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Reliability analysis of a system subjected to two-state operation process

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Abstract
The paper presents analytical and Monte Carlo simulation methods applied to the reliability evaluation of a system operating at two different operation states. A semi-Markov process is applied to construct the system operation model and its main characteristics are determined. Analytical linking of this operation model with the system reliability model is proposed to get a general reliability model of the system operating at two varying operation conditions and to find its reliability characteristics. The application of Monte Carlo simulation based on this general model to the reliability evaluation of this system is proposed as well. The results obtained from those two considered methods are evaluated.

Introduction

The reliability analysis of a system subjected to varying in time its operation process very often leads to complicated calculations and, therefore, it is difficult to implement analytical modeling, prediction and optimization, especially in the case when we assume the system multistate reliability model and the multistate model of its operation process [1, 2, 3, 4, 5]. On the other hand, the complexity of the systems' operation processes and their influence on changing in time the systems' reliability parameters are very often met in real practice [3, 6, 7, 8]. Thus, the practical importance of an approach linking the system reliability models and the system operation processes models into an integrated general model in reliability assessment of real technical systems is evident. The Monte Carlo simulation method [5, 9] is a tool that sometimes allows to simplify solving this problem [4, 10, 11]. All cited here publications presents general results obtained under a strong assumption that the system components have exponential conditional reliability functions at different operation states. To omit this assumption that narrows the investigation down and to get general solutions of the problem, at the beginning, we deal with the two-state reliability model of the system and two-state model of its operation process. The analytical approach to the reliability analysis of two-state systems subjected to two-state operation processes is presented and next the computer simulation modeling method for such systems reliability assessment is proposed.

System operation process

We assume that a system during its operation at the fixed moment t, $t \in \langle 0, +\infty \rangle$, may be at one of two different operations states z_b, $b = 1,2$. Consequently, we mark by $Z(t)$, $t \in \langle 0,+\infty \rangle$, the system operation process, that is a function of a continuous variable t, taking discrete values at the set $\{z_1, z_2\}$ of the system operation states. We assume a semi-Markov model [2, 3] of the system operation process $Z(t)$ and we mark by θ_{bl} its random conditional sojourn times at the operation states z_b, when its next operation state is z_l, $b,l = 1,2$, $b \neq l$. The exemplary realizations of the considered system operation process are presented in figure 1.

Consequently, the operation process may be described by the following parameters [4]:

– the vector $[p_b(0)]_{1\times 2}$, $b = 1,2$, of the initial probabilities of the system operation process $Z(t)$

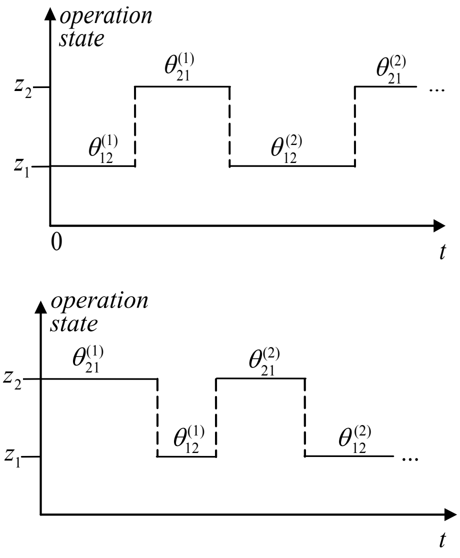

Fig. 1. The exemplary realizations of the system operation process

staying at the particular operation states at the moment $t = 0$;
- the matrix $[p_{bl}]_{2\times 2}$ of the probabilities of the system operation process $Z(t)$ transitions between the operation states z_b and z_l, $b,l = 1,2$, $b \neq l$;
- the matrix $[H_{bl}(t)]_{2\times 2}$ of the conditional distribution functions of the system operation process $Z(t)$ conditional sojourn times θ_{bl} at the operation states, $b,l = 1,2$, $b \neq l$.

We mark by:

$$\phi_1^{(n)}(t) = P(\theta_1^{(n)} < t),\ t \in \langle 0,\infty),\ n = 1,2,...,$$

the distribution functions of the random variables:

$$\theta_1^{(n)} = \theta_{12}^{(1)} + \theta_{12}^{(2)} + ... + \theta_{12}^{(n)},\ n = 1,2,...,$$

where the variables $\theta_{12}^{(i)}, i = 1,2,...,n,$ are independent random variables having identical distribution functions with the distribution of the sojourn time θ_{12}, i.e.:

$$P(\theta_{12}^{(i)} < t) = P(\theta_{12} < t) = H_{12}(t),\ i = 1,2,...,n,$$

and by:

$$\phi_2^{(n)}(t) = P(\theta_2^{(n)} < t),\ t \in \langle 0,\infty),\ n = 1,2,...,$$

the distribution functions of the random variables:

$$\theta_2^{(n)} = \theta_{21}^{(1)} + \theta_{21}^{(2)} + ... + \theta_{21}^{(n)},\ n = 1,2,...,$$

where the variables $\theta_{21}^{(i)}, i = 1,2,...,n,$ are independent random variables having identical distribution functions with the distribution of the sojourn time θ_{21}, i.e.:

$$P(\theta_{21}^{(i)} < t) = P(\theta_{21} < t) = H_{21}(t),\ i = 1,2,...,n.$$

Realizations $\theta_{12}^{(i)}$ and $\theta_{21}^{(i)}$ of the random variables $\theta_{12}^{(i)}$ and $\theta_{21}^{(i)}, i = 1,2,...,$ are illustrated in figure 1.

