PROBABILISTIC INVERSE ANALYSIS OF DATA OBTAINED FROM PAVEMENT DEFLECTION MEASURED BY FWD [1]

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FWD tests are one of the most common tests of pavement in the world. The elastic modulus of each pavement layers can be obtained by applying inverse analysis, which is useful not only in assessment of road pavement condition, but also in designing the overlayer. The author of this paper confronted probabilistic and deterministic approaches of input data for backcalculation (thickness and Poisson's ratios of each layer) for homogeneous experimental section. The results of the sensitivity analysis have shown that the most significant influence for the identified quantity had a thickness of bituminous layers. In this paper the measurement error of thickness of asphalt concrete layers was simulated for comparison the probabilistic and deterministic approaches, moreover, the results of laboratory tests were used for verification.

1. INTRODUCTION

Pavement deflections testing is the most frequently encountered method used for assessing the technical condition of pavements. An additional advantage of this study is its non-destructive nature. Inverse analysis constitutes a part of inference methods used for the parameters of pavement construction. It consists in identifying specific parameters by means of easily measurable pavement characteristics. Adjusting elastic modulus values to a pavement layer model is a process that demands optimization, and for the purposes of this paper, the Nelder-Mead algorithm is used. The well-known ambiguity issue of the results obtained in the procedure of inverse calculation may evoke limited confidence in this method of diagnostics. The paper discusses a more accurate method of obtaining identification results by implementing the probabilistic approach, in which chosen values are treated as random variables. This counters the traditional procedure for inverse calculations in which deterministic definitions of the values are adopted a priori. Assuming that the searched parameters are values of elastic modulus of pavement model layers, the thickness of the highest layer is a varia-

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ble that most significantly determines the elastic modulus. Other parameters, such as Poisson's ratio and the thickness of other layers, have a considerably smaller influence on the results of the identification. On the basis of the analysis of an actual pavement conducted in the paper, one can assume that with probabilistic analysis we can identify elastic modulus values of asphalt courses more accurately, which means that values are more similar to the values obtained in laboratory tests than those obtained with deterministic analysis.

2. LITERATURE REVIEW

When using the inverse analysis according to [7], the user cannot be sure if the identified values of elastic modulus are properly assigned to the corresponding layers. Moreover, the author emphasizes the necessity of learning the pavement structure for building an adequate model. Generally, in order to understand the pavement structure, core drilling is used. The basis for the inverse calculation procedure are objective functions, which can have numerous local minimums, as further explained in [14]. For this reason, inverse identification is often questionable. Research presented in paper [9] summarizes the results of parameter identification from various scientific centers. By performing an analysis of those results, the author of the paper draws conclusions about ambiguity of the identifications. Inverse analysis based on probabilistic approach by means of a dynamic model was used in paper [5], which was restricted to synthetic data exclusively. However, the experiences of the author of this paper show that an identification of 3-layer modulus does not pose any real problems, even with the implementation of static model [2]. Another interesting proposition for identification was shown in paper [4]. The author described an attempt to conduct inverse calculations based on displacement data obtained in a dynamic-to-static test, which, according to the author, improved the identification accuracy of the modulus of elasticity values of pavement layers. The authors of papers [12] and [8] have shown that there is a possibility of effective use of artificial neural networks for the identification of the number of layers in a pavement model and the thickness of each of them. An equally important aspect of inverse identification is the compensation effect of layers described in more detail in paper [3]. According to the author of paper [15], compensation effect is the result of assigning linear elastic modulus to materials that cannot always assume this characteristic. One of the solutions to this problem is defining a rigid layer below the pavement subgrade [6], and the method described in that article created opportunities for more accurate identification of unbounded layers.

3. BACKCALCULATION

Backcalculation is a procedure of calculating values that are arguments of a function, while the exact value of that function is known. In the present case, the following are known: the displacement of the surfaces to strictly specified distances from the axis of the load, the load value, the field through which the load is transferred to the pavement, and the data of the pavement construction. The values we are looking for are elastic modulus of each pavement course. Assuming that the courses are fully bond, and the surface materials are isotropic and homogenous, it is possible to calculate displacements at any point of the surface under the load with the flexible pavement mechanistic model (LET), as has been shown in formula 1.

$$w = \frac{1}{2} \cdot \int_0^{\infty} \{-A \cdot \sinh(kz) - B \cdot cpsh(kz) + C[(2 - 4 \cdot \nu) \cdot \cosh(kz) - kz \cdot \sinh(kz)] + D \cdot [(2 - 4 \cdot \nu) \cdot \sinh(kz) - kz \cdot \cosh(kz)]\} \cdot J_0(kr)dk \quad (1)$$

where:

υ – Poisson's ratio,

z – the vertical ordinate in the layer,

r – the horizontal distance from the axis of the load,

k – integral parameter,

J_0 – Bessel function of the first kind, zero order,

A, B, C, D – unknowns, dependent on the values of the integral parameter, determined on the basis of boundary conditions resulting from:

- the load,
- the bond between the layers and the entailing shear strain,
- equality of vertical displacements on the borders of the each layer.

