Advances in Science and Technology Research Journal

Advances in Science and Technology Research Journal 2023, 17(3), 145–153
https://doi.org/10.12913/22998624/165914
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Received: 2023.02.28
Accepted: 2023.05.11
Published: 2023.06.01

Study of Contact Parameters in Metal-On-Plastic Hip Endoprosthesis with the Analytical-Numerical Method of Contact Mechanics

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ABSTRACT

The purpose is to analyze the influence of the elastic characteristics of the materials of the tribological pair: acetabular–femoral head on the maximum pressures in the hip joint endoprosthesis. The paper analyzes the impact of elasticity characteristics (i.e. E and ν) on the contact parameters of a tribological pair (maximum contact pressure and contact angle), which is a hip endoprosthesis. The analysis was made on the basis of the author's calculation method. The study determined the value of contact pressures depending on the changes in the elastic characteristics of UHMW-PE and changes in the geometry of the tribological pair (variable radii of the Grade2TDN head and the UHMW-PE acetabulum). It was noticed that increasing E modulus of UHMW-PE increases its stiffness, which causes a noticeable increase in contact pressures in the endoprosthesis. The quantitative and qualitative regularities of this impact were determined. The developed method allows for earlier estimation of contact pressures depending on the external load (patient's body weight), and geometric parameters of the artificial hip joint (diameter of the endoprosthesis head and the acetabulum). This allows you to make a personalized endoprosthesis resistant to damage. It is very important in modern medicine because life expectancy in developed countries is longer and the durability of endoprostheses should be 10–12 years. Hospital statistics show that the need for total hip replacement concerns even younger people. Implantation of a hip joint prosthesis is an invasive and costly procedure, hence the need to produce prostheses with a long service life (over 15–20 years) before re-arthroplasty. The obtained test results showed that the use of an endoprosthesis cup made of UMHW-PE with higher stiffness (high E_1 value and low × value) results in an increase in maximum contact pressures p(0). Increasing the Poisson's ratio in the tested range causes an increase in the contact pressure p(0) even 1.1 times. Increasing the diameter of the head of the D_2 endoprosthesis decreases the contact pressure by an average of 2.32 times. On the other hand, a change in the radial clearance e in the tribological pair of the hip endoprosthesis from 0.1 mm to 0.2 mm causes an increase in contact pressures by an average of 1.35 times.

Keywords: hip endoprosthesis, calculation method, maximum pressures, tribological pair

INTRODUCTION

The statistics of arthroplasty show that in over 90% of hip endoprostheses the acetabular skull was made of UHMW-PE. In this type of endoprostheses, a combination of materials with a very high ratio of Young's modules, the femoral head materials made of stainless steel, CoCrMo alloy, titanium and ceramics to the UHMW-PE module. According to various data, it is 100 ÷ 200 times, and in the case of alumina - up to about 400 times. As for the Poisson's ratio, there is also a relatively small difference here.

The influence of the module change on the maximum contact pressures was partially tested in [1] on the basis of the calculation method and in [2] experimentally on the contact area and, according to FEM, on contact pressures. Moreover, the literature lacks wider studies of the influence of both Young's modulus and Poisson's ratio on contact parameters: maximum contact pressures, contact angle, radius and contact area.

Individual FEM studies of hip joint prostheses, known in the literature [3–13], provide differentiated information on the indicated contact parameters with different values of Young's modulus and Poisson's ratio (Table 1). UHMW-PE polyethylene has poor heat resistance and the given values of Young's modulus may be reduced to 15–20% [14, 15] at the temperature of the human body.

In this paper, an analysis of the influence of both UHMW-PE elasticity characteristics on the maximum contact pressure was carried out on the basis of the proprietary computational method of hip replacement prostheses.

RESEARCH METHOD

The hip joint endoprosthesis is a 3D system (Fig. 1), and can be modeled as a cylindrical joint (3D system) with an additionally defined effective radius. For the purpose of the research, this 3D system was reduced to a 2D planar system. This was achieved by reducing the total compressive load N (Fig. 2) on the femoral head of the prosthesis head to unity of the spigot diameter, i.e. $N' = N/D_2$. Correspondingly in this model joint $R_1 \approx R_2 = R$ (R_1 – sleeve radius 1, R_2 – disc radius 2). Both in endoprostheses and in the model system there will be radial clearance $\varepsilon = R_1 - R_2 \geq 0 << R$. Load N' causes both elements into contact in the zone defined by the angle $2\alpha_0$ and the emergence of contact pressures p_α.

