Pavel Záskalický
Technical University of Košice

TORQUE RIPPLE CALCULATION OF TWO-PHASE IM SUPPLIED BY THREE-LEG VSI INVERTER

Abstract: The proposed paper deals with steady state estimation of a electromagnetic torque ripples and a current waveform of a two-phase induction motor (IM), which is supplied by an three-leg voltage source IGBT bridge connected inverter (VSI). The complex Fourier series analysis of the inverter's output voltage was made, to obtain an analytical formula of supply voltage waveform. The space phasor theory has been applied to the two-phase induction machine model to obtain electromagnetic torque and supply current waveforms for various operation states.

Keywords: *two-phase motor, torque ripple, mathematical model, space phasor*

1. Introduction

The electrical low-power unites which are supplied by a single phase voltage are widely deployed by two-phase motors. They are settled in different industrial and domestic devices. They are very often deployed as a pumps drives in a washing machines and dishwashers, but also in a circulating pumps for central domestic heating. [1],[2]

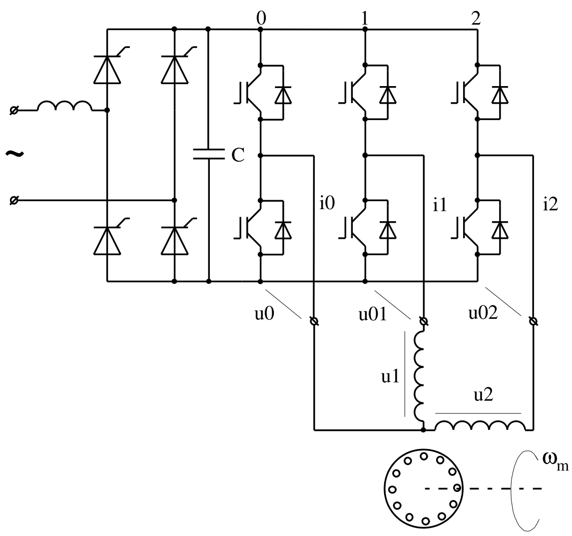

Fig.1 Two-phase IM drive topology

A two-phase motor by their characteristics no differs from the three-phase ones. Their advantage is simpler winding layout, which is of great importance for automated motor production.

Slovak Science Grant Agency VEGA is acknowledged. The paper was made under project No: VEGA 1/0121/15.

The two-phase motors are at present manufactured as either squirrel cage induction or permanents magnets synchronous motors. A permanent magnet in this case is water and lye resistant, which allows making a pump with an absolute waterproof. The stator winding can be configured in either a serial or parallel two-phase system. Normally, the winding are identical. The windings which form one phase are connected to induce opposite magnetic polarity.

Modern devices require drives enable speed variation. For this reason, desist from auxiliary phase with capacitor and two-phase motors are supplied by semiconductor converters too. There are several ways to create a two-phase supply voltage. One of possibility is shown in Fig.1.[5],

The voltage source inverter (VSI) processes represent the final stage in terms of generating the output voltage and frequency. This one guarantees good operating conditions throughout the whole speed range by adapting the output voltage to the load conditions. It is thus possible to maintain the magnetization of the motor at the optimal value. From the DC link circuit, the inverter is supplying variable DC voltage.

The main components are controlled semiconductors (IGBT transistors), arranged in pairs in three-legs, as shown in Fig. 1. A freewheeling diode is connected in parallel with each transistor, because high induced voltages can occur across the inductive output load.

Assume that variable voltages and frequencies are generated by a pulse amplitude modulation (PAM). PAM is used in converters with variable intermediate DC-link voltage. In a case where the rectifier is fully controlled, the amplitude of the output voltage is generated directly. This means that the output voltage for the motor is made available in the DC link circuit.[8]

2. Inverter mathematical model

To build a mathematical model of inverter the complex Fourier series is used. Fourier series are extremely useful as a way to break up an arbitrary periodic function into a set of simple terms that can be solved individually, and then recombined to obtain the solution to the original problem.

For inverter operation study we assume that all semiconductor components are ideal. This means that the switches can handle with unlimited current and voltage. Let's assume that the voltage drop and current leakage through switches are zero and that they are able turned on and off with no rise and fall times. We also assume that the inverter input capacity is sufficiently high, so we can suppose the converter input DC voltage to by constant for any output currents.

