S(x,p) = \begin{cases} 10^6(1 - \frac{\alpha(x)}{p}) + 10^{-2}\frac{\alpha(x)}{p}, & p \le \alpha(x) \\ 10^{-2}, & \alpha(x) < p < \beta(x) \\ 10^6(1 - \frac{\beta(x)}{p}) + 10^{-2}\frac{\beta(x)}{p}, & p \ge \beta(x) \end{cases}$$

with the functions $\alpha(x)$ and $\beta(x)$ being computed by equations (21) and (22). The response of the states of the perturbed singular system (30) under the designed RSVS controller for initial conditions $x_0 = \begin{bmatrix} 5 & -1 & -1 \end{bmatrix}^T$ are illustrated in Figs. 2 through 4. These figures verify the fact that the RSVS control law leads to asymptotic stability of the perturbed singular system (30). Also, Fig. 5 verifies that the proposed RSVS controller has met the condition of the constrained input, (32).

Second Example: Consider the singular system (34) such that the perturbation term $d(x,t)$ is of the non-vanishing type and the specifications are as follows.

$$E\dot{x}(t) = Ax(t) + bu(t) + d(x,t)$$

$$E = \begin{bmatrix} 1 & 1 & 1 \\ 5 & 7 & -1 \\ 2 & 3 & -1 \end{bmatrix}, \quad A = \begin{bmatrix} -1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & -5 & 0 \end{bmatrix}, \tag{34}$$

$$b = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad d(t,x) = \begin{bmatrix} \delta_1(t)\sin x_1 \\ \delta_2(t)\cos t \\ 0 \end{bmatrix}.$$

Suppose that $|\delta_i(t)| \leq 1$ for $i = 1, 2$. Therefore:

$$\|d(x,t)\| = \sqrt{(\delta_1(t)\sin x_1)^2 + (\delta_2(t)\cos t)^2} \leq \sqrt{2}.$$

Thus, $\delta = \sqrt{2}$. The RSVS control law is applied to the perturbed singular system (34) with initial conditions $x_0 = [5 - 1 - 1]^T$. Further, $\theta = 0.5$ is considered, and using $\hat{R}$ and $\hat{Q}$, which were proposed in Section 6.1, one obtains $\lambda_{\max}(\hat{R}) = 0.2387, \quad \lambda_{\min}(\hat{Q}) = 1$.

Figures 6 through 8 illustrate the responses of the singular system (34) with non-vanishing perturbations. As expected, the time responses of the state variables of the closed-loop singular system with non-vanishing perturbation converge to a band in the neighborhood of the origin. Control input $u(t)$ is presented in Fig. 9. The simulation results verify the fact that the designed control law leads to practical stabilization of the singular system in the presence of non-vanishing perturbations.

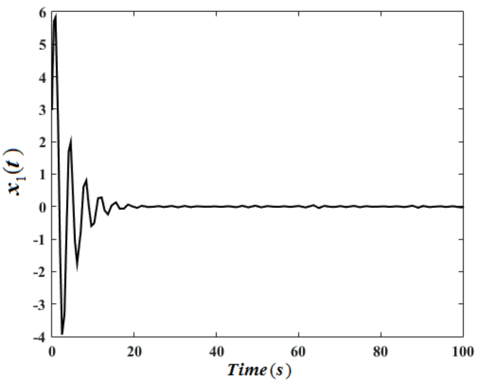

Figure 2. Time response of $x_1(t)$ under vanishing perturbations

7. Conclusions

This paper considered the robust stabilization of the perturbed singular systems consisting of vanishing and non-vanishing perturbations. For this purpose, the robust soft variable structure controller for the perturbed singular systems in the form of two theorems has been proposed. The first theorem proposed a robust control law for asymptotical stabilization of the singular systems under vanishing perturbations. Singular systems with non-vanishing perturbations do not have the equilibrium point(s), therefore practical stability is the best stability configuration for such systems. In the second theorem, the proposed RSVS controller results in practical stability of the closed-loop singular system. Finally, the effectiveness of the proposed technique was illustrated by computer simulations.