While searching for elasticity modulus values that would match the displacements, one should perform a series of displacement calculations for the assumed modulus and find a configuration that will most accurately describe the behavior of pavement constructions under load. For the purpose of this study, RMSE (Root Mean Square Error) was used, as described in formula 2.

$$RMSE = \sqrt{\frac{1}{n}\sum \frac{(w_c - w_m)^2}{w_m}} \cdot 100\% \quad (2)$$

where:

n – the number of displacement sensors,

w_c – deflection calculated from the pavement model,

w_m – deflection measured on the experimental pavement section.

In order to optimize the amount of deflection calculations, Nelder-Mead algorithm (described with more detail in [10]) was implemented, with RMSE used as the objective function. This optimization algorithm requires defining the starting point, which may be a factor influencing the final outcome. In the presented case, 5 independent starting points have been used, all resulting in the same outcome. Hence, the author assumed that the initial values do not affect the result of identification of the illustrated case, and they only have influence on the amount of iterations.

4. EXPERIMENTAL SECTION

The probabilistic inverse analysis was conducted on the data from the experimental section. The author discovered uniformity of the measurement section whose structure corresponds to the model in 1. Pavement model, which was developed under the assumptions of Layered Elastic Theory (LET), described in more details in [3]. The average measured values of thickness for each layer were, respectively: h1=12,1 cm – asphalt concrete layer (AC), h2 = 20,0 cm layer of compacted aggregate, the third layer was a substrate of infinite thickness.

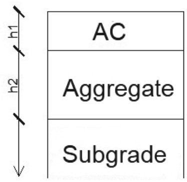

Figure 1. Pavement mechanistic model of the experimental section

Thirty-five measure points were set on the experimental section. During the construction of the pavement, there was an additional research conducted with Benkelman beam deflection method on the compacted aggregate layer. The results of these measurements were used to verify the earlier identified modulus of elasticity for the second and third layers of the surface model. On the surface of the asphalt concrete layer, FWD tests were carried out. They constituted the basis for the inverse analysis.

5. DEFINING RANDOM VARIABLE VALUES INTERVALS

In the inverse analysis of a pavement mechanistic model, it is almost always assumed that parameters such as thickness of individual layers or Poisson’s ratio are constant. In reality, layer thickness is not constant. This is confirmed by the results of thickness measurement using GPR (ground-penetrating radar), which is more accurate than measuring thickness based on cores drilled from the pavement. The uncertainty of thickness measurements of each pavement layer may also depend on the surface evenness of the upper and lower core layers. In such cases, it is common practice to average out the thickness from at least three measurements. Upon drilling a core, the thickness of a layer can be measured with a ruler or a caliper. The statistical distribution of values determined in this manner, is assumed to be uniform [1], and standard deviation can be derived from formula 3.

$$\sigma = \frac{\Delta}{\sqrt{3}} \tag{3}$$

where:

σ – standard deviation

Δ – boundary error value For assumed $\Delta = 1cm$, standard deviation equals σ = 0,58cm.

For assumed probability p=99%, the thickness measurement uncertainty for pavement layers equals 1.5 cm. The paper examines the thickness of model layers with an interval of 0.1 cm. Poisson's ratio is a material parameter, whose value can be variable depending on temperature and stress. For road construction materials, this parameter can vary from 0.2 to 0.45. For the purpose of this article, the ratio is analysed within this range with an interval of 0.02Analysis on surrounding objects

6. SENSITIVITY ANALYSIS

In order to select variables with fundamental influence on the values of the identification of elasticity modules, the author performed an analysis during the computation of inverse sensitivity. The sensitivity analysis was based on the assumptions described in [13], while using the method of factors prioritization. Exemplary and, at the same time, extreme values identified during the inverse calculations are shown in table 1. Elasticity modulus of the first layer was identified as the most ambiguous. The extreme values of this parameter have been summarized according to the change of one factor, in order to illustrate the scale of ambiguity of the determined parameter. RMSE values have been presented to show that in none of the cases can the identification results be excluded, and each time the discovered minimum is within the acceptable range from an engineering point of view.

