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OPTIMIZATION POTENTIAL TRANSPORT OF TRANSPORT COMPANY

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The article presents a method for determining the optimum number of vehicles transport company, depending on the size of the specific demand for transport services and depending on the performance properties of means of transport. Formulated the task of multi-criteria optimization and presents the results of its solution.

Keywords: the transport potential, stochastic process, multi-criteria optimization

1. Introduction

Important role of road transport in the economy should be seen in the fact that, among other modes, it stands out above all:

- mobility: you can get it anywhere where there are no rail, ship, etc.,
- high operability service, involving the availability of a relatively large number of means of transport,
- high availability: getting lower car prices and getting better technical parameters,
- timeliness and punctuality performance of services.

Unfortunately, the major disadvantages of this type of transport are:

- dependent on climatic conditions,
- not very eco-friendly,

- high rate of accidents,
- a relatively small volume of individual means of transport,
- not very low maintenance costs of vehicle.

In practice, however, the advantages of road transport outweigh its disadvantages, what is the reason for its continued presence in the transport market.

One of the basic problems of the transport company managed is to provide its continued presence in the market of transport services, mainly by providing a appropriate potential transport for the anticipated demand for transport services.

The transport potential of the transport company will be identified with the number of cars fit for the provision of transport services at any given time. Reducing the number of cars below a certain minimum (threshold value) will reduce the capacity of the transport company, and thus a loss of competitiveness and, consequently, falling out of the market of transport services.

The rest of this article will be considered a problem to provide the required level of transport potential of transport company, taking into account only the technical aspect of cars: the suitability of the vehicle for use at a time or its uselessness.

2. Description of the problem

Consider the transport company, which has $\boldsymbol{I}$ of the cars (means of transport) for the same destiny (e.g. lorries) and used to meet the demand for homogeneous type of transport services (e.g. transport of bulk cargo). Let $\boldsymbol{\mathcal{I}} = \{1, 2, ..., I\}$ be the set of numbers of cars that do not need to be the same, i.e. they do not have to have the same design solutions.

It is assumed that from the point of view of the transport company the process of each car can be considered as a succession over time of independent states:

- suitability of the vehicle for the implementation of transport services,
- incapacity of the vehicle for the implementation of transport services (car repair).

Thus, the process of exploitation of each car can be considered as a two-state stochastic process $\boldsymbol{X(t)}$ (Fig. 1), which is a sequence of consecutive (not overlapping in time) states fitness (rectangular pulses), separated states of unfitness. This process will be further referred to as a *square wave*. From the viewpoint of further consideration of the problem magnitude of the amplitude is not important.

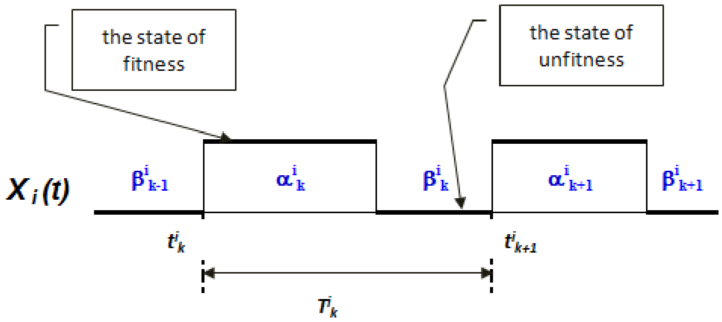

Figure 1. Example of a exploitation process of the ***i***-th, ($i \in \boldsymbol{I}$) of car

Fig. 1 are symbolized $\alpha_k^i, \alpha_{k+1}^i, \ldots,\ (k = 1,2,\ldots)$ durations of states fitness of ***i***-th car, and symbols $\beta_k^i, \beta_{k+1}^i, \ldots,\ (k = 1,2,\ldots)$ - durations of states unfitness.