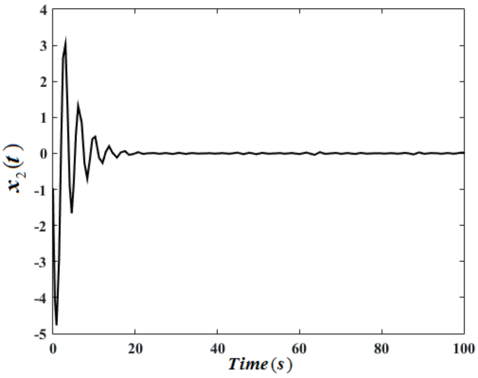

Figure 3. Time response of $x_2(t)$ under vanishing perturbations

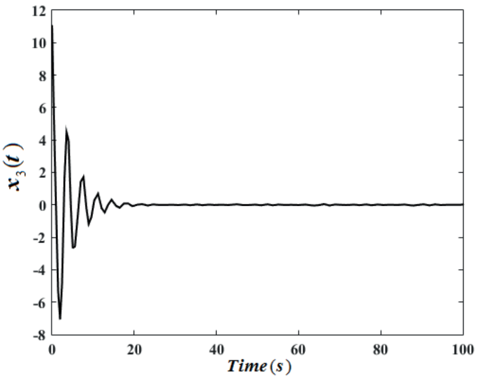

Figure 4. Time response of $x_3(t)$ under vanishing perturbations

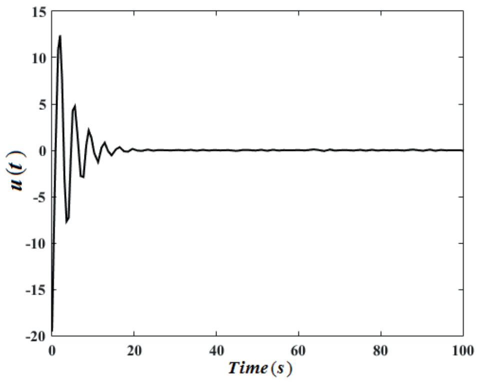

Figure 5. Time response of control input under vanishing perturbations

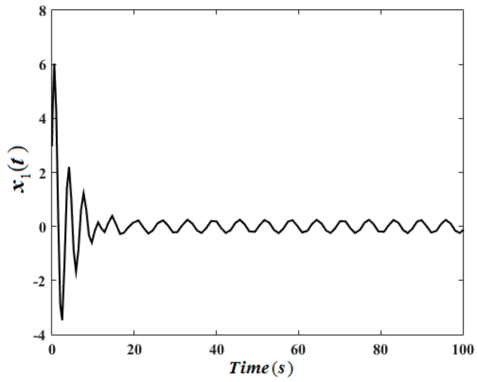

Figure 6. Time response of $x_1(t)$ under non-vanishing perturbations

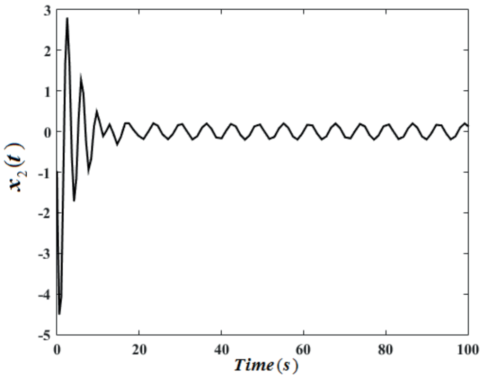

Figure 7. Time response of $x_2(t)$ under non-vanishing perturbations

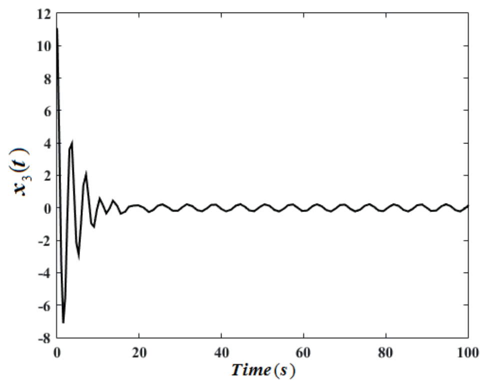

Figure 8. Time response of $x_3(t)$ under non-vanishing perturbations

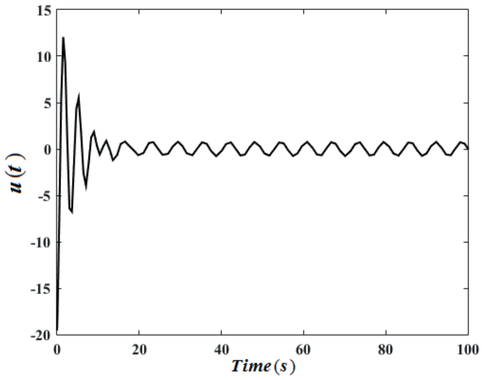

Figure 9. Time response of control input under non-vanishing perturbations

References

Adamy, J., and Flemming, A. (2004) Soft variable-structure controls: a survey. *Automatica,* **40** (11), 1821-1844.

Asadinia, M. and Binazadeh, T. (2016) Design of robust control law for stabilization of singular systems with time-varying delay in the presence of model uncertainty and input amplitude constraint. *Journal of Control*, **10**(3), 25-38.

Asadinia, M. S. and Binazadeh, T. (2017) Stabilization of time varying delay singular systems subject to actuator saturation. *Tabriz Journal of Electrical Engineering*, **74**(3), 843-855.

Ayasun, S., Nwankpa, CH.O. and Kwatny, H.G. (2005) An efficient method to compute singularity induced bifurcations of decoupled parameter-dependent differential algebraic power system model. *Applied Mathematics and Computation,* **167** (1), 435–453.

Balasubramaniam, P., Abdul Samath, J. and Kumaresan, N. (2007) Optimal control for nonlinear singular systems with quadratic performance using neural networks. *Applied Mathematics Computation,* **187** (2), 1535-1543.