The transistors in each leg are switched so that they form voltage impulses over the half period of the desired output frequency. The voltage impulse of the first transistors leg measured versus negative pole of the DC link can be expressed 12],[13]

$$u_0 = \frac{U_e}{2} + 2U_e \; Re\left(\sum_{k=1}^{\infty} c_k e^{j2k\pi\omega t} \right) \quad (1)$$

While: U_e is a DC link output voltage and c_k is a Fourier coefficient, defined as

$$c_k = \frac{U_e}{j2k\pi}\left(1 - e^{-jk\pi}\right)$$

valid for $k \neq 0$

The voltage impulses in other legs a shifted by $\pm\frac{\pi}{2}$

$$\begin{aligned} u_{01} &= \frac{U_e}{2} + 2U_e \; Re\left(\sum_{k=1}^{\infty} \mathbf{a}\, c_k e^{j2k\pi\omega t} \right) \\ u_{02} &= \frac{U_e}{2} + 2U_e \; Re\left(\sum_{k=1}^{\infty} \mathbf{a}^{-1} c_k e^{j2k\pi\omega t} \right) \end{aligned} \quad (2)$$

$\mathbf{a} = e^{-\frac{jk\pi}{2}}$ is a phase shifting factor.

Motor phase voltages are given by a difference of the legs voltages as follow

$$\begin{aligned} u_1 &= u_0 - u_{01} = u_{01} = 2U_e \; Re\left[\sum_{k=1}^{\infty} \left(1 - \mathbf{a}\right) c_k e^{j2k\pi\omega t} \right] \\ u_2 &= u_0 - u_{02} = u_{01} = 2U_e \; Re\left[\sum_{k=1}^{\infty} \left(1 - \mathbf{a}^{-1}\right) c_k e^{j2k\pi\omega t} \right] \end{aligned} \quad (3)$$

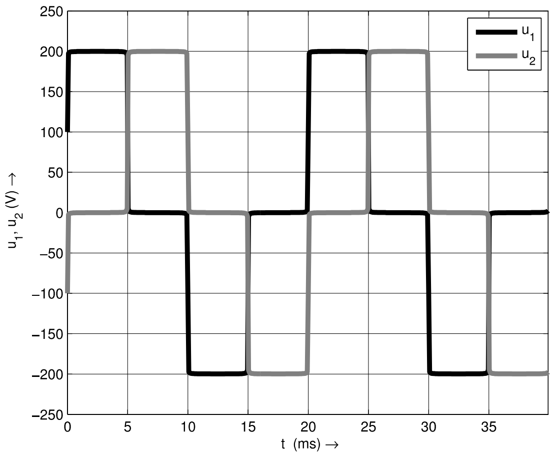

Fig.2 Phase motor voltages

Fig.2 shows the phase voltages u_1 and u_2 waveforms calculated on the base of equations (3). They are shifted by $\pi/2$ respectively.

To simplify the calculation of the AC motor quantities it is advantageously to employ space phasors [3], [6]. Output voltage space phasor of VSI $\underline{\mathbf{u}}$ is expressed as follow

$$\underline{\mathbf{u}} = u_1 + \mathbf{a_1} u_2 \quad (4)$$

While: $\mathbf{a_1} = e^{\frac{j\pi}{2}}$ is a space shifting factor

In Fig. 3 is shown the VSI output voltage trajectory. Calculation was made for DC-link voltage $U_e = 200V$ and frequency $f = 50\,Hz$.

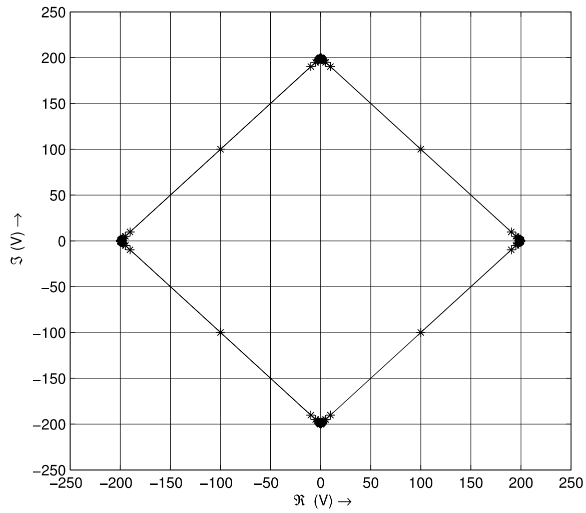

Fig.3 VSI output voltage trajectory

3. Current calculation

For the calculation stator and rotor space phasor trajectories, there was used the equivalent circuit of IM depicted in Fig. 4.