Let that $\alpha_k^i,\ (k = 1,2,\ldots)$ are realizations of continuous random variables, A_k^i respectively, with the same probability distributions. For simplify the notation, each of these random variables will be denoted by symbol A_i. Let that $\beta_k^i,\ (k = 1,2,\ldots)$ are realizations of continuous random variables, B_k^i respectively, with the same probability distributions. For simplify the notation, each of these random variables will be denoted by symbol B_i. By t^i_k and t^i_{k+1} denoted moments of two consecutive pulses (states fitness of ***i***-th car), and by T^i_k - the length of the interval between occurrences of two consecutive pulses (states fitness of ***i***-th car). Using the designations shown in Fig. 1, exploitation process of ***i***-th car you can be represented as a stochastic process, in which the condition is satisfied:

$$T_k^i = t_{k+1}^i - t_k^i > \alpha_k^i\ . \tag{1}$$

It is assumed that the processes of exploitation of all cars are stochastic processes, which are independent and stationary in a broader sense. Thus, for the ***i***-th car can be determined the expected length of time between occurrences of two consecutive pulses, which is expressed in the following formula:

$$ET_i = \int_0^\infty T \cdot f_i(T)\,dT \tag{2}$$

where $f_i(T)$ is the density function of the probability distribution of the random variable describing the length of time between occurrences of two consecutive pulses (states fitness) the process of exploitation the ***i***-th car.

Is assumed that are known density functions $f_i^\alpha(t)$ and $f_i^\beta(t)$ of probability distributions of random variables A_i and B_i respectively. It is also assumed that the random variables A_i and B_i are independent from each other and that have finite variances and finite expected values Ea_i and Eb_i expressed by the following formulas:

$$Ea_i = \int_0^\infty \alpha \cdot f_i^\alpha(\alpha) d\alpha,$$
$$Eb_i = \int_0^\infty \beta \cdot f_i^\beta(\beta) d\beta. \tag{3}$$

If the process exploitation of car is stationary, the probability that in randomly chosen time moment ξ there occurs pulse (the state of car fitness) is given by the formula:

$$p_i = \frac{Ea_i}{ET_i} = E\mu_i \cdot Ea_i \tag{4}$$

where $E\mu_i$ - expected frequency of occurrence pulse, wherein

$$E\mu_i = \frac{1}{ET_i} = \frac{1}{Ea_i + Eb_i}. \tag{5}$$

It is assumed that the transport company will have the required potential of lading when in the required period of time in a state of fitness would be no less cars than the threshold number ***r***.

Due to the fact that the transport companies can include a different number of different transport means and to exploit them under different conditions of the threshold number of means of transport in those companies will also be different. The threshold number of vehicles should be set so that:

- was the smallest possible under the given conditions,
- take into account parameters characterizing the evolution of the demand for transport services in the area of the company.

The independence of the process of cars exploitation, this means that it is possible that a randomly chosen moment in a state of alertness may also be more than one car. Let $X(t)$ is the resultant of a process exploitation of cars. It is a process binary (the state of fitness and the state of unfitness), in which the state of fitness, means the state referred to as ***TE*** (*technical efficiency*), formed by superposition of states fitness any car in number, at least equal to the threshold number of cars r, ($r = 1, 2, ..., I$). TE state will be taken as the desired state when its duration is not less than the established value τ. In other cases, the status of ***TE*** will be treated as a state indicating the inability to satisfy the demand for transport services at the required level; τ value is determined for each company separately. An example of the process $X(t)$ of cars exploitation is shown in Fig. 2.

