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# WAVE EFFECTS IN COUPLED PANTOGRAPH-TRACTION SYSTEMS

#### Abstract

The paper is devoted to the study of wave phenomena in one-dimensional continuous media caused by moving forces, exerted by contact with a discrete dynamical model. The discussion is connected with the interaction between pantograph and traction. In particular, coupling between the contact wires during travel of two trains on neighbouring tracks via the support system is examined. The influence of the travelling speeds on the solution is studied, maximal deflections, contact forces and loss of contact are monitored during numerical simulations.

### **INTRODUCTION**

In the analysis of transportation systems, interfaces between fast moving and immobile objects play a major role. In railway research, on the one hand, there is the contact between rails and wheels, which is essential for the support of the load of the vehicle, its guidance and the propulsion. On the other hand, the energy supply for the propulsion by an electrical locomotive or the engines of EMU trains is dependent on the contact between pantographs and a traction wire [1,7,8,10-13,16,17]. In the latter case, the contact forces are considerably smaller, however, a continuous electrical contact has to be maintained. The contact partner is here comparatively light and soft, as compared to the rail bedded on a typical subgrade. Since the train, in general, is moving with a certain speed, the forces by which its pantographs act on the wire generate dynamic deflections in the traction [1,3,7,8,16]. These are propagating along the wire, being reflected at supports and boundaries, transmitted to supporting wires and even parallel contact wires, causing nontrivial interactions with the given and possibly other pantographs.



Fig. 1. Segment of traction (post field).

In order to minimize fluctuations in the contact forces it is desirable to maintain a nearly constant height of the contact wire. To this end a messenger wire is spanned above the actual contact wire, thus keeping the pantograph in a nearly constant configuration. Both wires are periodically connected by droppers of varying lengths. Consequently, waves running in the contact wire, initiated by the travelling force, spread in both wires, which form a two-dimensional network together with the coupling droppers. A typical layout of a span width between two posts is presented in Fig. 1. Cross sections and the principle of the hangers can be seen in Fig. 2.

At the end of each span of the traction, weights are attached to maintain a tension in the wires that guarantees a wave speed larger than the speed of travel. This way critical cases as discussed in previous papers [7,8] are to be avoided. Despite this, and a security factor of more than 1.5, instability effects can still be observed in practice. In particular, the interaction between two or more pantographs, by means of the wire, can lead even to the destruction of pantograph and rupture of the contact wire. While for larger distances, the bending stiffness of the used wires, see Fig. 2 below, is mostly neglected, in the case of two pantographs mounted on the same locomotive bending effects may be essential.

In certain locations, e.g. at stations, the traction above several parallel tracks is suspended from wires perpendicular to the tracks, rather than from individual posts for each track. In this case, the two-dimensional network is extended in the third dimension. Coupling between neighboring lines has to be considered. A reduction of speed may be required, if trains running in opposite direction meet at such a place, or if one train is overtaking another one.

The problem of travelling loads has attracted large attention in the past, mostly the case of a priory known forces, constant or harmonic, was analyzed, cf. [2,5,12] and cited there sources. At first analytical methods were applied, then more and more numerical approaches were developed, e.g. [2,9]. Extensions to two-dimensional and nonlinear problems were studied as well [5,14]. The case of an oscillator moving along a track [3], proposed by the first author in 1976, was the prototype of the problem discussed here, where a MultiBody System (MBS) is coupled with a continuum. Recently, see e.g. [1,4,7,8,17], research of this aspect of discrete-continuous co-simulation gained momentum.

In publications [16,17] Matlab/Simulink solutions were developed to attempt an active control of the MBS. Wavelet based analytical tools have been proposed as well, [15]. The importance of nonlinear effects was discussed in [18], showing that the modeling of droppers needs special attention.

This present paper is organized as follows. In the first section, the traction is analyzed, some basic information on wave propagation, in particular in the context of travelling loads, is provided. Next in Section 2, a pantograph is modeled as a MBS. Then in Section 3 numerical experiments with the model composed of both components are carried out. The results are summarized in Sec. 4.

### **1. TRACTION**

In this section the continuous subsystem, with the main focus of the contact wire, will be discussed. For a single wire, allowing for bending stiffness, the equation of motion has the form [3,7,8]

$$\rho(x)\ddot{u}(x,t) = f(x,t) - (S(x)u(x,t)'' + Pu(x,t))''.$$
(1)

Here we denote by *S* the bending stiffness of the medium, by  $\rho$  the mass density per unit of length and by *P* the compressing force. Time is denoted by *t*, position along the wire by *x*, the corresponding partial derivatives are indicated by a superposed dot or an apostrophe, respectively.