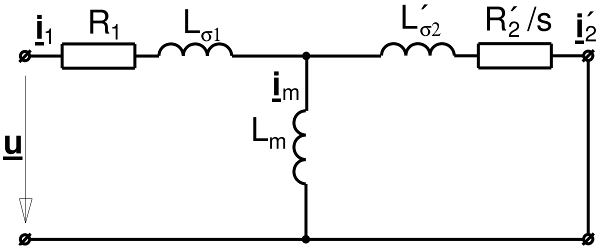

Fig.4 Equivalent circuit of IM

For further calculation will used following measured parameters of two-phase IM

$P_n = 50W; U_n = 2\times160V / 50Hz;$

$n_n = 1320\ rev / min;\ p = 2;$

$R_1 = 31\Omega;\ R_2' = 51\Omega;$

$L_m = 1.181H; L_{1\sigma} = L_{2\sigma}' = 0.15H;$

Referred to the equivalent circuit above, for the current space phasor $\underline{\mathbf{i}}_1$ the following equation is valid

$$\underline{\mathbf{i}}_1 = \sum_{k=1}^{\infty} \frac{\underline{\mathbf{u}}}{R_1 + \left[jk\omega L_{1\sigma} + \dfrac{jk\omega L_m \left(R_2'/s_k + jk\omega L_{2\sigma}' \right)}{R_2'/s_k + jk\omega \left(L_m + L_{2\sigma}' \right)} \right]} \quad (5)$$

taking: $s_k = \dfrac{k\omega_1 - sg\,\omega_m}{k\omega_1}$ is slip for k^{th} harmonic component

where: $sg = 1$ for positive, $sg = -1$ for negative sequence

Fig. 5 shows calculated the stator current space phasor trajectory. Computation was made for nominal motor speed.

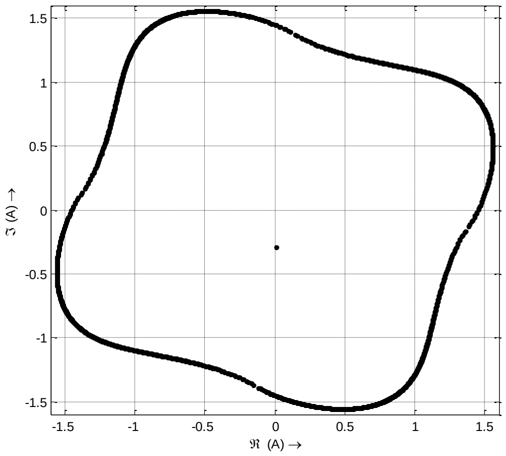

Fig.5 Stator current space phasor trajectory

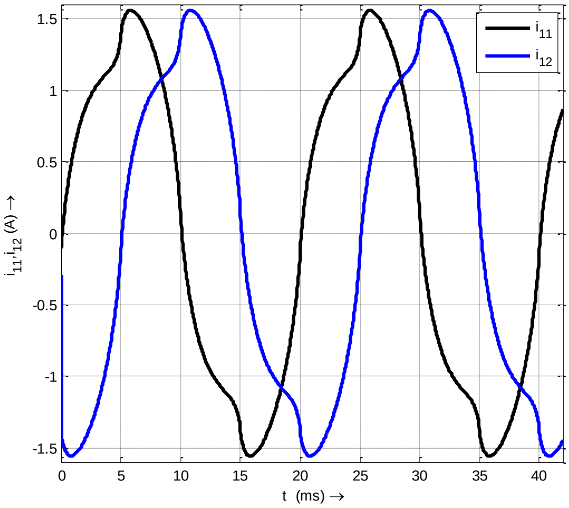

Fig.6 Stator currents waveforms

By decomposition of current space phasor to real and imaginary part, we receive stator currents waveforms. They are shown in Fig. 6. There a several possibilities to solve equivalent circuit for the current $\underline{\mathbf{i}}_2$. The easiest one is to determine “Thevenin” equivalent circuit. Thevenin equivalent circuit presents a simple series combination of elements, as shown in Fig. 7.