Tab 1. Sensitivity analysis for each of variables

variable	Max E1 [MPa]	Min E1 [MPa]	Max RMSE [%]	Min RMSE [%]
H1	14455	5332	2,26	1,85
H2	8405	8309	2,07	2,05
v1	8817	7318	2,07	2,07
v2	8097	9206	2,61	1,94
v3	8373	8204	2,07	1,93

As can be seen, the identified elastic modulus values of the highest layer depend primarily on its thickness. Other parameters do not have a significant effect on the identified values, so for time-saving reasons, they were assumed to be constant.

7. RESULTS OF THE IDENTIFICATION

The author concluded that the elastic modulus values for each layer of pavement models obtained by means of probabilistic and deterministic approaches are characterized by a normal distribution. This statistical hypothesis can be rejected on no grounds , due to $p>95\%$. The results are shown in table 2.

Tab 2. The results of identification by the use of different approaches

Identification	E1 [MPa]	E2 [MPa]	E3 [MPa]	RMSE [%]
Deterministic identification	7093	29	126	2,03
Probabilistic identification	7568	29	127	2,07

In order to verify the idea of probabilistic identification of pavement layers values, the parameters obtained by this method were confronted with the results of laboratory and field tests. The obtained results have been calculated with the assumption that the measurement of thickness of the first layer has been encumbered with an error of 1,5 cm. As a consequence, the obtained thickness of the first layer ranged between 9,6 and 13,6 cm. The range was split every 0,5 cm. The calculations of the obtained elastic modulus values of the asphalt concrete layers were conducted for thicknesses ranging from 9,6 to 13,6 cm, with 0,5 interval. Next, the results were confronted with the results of the laboratory tests based on the model created from beams cut from an experimental section AC layer combined. The laboratory test method is known under the name 4PBB (beam loaded with constant moment). A possibility to compare the results of 4PBB research and calculated elastic modulus values is presented in [11]. The results of the comparison are illustrated in the figure 2, in which a histogram of the results obtained by probabilistic method is also included.

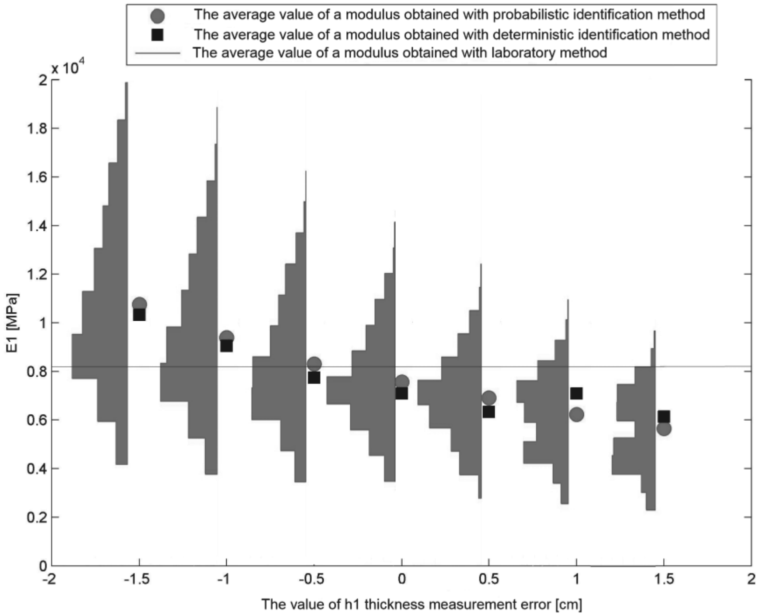

Figure 2. Summarized identification and laboratory results

Compared to the deterministic method, the values identified with the probabilistic method are characterized by identification error reduction (average 49%) with a relatively small thickness measurement error (ranging $\pm 0,\ 5cm$). Values of the elastic modulus layers 2 and 3 were also affected with compensation, which is mentioned in paper [15] (table 2). This article verifies the compensation phenomenon by controlling the results of Benkelman beam from in situ tests and the results of deflection calculation based on identified elastic modulus values. Although the author accepts the simplified answer, he analyses the harmfulness of the compensation effect on the identification of pavement layers modulus. The aim of the research is to identify displacements on the loaded surface on the axis of load surface. Displacements from different measuring points were averaged in order to obtain a reference value, and summarized with the values calculated basing on identified elastic modulus of layers 2 and 3, which had been loaded with the same conditions. The results are shown in the figure 3.