The solution to the contact problem is the introduction to the determination of the maximum contact pressures p_0, their decomposition p_α and the contact angle $2\alpha_0$. For such a flat problem of the theory of elasticity, the equation for contact pressures p_a has the form [16–21].

$$c_1 \int_{-\alpha_0}^{\alpha_0} \cot \frac{\alpha - \theta}{2} p'_\theta d\theta = c_2 p_\alpha + c_3 \int_{-\alpha_0}^{\alpha_0} p_\alpha d\alpha$$

$$+ \ c_4 \cos\alpha \int_{-\alpha_0}^{\alpha_0} p_\alpha \cos\alpha d\alpha + \frac{\varepsilon}{R^2} \qquad (1)$$

where: $p'_\theta = dp/d\theta$;

α – polar angle; $0 \leq \alpha \leq \theta$, $-\alpha_0 \leq \theta \leq \alpha_0$;

$$c_1 = \frac{1}{8\pi R}\left(\frac{1+\kappa_1}{G_1} + \frac{1+\kappa_2}{G_2}\right);$$

$$c_2 = \frac{1}{4R}\left(\frac{1-\kappa_1}{G_1} - \frac{1-\kappa_2}{G_2}\right);$$

$$c_3 = \frac{1+\kappa_1}{8\pi G_1 R}; \ c_4 = \frac{1}{2\pi R}\left(\frac{\kappa_1}{G_1} + \frac{1}{G_2}\right);$$

G_1, G_2 – modulus of material elasticity;

ν_1, n_2 – their Poisson's ratios;

$\kappa = 3 - 4\nu$ – plane deformation state.

When calculating the model cylindrical joint as a 2D replacement system for an endoprosthesis as a 3D system with radiuses R_1, $R_,$ elements, the effective radius was used $R_* = 0.5\sqrt{R_1 R_2} = 0.5\sqrt{(R_2 + \varepsilon)R_2}$.

Approximate solution of equation (1) is effectively implemented according to the collocation method [22]. In order to determine the elasticity characteristics of UHMW-polyethylene on contacr pressures in hip joint endoprostheses, the author's IT application "Contact pressure in hip joint endoprosthesis" was developed (Fig. 3). The application was created in the MATLAB R2022 system. The original calculation method described in the papers [23–25] was used to create it.

RESULTS AND DISCUSSION

The above-mentioned contact parameters in the analyzed metal-polymer (MoP) hip joint endoprosthesis were assumed to be the following input data:

- femoral head diameters: D_2 = 28, 38, 48 mm;
- the total pressure force N that acted on the endoprosthesis head was assumed for a man with a weight of K = 700 N while walking [26–28]: N_{max} = 2900 N, N_s = 1900 N;
- radial clearance: ε = 0.1, 0.2 mm;
- femoral head 2 endoprosthesis was made of thermo-diffusion nitrided titanium (GRADE2 TDN), for which E_2= 112 000 MPa, ν_2 = 0.32;
- acetabular cup material 1 – ultra-high molecular weight polyethylene (UHMW-PE);

Table 1. Parameters of the UHMW-PE elasticity characteristics used by other authors

Authors	[2]	[3]	[3]	[4]	[5]	[6]	[7]	[8]	[11]	[12]	[13]	[14]
$E_1 \times 10^3$ MPa	1.0	0.84	0.68	1.4	0.88	0.5	0.85	1.0	2.2	1.0	1.4	0.75
ν_1	0.4	0.44	0.425	0.46	0.4	0.4	0.4	0.4	0.3	0.45	0.4	0.46

Fig. 1. Hip joint endoprosthesis

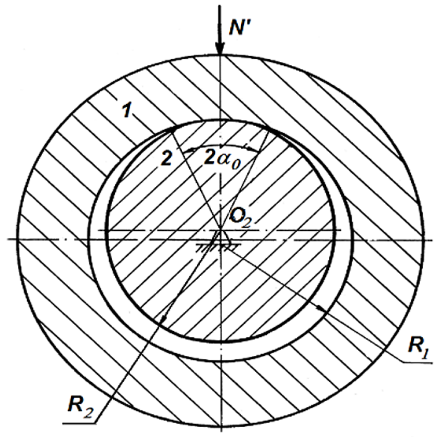

Fig. 2. Calculation diagram of a cylindrical joint

appropriately selected E_1= 500, 1000, 1500, 2000, 2500 MPa, ν_1= 0.3, 0.35, 0.4, 0.45, 0.5.