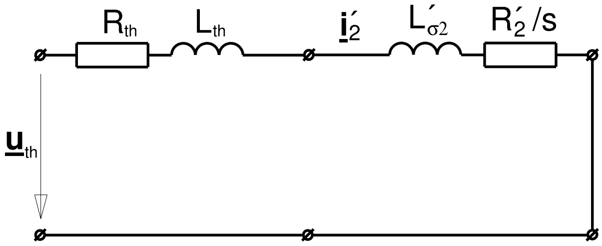

Fig.7 Thevenin equivalent circuit of IM

To find the Thevenin voltage, we must recalculate the DC-link voltage for requested frequency and motor parameters as follow

$$U_{th} = U_e \frac{\omega L_m}{\sqrt{R_1^2 + \omega^2 \left(L_{1\sigma} + L_{2\sigma}' \right)^2}} \quad (6)$$

$\omega = 2\pi f$ is an angular frequency

The Thevenin impedance is given by

$$Z_{th} = R_{th} + j\omega L_{th} = \frac{j\omega L_m \left(R_1 + j\omega L_{1\sigma} \right)}{R_1 + j\omega \left(L_m + L_{1\sigma}' \right)} \quad (7)$$

Because $L_m \gg L_{1\sigma}'$ and $\omega\left(L_m + L_{1\sigma}' \right) \gg R_1$, the Thevenin resistance and inductance are approximately given by

$$R_{th} \approx R_1 \frac{L_m}{L_m + L_{1\sigma}'} \quad (8)$$

$$L_{th} \approx L_{1\sigma}$$

On the base of Thevenin equivalent circuit for the rotor current space phasor following equation is valid

$$\underline{\mathbf{i}}_2 = -\sum_{k=1}^{\infty} \frac{\mathbf{u}_{th}}{R_2'/s_k + R_{th} + jk\omega\left(L_{th} + L_{2\sigma}'\right)} \qquad (9)$$

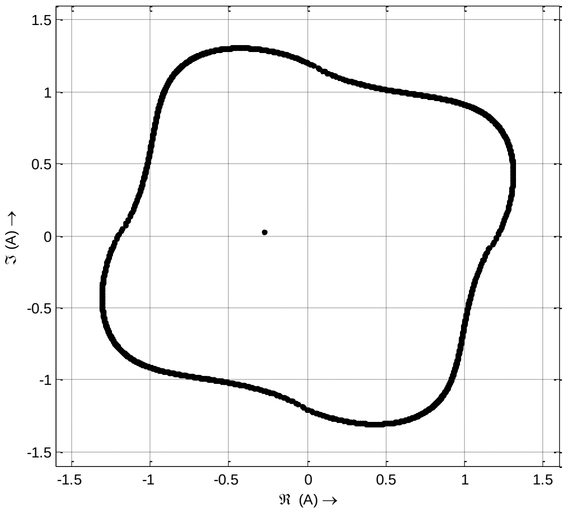

Fig.8 Rotor current space phasor trajectory

Fig. 8 shows the rotor current space phasor trajectory calculated on the base of the equation (9). Corresponding two dimensional rotor current components are shown in Fig. 9.

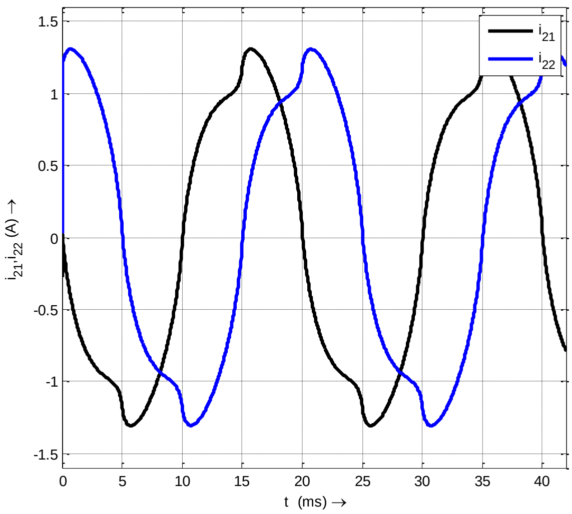

Fig.9 Rotor current waveforms

4. Torque calculation

Electromagnetic torque of two-phase induction motor depends on the stator and rotor currents. Generally following equation is valid

$$M_{em} = p\left(\psi_{1\alpha} i_{1\beta} - \psi_{1\beta} i_{1\alpha}\right) = p\left(\psi_{2\alpha} i_{2\beta} - \psi_{2\beta} i_{2\alpha}\right) \quad (10)$$