This equation, describing a Bernoulli-Euler Beam, is degenerate parabolic, and it admits wave type solutions. Depending on the type of boundary conditions, standing as well as running waves may be observed. In general, dispersion occurs, i.e., the propagation speed of waves depends of their frequency.



Fig. 2. Cross sections of supporting and contact wires

The external force f may be as a first approach modeled as a Dirac distribution, which is concentrated in each moment t at the point Vt, where V is the travelling speed of the pantograph. In a typical traction wire, S may be neglected in a first approximation, and P is negative. This leads in the case of very flexible media to the simplified equation

$$\rho \ddot{u}(\mathbf{x},t) - c^2 u(\mathbf{x},t)'' = \mathbf{f}(\mathbf{x},t)$$
(2)

The coefficient  $c^2$  is introduced for the ratio of tension *P* to mass density  $\rho$ . Notice that often there appears the parameter *T* (for tension), instead of *P*, in the literature on beams. We prefer to reserve *T* for the duration of transient processes and to use the parameter name *P* as it is being used in papers on columns.

Technically, the tension is maintained by concrete weights of about two metrical tons of mass, see Fig. 3.



Fig. 3. Applying tension at the end of wires

Physically, each wire has a length of the order of one kilometer, which would allow solving initial-boundary value problems on a finite domain. However, additionally to the boundary conditions, in models of traction interface conditions at suspenders and hangers have to be included, see Fig. 4.



Fig. 4. Structure of a suspension

Further, at the end of a length of wire, the next one is guided in, while the previous one is lead to the side and slightly up, and next to the post with the weight, see Fig. 5.



Fig. 5. Transition between sections



Fig. 6. Two tracks with common suspensions

More details on geometry and mechanical parameters can be found e.g. in [10,11,13] and [18].

#### 2. PANTOGRAPH

The simplest way to model the interaction between a moving object and its immobile contact partner like a rail or a contact wire is to assume a constant or periodic force, acting at a point located at position x=Vt, where x is the coordinate along the track, V is the traveling speed and t is the elapsed time. While e.g. in the case of rail-wheel contact in certain situations quite acceptable, for the interaction with a wire, i.e. an object with very small bending stiffness and rather large distance between supports, realistic results cannot be expected that way. It is essential to allow for a quick reduction of the modulus of the vertical force, when the wire is elevated at the momentary vehicle position x. Further, the dynamics of the pantograph has to be taken into account, see Fig. 7. Within reasonable accuracy this can be achieved by the introduction of three degrees of freedom, assuming linear-viscoelastic forces in the joints and in the contact. This adds six more equations of motion to the system obtained so far from the discretization of the contact wire.



Fig. 7. Pantograph as MBS

It is acceptable and recommendable to consider the rods and lines constituting the pantograph as inflexible, respectively not stretchable, otherwise e.g. elastic multibody systems would have to be introduced. This would further enlarge the number of unknowns in the model. Obviously, in the case of multiple pantographs, or more than one train, e.g. two trains running on parallel tracks at different velocities, or in the case of different constructions of the pantograph, the number of degrees of freedom has to be modified in a suitable way.

Assume the generalized coordinates, which describe the angles of the arms of the pantograph against the vehicle's roof, and the distance between the contact wire and the end of the upper arm, are denoted by  $q_1$ ,  $q_2$  and  $q_3$ . Together they form the vector of generalized positions q, the corresponding generalized velocities are  $\dot{q}$ . Introducing generalized masses and forces, one can easily formulate the equations of motion in the form

$$\frac{a}{dt} \left( M \dot{q}(t) \right) = Q \left( t, q(t), \dot{q}(t), u(x(t), t), \dot{u}(x(t), t), u'(x(t), t) \right)$$
(3)

where on the left-hand side, there are the time derivatives of the generalized moments, while on the right-hand side, the forces depend on positions and velocities of the contact partners in the point of contact.

#### **3. NUMERICAL CALCULATIONS**

Solutions of the hyperbolic partial differential equation of second order (2) can be obtained in terms of the initial deflections and integrals over initial speed and external force. Assuming constant coefficients and zero forces, any solution of equation (2) on the whole real axis  $x \in R$  has the following form:

$$u(\mathbf{x},\mathbf{t}) = \varphi(\mathbf{x} + c\mathbf{t}) + \psi(\mathbf{x} - c\mathbf{t})$$
(4)

An initial displacement, concentrated around the origin, splits into two waves, one running left, the other right, with equal but opposite speeds. The functions  $\Phi$  and  $\Psi$  can be identified from initial conditions  $u(\cdot, 0)$  on the deflection and on the initial lateral, i.e. upward, speed  $v(\cdot, 0) = \dot{u}(\cdot, 0)$ 

In the case of the forth order equation (1), solutions are assumed in the form of running waves with a speed depending on the frequency, hence infinite series have to be discussed. We refer to our paper [7].