The results of solving the problem for determining the maximum contact pressures p(0) with an average load of N_s = 1900 N are presented in Tables 2–4 and in Figures 3 and 4. Correspondingly, for the diameter D_2 = 28 mm of the head, the results of calculations p(0) are given in the Table 2, for the diameter D_2 = 38 mm – in Table 3 and for the diameter D_2 = 48 mm – in Table 4.

The results of the analytical tests showed that the effect of Young's modulus E_1, Poisson's coefficient ν_1 of the tested UHMW-PE polymer, endoprosthesis head diameter D_2 and radial clearance of the tribological pair on the value of the maximum contact pressure p(0) in the endoprosthesis was visible:

Young's modulus E_1

With a fivefold increase in module (E_1 = 500 ÷ 2500 MPa) the pressures increase about two

Table 2. Maximum contact pressures (D_2 = 28 mm, ε = 0.1 / 0.2 mm)

E_1 [MPa]	ν_1 = 0.3	ν_1 = 0.35	ν_1 = 0.4	ν_1 = 0.45	ν_1 = 0.5
	p(0) [MPa]	p(0) [MPa]	p(0) [MPa]	p(0) [MPa]	p(0) [MPa]
500	5.59 / 7.29	5.66 / 7.41	5.74 / 7.55	5.84 / 7.73	5.97 / 7.94
1000	7.31 / 9.99	7.42 / 10.16	7.57 / 10.37	7.74 / 10.62	7.95 / 10.94
1500	8.75 / 12.09	8.9 / 12.31	9.07 / 12.56	9.29 / 12.88	9.56 / 13.26
2000	9.99 / 13.87	10.16 / 14.12	10.36 / 14.41	10.62 / 14.78	10.93 / 15.22
2500	11.08 / 15.44	11.27 / 15.71	11.5 / 16.04	11.78 / 16.44	12.13 / 16.93
2500/500	1.982 / 2.118	1.991 / 2.120	2.004 / 2.125	2.017 / 2.127	2.032 / 2.132

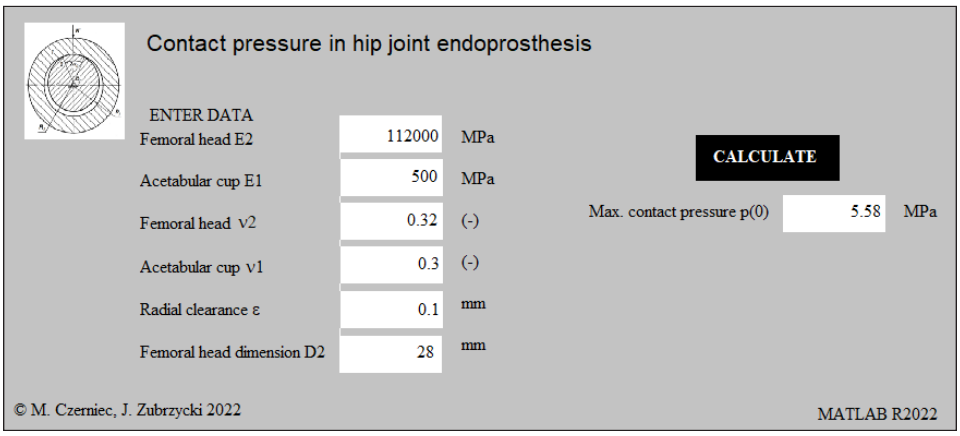

Fig. 3. The communication interface of the application user for calculating the elastic characteristics of the tribological pair of the hip joint endoprosthesis

Table 3. Maximum contact pressures (D_2 = 38 mm, ε = 0.1 / 0.2 mm)

E_1 [MPa]	ν_1 = 0.3	ν_1 = 0.35	ν_1 = 0.4	ν_1 = 0.45	ν_1 = 0.5
	p(0) [MPa]	p(0) [MPa]	p(0) [MPa]	p(0) [MPa]	p(0) [MPa]
500	3.38 / 4.55	3.43 / 4.62	3.5 / 4.72	3.57 / 4.83	3.68 / 4.97
1000	4.55 / 6.29	4.63 / 6.39	4.72 / 6.53	4.83 / 6.69	4.97 / 6.89
1500	5.48 / 7.63	5.58 / 7.77	5.69 / 7.93	5.83 / 8.13	6.0 / 8.38
2000	6.28 / 8.76	6.38 / 8.92	6.52 / 9.11	6.68 / 9.34	6.88 / 9.62
2500	6.97 / 9.76	7.09 / 9.93	7.24 / 10.14	7.42 / 10.4	7.64 / 10.71
2500/500	2.056 / 2.145	2.061 / 2.147	2.069 / 2.149	2.078 / 2.152	2.087 / 2.155