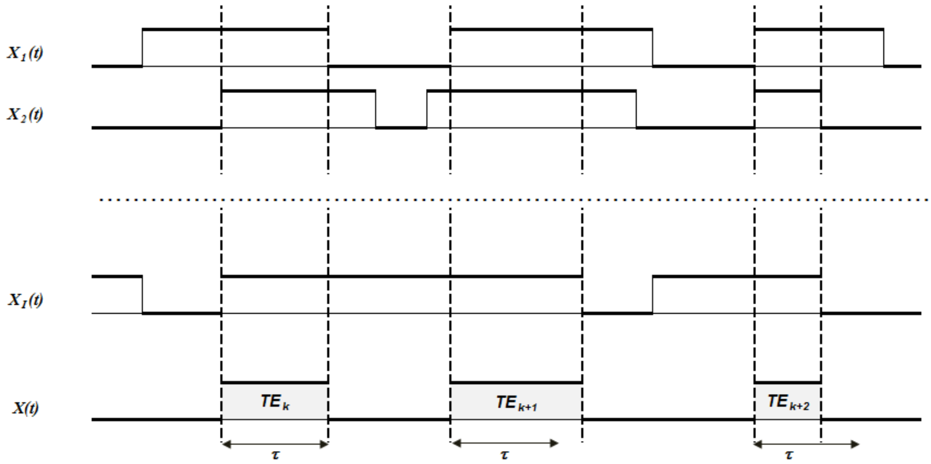

Figure 2. The process of formation of the resultant process ***X(t)*** exploitation of cars the company (the duration of the state of $\boldsymbol{TE_{k+2}}$ is shorter than τ which means that transport company does not have the adequate potential transport at this time)

3. Formulation of optimization problem

Considered further optimization task will concern to determine the minimum threshold number of cars ***r***, (***r = 1,2, ..., I***), ensuring the satisfaction of the demand for transport services.

The choice of this size would be made taking into account following criteria of minimizing:

- the expected frequency of the occurrence of states of fitness ***TE*** process ***X(t)*** exploitation the cars in transport company,
- the expected duration the state of fitness ***TE***

taking into account the following restrictions:

- it is known for the number of cars (means of transport) ***I*** in the transport company,
- the threshold number of cars (means of transport) can not be greater than the number of cars exploited by the transport company,
- expected value the duration of the state of ***TE*** may not be less than a predetermined value τ.

To solve the optimization task is required an ability to determine the expected duration of the ***TE*** state of the process ***X(t)***.

Let $\boldsymbol{Y_I(t)}$ is the stochastic process of the form [11]:

$$\boldsymbol{Y_I(t) = \sum_{i=1}^{I} X_i(t).} \tag{6}$$

For the assumptions regarding the exploitation processes of cars, an event that in the random moment ξ k of cars from among cars owned by the company is able to fitness can be written as:

$$Y_I(\xi) = k, \quad k = 0,1,2,\ldots, I. \tag{7}$$

The probability of this event is expressed by formula [3, 9]:

$$\gamma_{I,k} = \frac{1}{k!}\frac{d^k}{dx^k}\prod_{i=1}^{I}(q_i + xp_i)\Bigg|_{x=0}, \quad k = 0,1,2,\ldots, I, \tag{8}$$

at the condition

$$\sum_{k=0}^{I}\gamma_{I,k} = 1, \tag{9}$$

where p_i is expressed by equation (4), and $q_i = 1 - p_i$.

If the company has cars, which exploitation processes have the same characteristics, there is

$$\forall_{i=1,2,\ldots,I} \quad p_i = p$$

and

$$\gamma_{I,k} = \binom{I}{k} \cdot p^k \cdot (1-p)^{I-k}, \quad k = 0,1,2,\ldots, I. \tag{10}$$

For a further consideration is the important size of the expected frequency $E\mu_{I,k}(\tau)$ the occurrence of states TE of length (duration) of not less than τ, which was formed by the superposition of the states of fitness k, ($k = 0,1,2, \ldots, I$) of any among I of the cars exploited by the company. Using (8) can be $E\mu_{I,k}(\tau)$ expressed by the following formula:

$$E\mu_{I,k}(\tau) = -\frac{d}{d\tau}\gamma_{I,k}(\tau), \quad k = 0,1,2,\ldots, I, \tag{11}$$

wherein $\gamma_{I,k}(\tau)$ is the probability that in a randomly chosen moment of the time occurs the state TE of length (duration) of not less than τ, which was formed by the superposition of the states of fitness k, ($k = 0,1,2, \ldots, I$) of any among I of the cars exploited by the company.