Parameters for numerical calculations can be found in [1], [7,8], [13], [18]. Typical tensions are around 20 kN, the mass density of the contact wire is around 1.33 kg/m. See Fig. 2 for the cross sections of messenger and contact wires. The mean value of the lateral force is set to 200N for the present calculations.

Figure 8 presents a typical result, as previously shown in [7], when equation (1) is solved without any interface conditions and the load being applied by a massless spring. There are no supports in this case, yet the solution stays bounded due to the reaction of the pantograph's decreasing contact force, when the wire moves away from it. In the pure string case of equation (2), the solution is a fuzzy version of the characteristic cone of the classical hyperbolic problem. When allowing for bending stiffness, ripples appear in front and behind the disturbed area, see Fig. 8.



Fig. 8. Wave type traveling force solution to equation (1) neglecting interface conditions

In [7] and [8], also the case of several pantographs pressed to the same contact wire was discussed in this academic setting, i.e. neglecting realistic interface conditions. It was shown that the second pantograph runs into the wake of the leading one, but also disturbances from a trailing pantograph may reach the leading one and cause considerable fluctuations in the contact forces.

In realistic constructions as e.g. in Figs. 1, 5 or 6, analytical considerations become insufficient due to the heterogeneous structure of the system and the large number of reflections overlaying each other. Consequently, numerical approximations are required. In the following Figs. 9 and 10, a direct computation by a fifth order ODE solver applied to equations obtained from (1) by the method of lines, and then coupled with (3), was used.



Fig. 9. Deflections at various speeds for one and two pantographs in contact



Fig. 10. Deflections at various speeds for pre-stressed system

In both figures, the elevation of the contact wire is shown, in Fig. 9 the case of one and two pantographs are compared, the three curves correspond to the recommended maximum speed, the critical and a supercritical velocity of the vehicle. In Fig. 10, the sensitivity to the dropper model is shown. Here once more the case of a single pantograph, but with pre-stressed droppers, was simulated, [18].

#### **SUMMARY**

Modeling the coupled motion of one or several pantographs in contact with the traction over one or several railway vehicles requires a suitable combination of methods developed for the description of one-dimensional continuous media like strings and beams with solution techniques for mechanical multibody systems. A major problem are the countless couplings with the suspension system. As opposed to the very regular periodic support by sleepers in the case of traveling loads of railway vehicles on their track, the droppers from messenger wire to contact wire feature different lengths, so that the lateral stiffness of the catenary is variable in a complex way, cf. [7]. Numerical tests show that the nonlinearity of the characteristic of the droppers must not be neglected, i.e., there should be no pushing forces in slack wires.

Further, the traveling load, which in models of moving forces on railway track can be assumed a harmonic function in the time variable *t*, needs to be calculated by simulating the motion of the pantograph, which features at least two arms connected by a joint and a spring-damper element maintaining contact with the wire. In a first approximation, this sub-system may be treated as linear in the vicinity of the working point, as far as small variations in the height of the contact wire and small deflections can be assumed.

Analytical solutions to problems of wave propagation in ideally flexible as well as in bending stiff wires, which can be studied by classical formulas, give some hints on stability constraints in the present case. The most essential factor is the relation between traveling speed of the vehicle and the velocity of wave propagation in the continuous medium. However, in real world problems the interface conditions and nonlinearity effects make a full analytical study unfeasible. Instead, a dynamical approach in the time domain as well as an analysis of eigenforms and eigenfrequencies of the system have to be applied in order to evaluate the amplitude of deflections by numerical methods.

In the present analysis, the dynamics of the pantograph, modeled as a multi-body system, is coupled with a finite difference discretization of the contact and messenger wires. Now, the traveling force is no longer pre-assigned, it is dependent on the history of the solution for the wire deflection in the point of contact. This is of particular importance if multiple pantographs in a short distance between each other are considered, and when there is a coupling between several parallel contact wires, e.g. with trains running at high speed in opposite directions.

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# EFEKTY FALOWE W SPRZĘŻONYCH UKŁADACH PANTOGRAFU I TRAKCJI

#### Streszczenie

W pracy zbadane są zjawiska falowe w jednowymiarowych ośrodkach ciągłych, wywołane siłami ruchomymi, którymi sprzężony układ dyskretny oddziałuje z układem jednowymiarowych elementów ciągłych. Układy tego typu mają znaczenie w modelowaniu interakcji pomiędzy pantografem a kolejową siecią zasilającą.

W szczególności analizowany jest efekt sprzężenia zachodzący podczas przejazdu dwóch pociągów po sąsiadujących torach. Wpływ prędkości poruszających się pociągów na maksymalne przemieszczenia, siły kontaktowe oraz utratę kontaktu odbieraka prądu z siecią badany jest za pomocą numerycznych symulacji komputerowych.

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