Table 4. Maximum contact pressures (D_2 = 48 mm, ε = 0.1 / 0.2 mm)

E_1 [MPa]	ν_1 = 0.3	ν_1 = 0.35	ν_1 = 0.4	ν_1 = 0.45	ν_1 = 0.5
	p(0) [MPa]	p(0) [MPa]	p(0) [MPa]	p(0) [MPa]	p(0) [MPa]
500	2.34 / 3.18	2.37 / 3.23	2.42 / 3.30	2.47 / 3.38	2.54 / 3.48
1000	3.18 / 4.41	3.23 / 4.49	3.3 / 4.59	3.38 / 4.7	3.48 / 4.84
1500	3.84 / 5.37	3.91 / 5.46	3.99 / 5.58	4.09 / 5.72	4.21 / 5.89
2000	4.40 / 6.17	4.48 / 6.28	4.58 / 6.41	4.69 / 6.57	4.83 / 6.77
2500	4.90 / 6.87	4.98 / 6.99	5.09 / 7.14	5.22 / 7.32	5.38 / 7.54
2500/500	2.094 / 2.160	2.101 / 2.164	2.103 / 2.164	2.113 / 2.166	2.114 / 2.167

times, namely by 1.982÷2.114 times, depending on the D_2 head diameter, where the radial clearance ε = 0.1 mm; and by 2,118÷2,167 times – when ε = 0.2 mm.

Change in contact pressures with the variation of the Poisson's coefficient (ν_1= 0.3÷0.5) for the indicated individual values E_1 is small (D_2 = 28 mm: 2.5% – ε = 0.1 mm, 0.7% – ε = 0.2 mm; D_2 = 38 mm: 1.5% – ε = 0.1 mm, 0.5% – ε = 0.2 mm; D_2 = 48 mm: 1.0% – ε = 0.1 mm, 0.32% – ε = 0.2 mm).

Poisson's coefficient ν_1

Increase ν_1 in 1,667 times it causes a slight increase in p(0) within 1,087 – 1,1 times. For permanent ν_1 a variable E_1 = 500÷2500 MPa pressures p(0) increase to 2,167 times depending on the value of D_2, ε.

Diameter D_2 of the femoral head

Increasing the D_2 of the head diameter from 28 to 48 mm (by 1.714 times) causes a significant reduction of the contact pressures p(0) within 2.25÷2.39 times, depending on the value E_1, ν_1.

Radial clearance ε in an endoprosthesis

Doubling the radial clearance e causes an increase of contact pressures p(0) within the limits of 1.3÷1.37 times (E_1 = 500 MPa, ν_1= 0.3 ÷ 0.5) and in 1.39÷1.4 times (E_1 =2500 MPa, ν_1= 0.3÷0.5). Enlarging value of radial clearance (ε = 0.2 mm) there is a slightly more intense increase in the contact pressure p(0), accordance to lower value for ε = 0.1 mm, as shown in Fig. 4 and 5.

The research has shown (Tables 2–4, Figs. 4, 5) that in endoprostheses with the lowest UHMWPE stiffness (E_1 = 500 MPa, ν_1= 0.5) and the highest stiffness (E_1 = 2500 MPa, ν_1= 0.3) the difference of the contact pressures p (0) is significant, namely:

- *D_2* = 28 mm: 1,860 times – ε = 0.1 mm, 1.940 times – ε = 0.2 mm;
- *D_2* = 38 mm: 1,894 times – ε = 0.1 mm, 1.964 times – ε = 0.2 mm;
- *D_2* = 48 mm: 1.930 times – ε = 0.1 mm, 1.970 times – ε = 0.2 mm.

The enlarging of Young's modulus and Poisson's ratio increase the pressure relative to the lowest value accepted for calculations E_1 = 500 MPa, ν_1 = 0.3. In the case of the same diameter D_2 of the prosthesis head, the qualitative increase in the maximum contact pressure p(0) is similar for both values radial clearance. On the other hand, when the D_2 diameters are reduced, an intense increase in pressure is observed with the increase in Young's modulus.