Taking into account the previously mentioned assumptions concerning the exploitation processes of cars we obtain the following expression for the probability $\gamma_{I,k}(\tau)$ [3, 9]:

$$\gamma_{I,k}(\tau) = \frac{1}{k!}\frac{d^k}{dx^k}\prod_{i=1}^{I}\left(Q_i(\tau) + xP_i(\tau)\right)\Bigg|_{x=0}, \quad k = 0,1,2,\ldots, I, \tag{12}$$

where, taking into account (5)

$$P_i(\tau) = E\mu_i \int_\tau^\infty (x-\tau)\, f_i^\alpha(x)\,dx = E\mu_i \int_\tau^\infty dx \int_x^\infty f_i^\alpha(y)\,dy,\ \ i = 1,2,...,\ \ I,$$

$$Q_i(\tau) = E\mu_i \int_\tau^\infty (x-\tau)\, f_i^\beta(x)\,dx = E\mu_i \int_\tau^\infty dx \int_x^\infty f_i^\beta(y)\,dy,\ \ i = 1,2,...,\ \ I\,.$$

Taking into account (11) and (12) finally obtained:

$$E\mu_{I,k}(\tau) = -\frac{1}{k!}\frac{\partial^{k+1}}{\partial x^k\,\partial\tau}\prod_{i=1}^{I}\left(Q_i(\tau) + xP_i(\tau)\right)\Bigg|_{x=0},\ \ k = 0,1,2,...,\ I. \qquad (13)$$

Let $E\lambda_{I,k}(\tau)$ is the expected length (duration) the of state ***TE*** of length (duration) of not less than $\tau > 0$, which was formed by the superposition of the states of fitness k, ($k = 0,1,2,\ ...,\ I$) of any among I of the cars exploited by the company. It expresses by the following formula:

$$E\lambda_{I,k}(\tau) = \int_0^\infty \tau f_{I,k}^\alpha(\tau)\,d\tau, \qquad k = 0,1,2,\dots,I, \qquad (14)$$

where $f_{I,k}^\alpha(\tau)$ is the density function of the probability distribution of the duration of the state TE created by the superposition of the states of fitness k, ($k = 0,1,2,\ ...,\ I$) of any among I of the cars that is not less than a certain value $\tau > 0$. Taking into account (11) function $f_{I,k}^\alpha(\tau)$ can be expressed by the following formula:

$$f_{I,k}^\alpha(\tau) = \frac{1}{E\mu_{I,k}(\tau)}\frac{d^2}{d\tau^2}\gamma_{I,k}(\tau), \qquad k = 0,1,2,\dots,I\,. \qquad (15)$$

Taking into account (14) i (15) is obtained:

$$E\lambda_{I,k} = \frac{\gamma_{I,k}}{E\mu_{I,k}(0)}, \qquad k = 0,1,2,\dots,I\,. \qquad (16)$$

If the company has cars, which exploitation processes have similar characteristics, $E\lambda_{I,k}$ expresses the relationship:

$$E\lambda_{I,k} = \frac{1}{E\mu}\frac{p(1-p)}{(I-k)p + k(1-p)}, \qquad k = 0,1,2,\dots,I\,, \qquad (17)$$

wherein p and $E\mu$ express dependencies (4) and (5), respectively, and are the same for each car.

From a practical point of view, may be interesting the following two cases:

- $E\lambda_{I,I}$ - expected length the state of fitness ***TE***, when are taken into account all the cars exploited in the company ($k = I$),
- $E\theta_{I,I}$ - expected length the state of unfitness ***TN***, when are taken into account all the cars exploited in the company ($k = I$).

Formulas defining the above the sizes mentioned are of the form:

$$E\lambda_{I,I} = \left(\sum_{i=1}^{I} \frac{1}{Ea_i} \right)^{-1}, \tag{18}$$

$$E\theta_{I,I} = \left(\sum_{i=1}^{I} \frac{1}{Eb_i} \right)^{-1}. \tag{19}$$

Based on the previously adopted assumptions and using formulas (13) and (16) can be obtain the solution of a problem the receipt of the expected frequency the occurrence of the pulses ***TE*** and the expected their length, created by the superposition of the states of fitness of cars in number is not less than set their threshold r. Thus, the transport company will have the required potential of lading, when at a random time interval of length t will fulfilled the inequality:

$$Y_I(t) \geq r, \tag{20}$$

where $Y_I(t)$ expressed by equation (6).