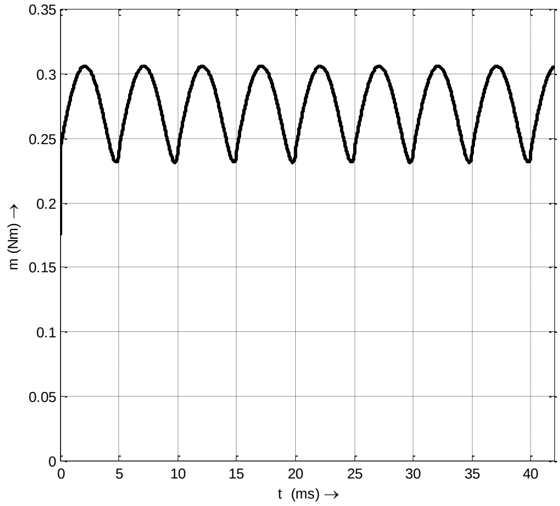

Fig.10 Electromagnetic torque waveform

There is a possibility to express stator and rotor linked fields in terms of product of inductance and current

$$M_{em} = p\,L_m\left(i_{1\beta} i_{2\alpha} - i_{1\alpha} i_{2\beta}\right) \qquad (11)$$

Using space phasor relationship, the equation (11) comes into the form

$$M_{em} = p\,L_m\, Im\left(\underline{\mathbf{i}}_1\, \underline{\mathbf{i}}_2^{*}\right) \qquad (12)$$

Fig.10 shows the calculated time plot of electromagnetic torque of a two-phase IM supplied by a rectangular voltage.

5. Conclusion

The proposed contribution shows possibility of calculation electromagnetic torque of a two-phase IM using space phasor theory. The computation is simple and gives sufficiently accurate values. The proposed mathematical method is easily applicable to multiphase IM, too. The condition is that we are able analytically express course of supply input voltages.

The method of the mathematical representations of the voltages be done may be different. This one can be done in the form of infinite series, but also by means of trigonometric functions.

6. References

[1] S.J. Chapman, "Electric machinery fundamentals," Mc-Graw-Hill, 4th edition, (2005)

[2] J.Chatelein, "Machines électriques," Vol. X, Edition Georgi, (1983)

[3] G. J.Machowski, J.W. Bialek, J.R. Bumby, « Power system Dynamics: Stability and control," John Wiley & Sons, (2011)

[4] B. Robyns, B. Francois, P. Dagobert, J.P. Hautier, "Vector Control of Induction Machines, " Springer (2012)

[5] N. Mohan, T.M. Undeland, W.P. Robbins, " Power electronics : Converters, Applications and Design, "John Wiley & Sons, (1993)

[6] H.M. Ryu, S.K. Sul, "ultiple d-q space concept for multi-phase AC motor drive," ICPE-04, pp.670-674, (2004)

[7] M. Trabelsi, K.N. Ngac, E. Semail, "Real-Time Switches Fault Diagnosis Based on Typical Operating Characteristics of Five-Phase Permanent-Magnetic Synchronous Machines," IEEE Trans. on Ind. Electronics, Vol. 63, No. 8, pp. 4683-4694 (2016)

[8] J.O. Estima, Marques A.J. Cardoso, "A new algorithm for real-time multiple open-circuit fault diagnostic in voltage-fed PWM motor drives by the reference currents errors," IEEE Trans. on Ind. Electronics, Vol. 28, No. 5, pp. 3496-3505 (2013)

[9] E. Levi, "Multiphase electric machines for variable-speed applications," IEEE Trans. on Ind. Electronics, Vol. 55, No. 5, pp. 1893-1909 (2008)

[10] Lu Shuai, K. Corzine, "Direct torque control of five-phase induction motor using space vector modulation with harmonics elimination and optimal switching sequence," APEC 06, (2006)

[11] C.Gh. Nistor, Gh. Scutaru, R. Campeanu, M. Cernat, „Noise and Vibration Monitoring for Premium Efficiency IE 3 Three-Phase Induction Motors", Advances in Electrical and Computer Engineering, Vol.15, No.3, pp. 117-122, Romania, (2015)

[12] B. Dobrucký, P. Záskalický, S. Kaščák, M. Praženica, "Comparison of modified sinusoidal and space vector modulation for symmetrical two-phase induction motor under field oriented control", IREMOS Int. Review of Modeling and Simulation, Vol. 5, No. 6, pp. 2445-2451 (2012)

[13] P. Záskalický, B. Dobrucký, "Complex Fourier series mathematical model of a three-phase inverter with Improved PWM output voltage control", Elektronika ir Elektrotechnika, No. 7 (123), pp. 65-68, KTU, Lithuanie, (2012)

[14] M. Chomát, L. Schreier, "Effect of Stator Winding Configuration on Operation of Converter Fed Five-Phase Induction Machine", IEEE Electric Drives and Power Electronics Conf. (EDPE), High Tatras, pp. 488-496, Slovakia, (2015)