As a result of the research, it was found that the change in the elasticity characteristics of the

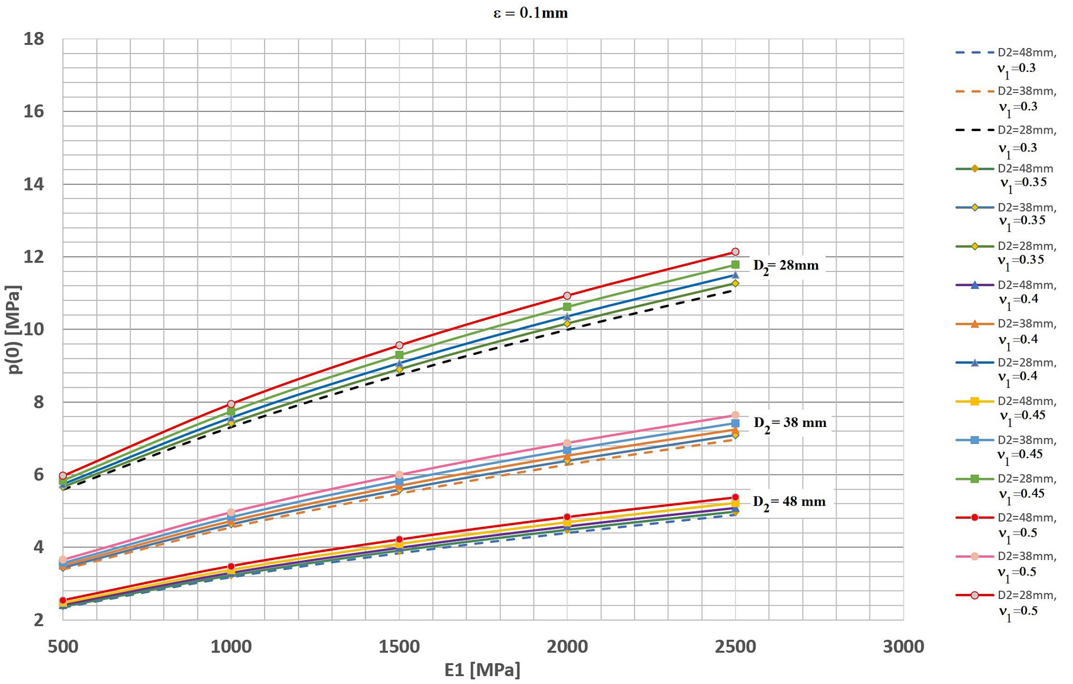

Fig. 4. Influence of the UHMWPE elasticity characteristics on the contact pressure (ε = 0.1 mm)