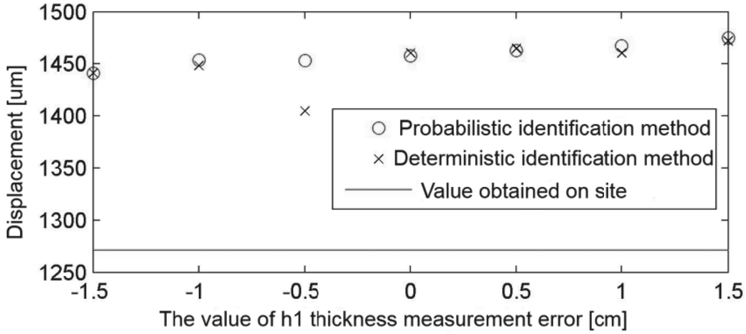

Figure 3. Displacements calculated and measured on site on top of the aggregate layer

Displacement values calculated from the model are larger than the ones measured by Benkelman beam; however, they are relatively low sensitive to thickness measurement error of asphalt concrete layer. Displacement values of Compacted Aggregate Layer obtained with probabilistic and deterministic method, as well as the ones acquired on site, are similar. From the engineering point of view, an error of 15% in displacement calculations can, in this case, be considered acceptable due to major divergence of the results obtained during in situ measurements, reaching up to 30%.

8. CONCLUSIONS AND FUTURE RESEARCH

Based on the analysis prepared for the experimental section, one can formulate the following conclusions:

The measurement uncertainty of the pavement layer thickness of ±1.5 cm is common, therefore, disregarding this fact can lead to errors in the identification of elastic modulus of pavement layers. The only value significantly affecting the result of identification of elastic modulus of pavement layers is asphalt layer thickness, whereas base course thickness in the range of ±1.5 cm, and the value of Poisson's ratio for each layer has less significance for a static analysis.

Compared to the deterministic method, the values identified with the probabilistic method of the identification of elastic modulus of asphalt layers are closer to the results of laboratory research. At the assumed 5% measurement uncertainty of the asphalt layer thickness, the average error reduction in identification of modulus value was 49%.

The compensation effect occurs when both deterministic and probabilistic methods are employed.

When confronted with the results of deflections measured in on site studies, the compensation effect is not significant enough to eliminate the results of the inverse analysis from the inferences made about the elasticity modulus of pavement layers.

The time of calculations is longer with the probabilistic method than it is when the deterministic one is used. This opens new areas for researching the appropriate tool to shorten the time in these tasks.

Summing up, the use of probabilistic analysis makes sense due to the possibility of including factors which the deterministic analysis disregards. Thanks to the implementation of probabilistic data, inverse analysis allows more precise identification of searched values.

Elimination of the compensation effect can probably be achieved by means of dynamic models, but the parameters of these models are still to be verified.

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ANALIZA ODWROTNA REZULTATÓW UZYSKANYCH PRZY ZASTOSOWANIU URZĄDZENIA FWD Z WYKORZYSTANIEM METODY PROBABILISTYCZNEJ

Streszczenie

Urządzenie typu FWD należy do jednych z najczęściej wykorzystywanych w badaniach nośności nawierzchni. Wyniki uzyskane za pomocą urządzenia FWD mogą posłużyć do obliczenia modułów sprężystości poszczególnych warstw nawierzchni, co jest niezwykle pomocne zarówno w diagnostyce drogowej jak i przy wymiarowaniu wzmocnienia konstrukcji nawierzchni jezdni. W niniejszej pracy porównany wyniki obliczeń odwrotnych metodą probabilistyczną i deterministyczną, gdzie danymi wejściowymi były grubości warstw i współczynniki Poissona. Analiza wrażliwości pozwoliła stwierdzić, że grubość najwyższej warstwy nawierzchni jest najbardziej istotna spośród wszystkich parametrów będącymi danymi wejściowymi do obliczeń odwrotnych. W pracy zasymilowano błąd pomiarowy określenia grubości najwyżej warstwy nawierzchni i przeprowadzono obliczenia modułów sprężystości poszczególnych warstw nawierzchni. Wyniki obliczeń uzyskane zarówno metodą deterministyczną jak i metodą probabilistyczną porównano z wartościami uzyskanymi na drodze badań laboratoryjnych.

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