Let $E\mu^*_{i,r}$, ($r=1,2,...,I$) is the expected frequency of occurrence of the ***TE*** state, formed by the superposition of states of fitness at least r cars. This value is calculated from the formula:

$$E\mu^*_{I,r} = \sum_{k=0}^{r-1} (-1)^{r+k+1}\, E\mu_{I,k}, \qquad r = 1,2,\ldots,I, \tag{21}$$

where $E\mu_{I,k}$, ($k=1,2 ....,r-1$) is the expected frequency of superposition k of states of fitness of the cars from the among I cars, is expressed by equation (11) with the condition $\tau = 0$, i.e.:

$$E\mu_{I,k}(\tau) = -\frac{d}{d\tau}\gamma_{I,k}(\tau)\Big|_{\tau=0} = E\mu_{I,k}, \quad k = 0,1,2,..., \ I. \tag{22}$$

Let $E\lambda^*_{I,r}$, ($r=1,2 ....,I$) is the expected length (duration) of the ***TE*** state, formed by the superposition of states of fitness at least r cars. This value, taking into account (8), you can express by formula:

$$E\lambda^*_{I,r} = \frac{1 - \sum_{k=0}^{r-1} \gamma_{I,k}}{E\mu^*_{I,r}}, \quad r = 1,2,...,I. \tag{23}$$

If the company has cars, which exploitation processes have similar characteristics, it is possible to adopt the following assumptions:

$$\begin{aligned} &\forall_{i\in I} \quad Ea_i = Ea, \\ &\forall_{i\in I} \quad Eb_i = Eb, \end{aligned} \tag{24}$$

formulas (21) and (23) take the form of:

$$E\mu^*_{I,r} = C_I^r \cdot r \cdot p^{r-1} \cdot (1-p)^{I-r} \cdot E\mu\,, \tag{25}$$

$$E\lambda^*_{I,r} = \frac{1-\sum_{k=0}^{r-1} C_I^k \cdot p^k \cdot (1-p)^{I-k}}{E\mu^*_{I,r}}\,, \tag{26}$$

where p and $E\mu$ are expressed by the formulas, respectively (4) and (5) provided (24).

Optimization task [5, 7, 8], formulated at the beginning of this point can be clarified as follows:

*for car company, which uses **I** of cars to determine a minimum threshold number of vehicles r, (**r=1,2,...,I**) from the point of view of maximizing of expected frequency of superposition of states of fitness at least **r** from the among **I** of the cars and of maximizing of expected duration this superposition with the following conditions:*

- *number of vehicle fleet of the company is known and equal to **I**,*
- *the expected duration of superposition of states of fitness at least **r** of cars not less than a specified value τ.*

This optimization task formulated verbally takes the following form formal [1]:

$$(\Omega, \varphi, R), \tag{27}$$

where:

- Ω - the set of feasible solutions as:

$$\Omega = \left\{ r: \quad r = 1,2,\ldots,I;\ E\lambda^*_{I,r} \geq \tau \right\} \tag{28}$$

- φ - vector criterion as:

$$\varphi = \left(r, E\lambda^*_{I,r}, E\mu^*_{I,r} \right) \tag{29}$$

- R - the conical relationship dominating as:

$$R = \left\{ (y_1, y_2) \in Y \times Y: \quad y_1^1 \leq y_2^1, y_1^2 \geq y_2^2, y_1^3 \geq y_2^3 \right\}. \tag{30}$$

4. Solution of optimization problem

The task formulated out in point 3 is a nonlinear multi-objective optimization task. The solution of this task will involve the appointment of a set of non-dominated solutions (Pareto-optimal) by the relation of domination (30).

Determination of the set of non-dominated solutions in this case may be difficult due to the non-linear criterion. This makes it impossible to direct application of known methods for solving multi-criteria optimization task [e.g. 2,6,10]. Most often, a small cardinality of set $\boldsymbol{I}$ will be used for a full overview of possible acceptable solutions on the basis of which will be designated non-dominated set of solutions, or in the best case solution dominant. In the event that a full overview will not be out of the question, use the methods of representation or random.