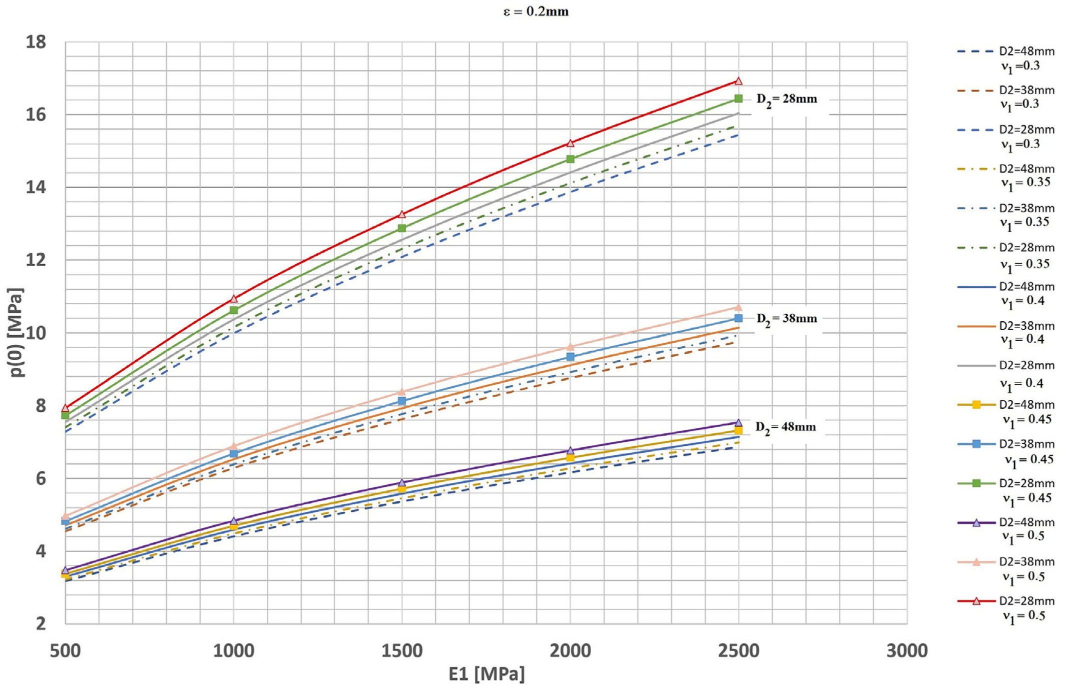

Fig. 5. Influence of the UHMWPE elasticity characteristics on the contact pressure (ε = 0.2 mm)

UHMW-PE acetabulum of the endoprosthesis has a significant impact on the contact pressures. As can be seen in Fig. 4 and 5 successive changes in the value of the elastic characteristics (increase of Young's modulus and Poisson's ratio) cause an increase in contact pressures. With the assumed Young's modulus of the endoprosthesis head material E_2 = 112000 MPa, the E_1/E_2 ratio changes 5 times from 0.0045 (0.45%) to 0.0223 (2.23%), which explains the increase in contact pressure p(0). Along with the increase of the Young's modulus for UHMW-PE, its stiffness increases, which results in increased contact pressures. It should also be emphasized that the change of the Poisson number influences to some extent the change of the pressures p(0).

Apart from the elasticity characteristics of UHMW-PE, the contact pressure is influenced by: radial clearance ε and the diameter D_2 of the prosthesis head. On the basis of the obtained test results, it was clearly shown that the increase in radial clearance has a negative effect on the contact pressures. It is an increase of approx. 130–140% depending on the diameter of the head. On the other hand, increasing the diameter of the head causes a significant reduction in pressure.

Figures 6–8 shows the juxtapositions of the contact pressures p(0) depending on the change in the elastic characteristics of polyethylene of the endoprosthesis socket, diameter D_2 of the

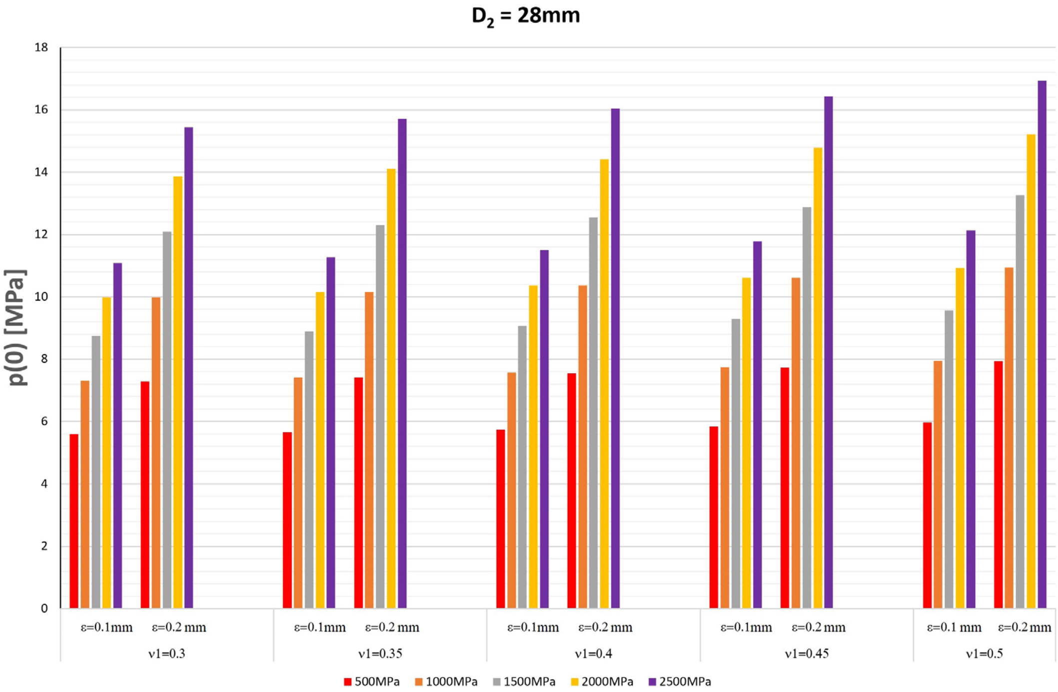

Fig. 6. Change of contact pressures p(0) in hip joint endoprosthesis due to changes in elastic characteristics (D_2 = 28 mm)