Dai, L. (1989) *Singular Control Systems.* Springer, Berlin.

Duan, G.R., Yu, H.H., Wu, A.G. and Zhang, X. (2012) *Analysis and Design of Descriptor Linear Systems.* Science Press, Beijing.

Gani, R. and Cameron, I. T. (1992) Modelling for dynamic simulation of chemical processes: the index problem. *Chemical Engineering and Science,* **47** (5), 1311-1315.

Guo, J.F. and Gao, C. (2007) Variable structure control of discrete uncertain singular systems with multiple time-delays. *Control Decision,* **22** (1), 95-99.

Ishihara, J.Y. and Terra, M.H. (2002) On the Lyapunov theorem for singular systems. *Automatic Control, IEEE Transactions,* **47** (11), 1926-1930.

Jafari, E. and Binazadeh, T. (2017) Modified composite nonlinear feedback for nonstep output tracking of multi-input multi-output linear discrete-time singular systems with actuator saturation. *IEEE, 5th International Conference on Control, Instrumentation, and Automation (ICCIA)*, 2017, 114-119.

Krishnan, H. and McClamroch, N.H. (1994) Tracking in nonlinear differential-algebraic control systems with applications to constrained robot systems. *Automatica,* **30** (12), 1885-1897.

Luenberger, D.G. and Arbel, A. (1977) Singular dynamic Leontief systems. *Econometrica,* **45** (4), 991–995.

Liu, Y. and Wen, X. (1997) *Variable Structure Control for Singular Systems.* South China University of Technology Press, Guangzhou.

Lin, J.Y. and Yang, Z.H. (1988) Optimal control problems for singular systems. *International Journal of Control,* **47** (6), 1915-1924.

Lin, K.J. (2012) Sliding Mode Control Design for Uncertain Singular Systems. *Applied Mechanics and Materials,* **145**, 16-20.