Table 1 shows an examples of compromise solutions (Pareto-optimal) for the case when all the cars of the company have the same exploitation characteristics: $\boldsymbol{Ea = 20}$ [unit time], $\boldsymbol{Eb = 2}$ [unit time].

Table 1. Examples of non-dominated solutions under consideration optimization task

r	$E\mu^{*}_{I,r}$	$E\lambda^{*}_{I,r}$	τ
I = 10			
5	0,026424	150,4127	8
6	0,070464	36,60317	
7	0,120796	12,47619	
I = 14			
5	0,000336	11908,47	10
6	0,002015	1653,399	
7	0,009213	309,3873	
8	0,032244	76,63253	
9	0,085985	24,71084	
10	0,17197	10,11988	
I=30			
15	0,000134	7455,133	20
16	0,000537	1863,45	
17	0,001878	532,0571	
18	0,005744	173,5571	
19	0,015317	64,66726	
20	0,035471	27,46995	

As can be seen from the table above, the threshold efficient means of transport below, which the transport company may lose a substantial part or all its potential transport grows disproportionately more slowly than the number of means of transport in the company at all. This means that larger companies with greater number of means of transport, can more easily maintain a reasonable level of transport potential than smaller companies, and this is confirmed by the fact, which indicates the usefulness of considered model.

5. Conclusions

In the case of large companies activities exploiting a large number of cars formulas (25) and (26) can be approximated respectively by the following:

$$E\mu^*_{I,r} \approx I \cdot E\mu \cdot \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{x_r^2}{2\sigma^2}}, \tag{31}$$

$$E\lambda^*_{I,r} \approx \frac{1}{I \cdot E\mu} e^{\frac{x_r^2}{2\sigma^2}} \int_{x_r}^{\infty} e^{-\frac{x^2}{2\sigma^2}} dx, \tag{32}$$

where

$$x_r = r - I \cdot p, \qquad \sigma = \sqrt{I \cdot p \cdot (1-p)}. \tag{33}$$

Fig. 3 is a graph showing the course of size described formulas (31) and (32) depending on the threshold value r for the case when a company uses ***100*** cars and they all have similar exploitation characteristics: $Ea = 100$ [unit time], $Eb = 25$ [unit time].

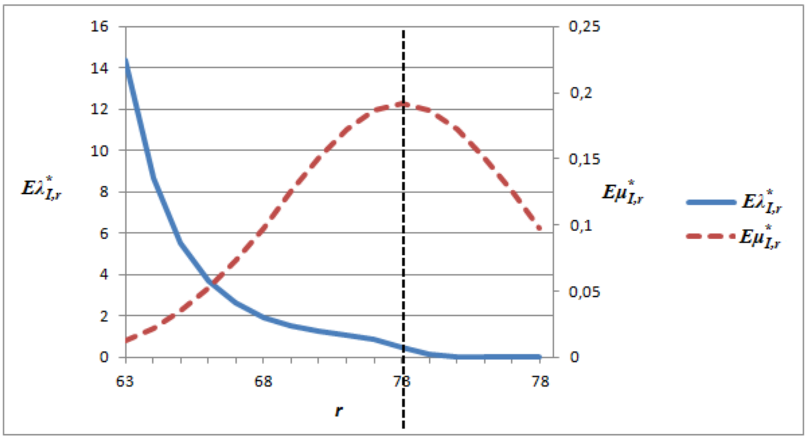

Figure 3. The course of size described formulas (31) and (32) depending on the threshold value ***r*** for the above-described data

Fig. 3 dashed lines indicate the right border of the interval containing solutions not-dominated. To the right of this border are dominated solutions by indicated previously: they are characterized by worse values for the $\boldsymbol{E\lambda^*_{I,r}}$, $\boldsymbol{E\mu^*_{I,r}}$ and $\boldsymbol{r}$ in the sense of accepted relationship of domination (30).

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