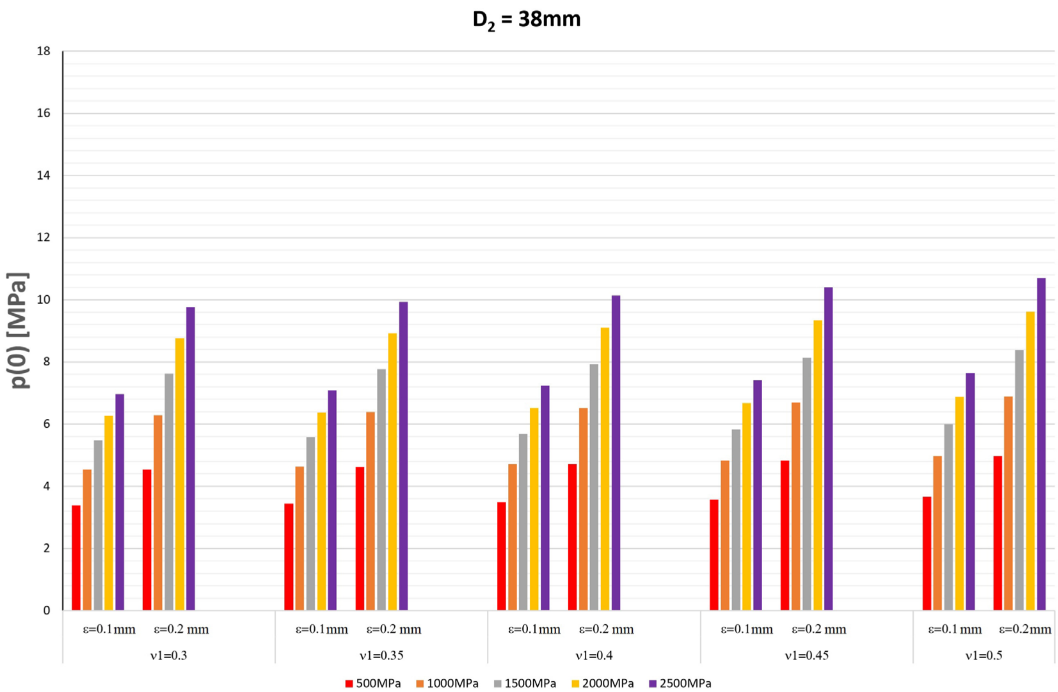

Fig. 7. Change of contact pressures p(0) in hip joint endoprosthesis due to changes in elastic characteristics (D_2 = 38 mm)