Liu, Y.Q. and Wen, X.C. (1997) *The Theory and Application of Large-Scale Dynamic Systems: Variable Structure Control for Singular Systems.* South China University of Technology Press, **6**. Guangzhou.

Liu, Y., Zhang, C. and Gao, C. (2012) Dynamic soft variable structure control of singular systems. *Communications in Nonlinear Science and Numerical Simulation,* **17** (8), 3345-3352.

Liu, Y., Kao, Y., Gu, Sh. and Karimi, H.R. (2015) Soft variable structure controller design for singular systems. *Journal of the Franklin Institute,* **352** (4), 1613-1626.

Lu, R., Su, H. and Xue, A. (2008) *The Robust Control Theory of Singular Systems.* Science Press, Beijing.

Marquez, H.J. (2003) *Nonlinear Control Systems: Analysis and Design.* John Wiley Interscience.

Men, B., Zhang, Q., Li, X., Yang, C. and Chen, Y. (2006) The stability of linear descriptor systems. *International Journal of Information and Systems Sciences,* **2** (3), 362-374.

Shu, Y. and Zhu, Y. (2017) Stability and optimal control for uncertain continuous-time singular systems. *European Journal of Control.* 34, 16-23.

Uezato, E. and Ikeda, M. (1999) Strict LMI conditions for stability, robust stabilization, and H_∞ control of descriptor systems. *Proceedings of the 38th Conference on Decision and Control, Phoenix, Arizona USA,* 4, 4092-4097.

Wu, A.G., Duan, G.R. and Zhou, B. (2008) Solution to singular Sylvester matrix equations. *Automatic Control, IEEE Transactions,* **53** (3), 811–815.

Wu, L. and Ho, D.W.C. (2010) Sliding mode control of singular stochastic hybrid systems. *Automatica,* **46** (4), 779-783.

Wu, L. and Zheng, W.X. (2009) Passivity-based sliding mode control of uncertain singular time-delay systems. *Automatica,* **45** (9), 2120-2127.

Xiaoping L. and Celikovsky S. (1997) Feedback control of affine nonlinear singular control systems. *International Journal of Control,* **68** (4), 753-774.

Xu, S.Y. and Lam, J. (2006) *Robust Control and Filtering of Singular Systems.* Springer-Verlag, Berlin.

Xu, S., Van Dooren, P., Stefan, R. and Lam, J. (2002) Robust stability and stabilization for singular systems with state delay and parameter uncertainty. *IEEE Transactions on Automatic Control,* **47** (7), 1122-1128.

Yang, X., Li, X. and Cao, J. (2018) Robust finite-time stability of singular nonlinear systems with interval time-varying delay. *Journal of the Franklin Institute*, **355**(3), 1241-1258.

Yang, Ch., Zhang, Q. and Zhou, L. (2013) *Stability Analysis and Design for Nonlinear Singular Systems. Applied Lecture Notes in Control and Information Sciences*, Springer.

Yang, C., Sun, J., Zhang, Q. and Ma, X. (2013) Lyapunov stability and strong passivity analysis for nonlinear descriptor systems. *Circuits and Systems, IEEE Transactions,* **60** (4). 1003-1012.

Yang, C., Zhang, Q., Lin, Y., and Zhou, L. (2006). Practical stability of closed-loop descriptor systems. *International Journal of Systems Science,* 37 (14). 1059-1067.

Yip, E.L. and Sincovec, R.F. (1981) Solvability, controllability, and observability of continuous descriptor systems. *Automatic Control, IEEE Transactions,* **26** (3), 702-707.

You, L. Sh. and Chen, B.S. (1993) Tracking control designs for both holonomic and non-holonomic constrained mechanical systems: a unified viewpoint. *International Journal of Control,* **58** (3), 587-612.

Zhang, Q. and Yang, D. (2003) *Analysis and Synthesis for Uncertain Descriptor Systems.* Northeast University Press, Shenyang.

Zhang, G. and Yu, P. (2016) Eigenstructure Assignment and Impulse Elimination for Singular Second-Order System via Feedback Control. *IET Control Theory and Applications*, **10** (8), 869-876.