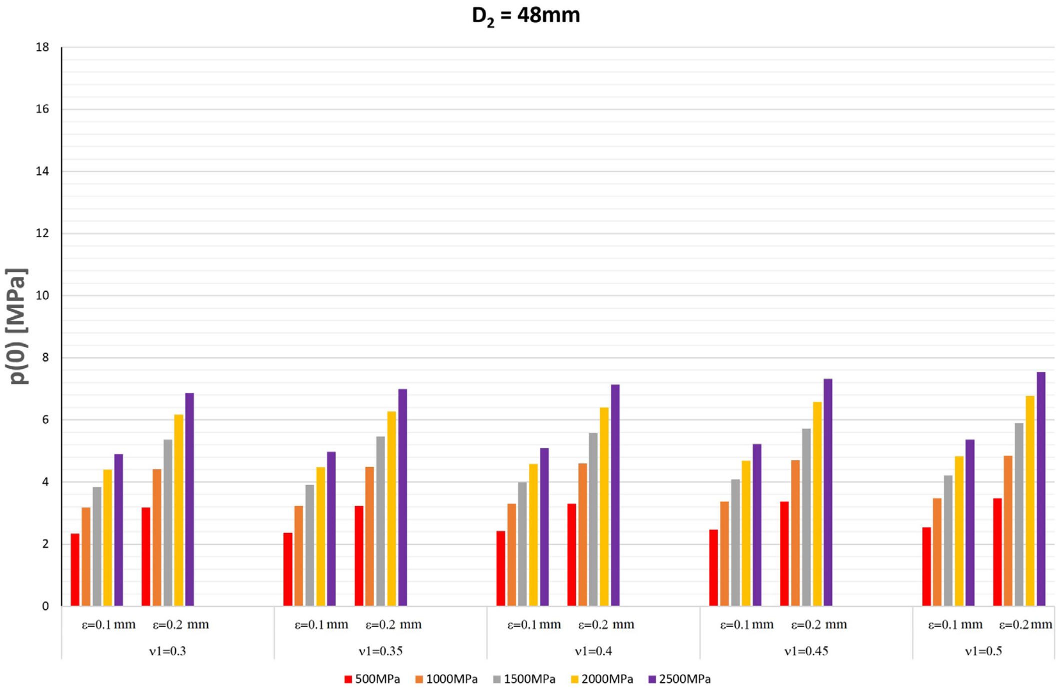

Fig. 8. Change of contact pressures p(0) in the endoprosthesis of the hip joint endoprosthesis due to changes in the elastic characteristics (D_2 = 48 mm)

endoprosthesis head and radial clearance ε in the endoprosthesis.

The main aim of this study was to develop an explicit analytical method to predict the contact pressure of an artificial hip joint. Obtained predicted maximum contact pressure, based on the authors' analytical method, overlapped with the result using the numerical method by Bartel et al. [1], Hua et al. [2] and Grushko et al. [6].

The presented research results are consistent with the results of other authors who conducted FEM numerical studies. Table 5 shows the p_{max} values obtained in [6] and [8], which most closely correspond to the results of the research. Similar results of numerical research performed by the finite element were received by Gao et al. [11] who analyzed of sliding distance and contact mechanics of hip implant under dynamic walking conditions. Their research tested the effects of the metal-on-ultra-high-molecular weight polyethylene (UHMW-PE) tribological pair. Also, in the works [13, 21, 28] obtained contact pressure values obtained in FEM numerical studies correspond to the analytical results obtained in this work.

Future studies will also focus on the application of the analytical method to the determine optimal body weight for the selected construction solution of the endoprosthesis as a tribological pair.

Table 5. Results from tests of maximum contact pressures

Author	p_{max} / $p(0)$ MPa	Prosthesis parameters
[5]	8.77 / 10.65 11.81 / 13.75	N = 2650 N; D_2 = 28 mm; ε = 0.1 mm; Ti - UMHWPE N = 2650 N; D_2 = 28 mm; ε = 0.2 mm; Ti - UMHWPE E_1 = 880 MPa; ν_1 = 0.4
[7]	8.84 / 8.5 9.07 / 8.9	D_2 = 32 mm; ε = 0.098 mm; CoCrMo - UMHWPE N = 1900 N N = 2000 N E_1 = 850 MPa; ν_1 = 0.4

Note: p_{max} – maximum contact pressures acc. to [6, 8]; p(0) – maximum contact pressures according to the presented method.

CONCLUSIONS

1. The qualitative and quantitative regularities of the influence of changes in the elastic characteristics of the UMHW-PE, the diameter of the iliac head and the radial clearance on the maximum contact pressure in MoP hip endoprostheses were established.
2. In a skull endoprosthesis with UMHW-PE with higher stiffness (higher values of E_1 and lower values ν_1), higher maximum contact pressures are created. Change in the elastic characteristics of the UMHWPE acetabular hip joint prosthesis and its geometry increases the contact pressure in the endoprosthesis by about two times.
3. Increasing only the Poison's coefficient in the tested range causes an increase in contact pressures up to 1.1 times. When the head diameter is increased, the contact pressure is reduced by an average of 2.32 times.
4. On the other hand, increasing the radial clearance by 2 times in the system causes their increase by an average of 1.35 times.
5. The developed method allows for an effective assessment of the maximum contact pressures in endoprostheses as a result of changes in elasticity characteristics, taking into account the radial play and the diameter of the iliac head.
6. Changing the elasticity characteristics and geometry of the joint of the endoprosthesis of the hip joint causes an increase in contact pressures in the endoprosthesis (Fig. 6–8) by up to 216%.
7. Uncontrolled increase of contact pressures will cause gradual deformation of the hip endoprosthesis acetabular cup until it is damaged – which necessitates re-implantation of a new endoprosthesis
8. Comparative analysis of the results of the p(0) assessment according to the author's method indicates compliance with the results known in the literature obtained by FEM numerical tests.
9. The use of UHMW-PE as a material for endoprosthesis cups requires a careful approach due to its elasticity characteristics. All test results presented in the paper were performed for one average load $N_s = 1900$ N and for different diameters of endoprosthesis heads.

Acknowledgements

The research was financed in the framework of the project Lublin University of Technology Excellence Initiative, Regional funded by the Polish Ministry of Science and Higher Education (contract no. 030/RID/2018/19)

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