Consequently, we get:

$$\phi_1^{(1)}(t) = H_{12}(t),$$

$$\phi_1^{(n)}(t) = \int_0^t \phi_1^{(n-1)}(t-u)\,\mathrm{d}H_{12}(u),\ n = 2,3,...,$$

$$\phi_2^{(1)}(t) = H_{21}(t),$$

$$\phi_2^{(n)}(t) = \int_0^t \phi_2^{(n-1)}(t-u)\,\mathrm{d}H_{21}(u),\ n = 2,3,....$$

Moreover, we mark by:

$$\psi^{(n)}(t) = P(\theta^{(n)} < t),\ t \in \langle 0,\infty),\ n = 1,2,...,$$

the distribution functions of the random variables:

$$\theta^{(n)} = \theta_1^{(n)} + \theta_2^{(n)},\ n = 1,2,...,$$

and we have:

$$\psi^{(n)}(t) = \int_0^t \phi_1^{(n)}(t-u)\,\mathrm{d}\phi_2^{(n)}(u)$$
$$t \in \langle 0,\infty),\ n = 1,2,..., \tag{1}$$

If we denote by $N(t)$ the number of changes of the system operation process' states before the moment t, by $N_b(t)$, $b = 1,2$, the number of changes of the system operation process' states before the moment t when its operation process at the initial moment $t = 0$ was at the operation state z_b, $b = 1,2$, for $t \in \langle 0,+\infty)$, we immediately get the following results [5].

Proposition 1

The distribution of the number $N(t)$ of changes of the system operation process' states before the moment t, $t \in \langle 0,+\infty)$, are given by:

$$P(N(t) = 2n) = $$
$$= \psi^{(n)}(t)\Big[p_1(0)\Big(1 - \int_0^t \psi^{(n)}(t-u)\,\mathrm{d}H_{12}(u)\Big) + \tag{2}$$
$$+ p_2(0)\cdot\Big(1 - \int_0^t \psi^{(n)}(t-u)\,\mathrm{d}H_{21}(u)\Big)\Big]$$

$$P(N(t) = 2n+1) = p_1(0)\cdot \int_0^t \psi^{(n)}(t-u)\,\mathrm{d}H_{12}(u)\cdot$$
$$\cdot\Big(1 - \int_0^t \psi^{(n+1)}(t-u)\,\mathrm{d}H_{21}(u)\Big) +$$
$$+ p_2(0)\cdot \int_0^t \psi^{(n)}(t-u)\,\mathrm{d}H_{21}(u)\cdot \tag{3}$$
$$\cdot\Big(1 - \int_0^t \psi^{(n+1)}(t-u)\,\mathrm{d}H_{12}(u)\Big)$$

for $t \in \langle 0,+\infty)$, $n = 0,1,2,...,$ where $\psi^{(0)}(t) = 1$ and $\psi^{(n)}(t)$ for $n = 1,2,...$, are determined by (1).

System reliability subjected to two-state operation process

We assume that the considered two-state system reliability depends on its operation state it is operating and on the number of changes of the operation process states. We define the system conditional reliability function at the operation state z_b, $b = 1,2$, after k, $k = 0,1,...,$ changes of its operation process states:

$$R_k^{(b)}(t) = P(T_k^{(b)} > t)$$
$$t \in \langle 0,\infty), \quad b = 1,2, \; k = 0,1,..., \tag{4}$$

where $T_k^{(b)}$, $b = 1,2$, $k = 0,1,...,$ is the lifetimes of the system at the operation state z_b, $b = 1,2$, after k, $k = 0,1,...,$ changes of its operation process states with the conditional distribution functions:

$$F_k^{(b)}(t) = P(T_k^{(b)} \le t) = 1 - R_k^{(b)}(t)$$
$$t \in \langle 0,\infty), \quad b = 1.2, \; k = 0,1,....$$

Under those assumptions, we want to find the unconditional reliability function of the system subjected to two-state operation process:

$$\boldsymbol{R}(t) = P(T > t), \; t \in \langle 0,\infty),$$

where T is the unconditional lifetime of the system with the unconditional distribution function:

$$\boldsymbol{F}(t) = P(T \le t), \; t \in \langle 0,\infty).$$

Analytical approach to system reliability evaluation

The application of Proposition 1 results in the following proposition.

Proposition 2

The unconditional reliability function of the system subjected to two-state operation process is given by:

$$\boldsymbol{R}(t) = \sum_{k=0}^{\infty} P(N(t) = k) R_k^{(b)}(t), \; t \in \langle 0,\infty),$$

where the distribution $P(N(t) = k)$, $t \in \langle 0, \infty)$, $k = 0,1,...,$ is determined by (2)–(3) and $R_k^{(b)}(t)$, $t \in \langle 0, \infty)$, $b = 1,2$, $k = 0,1,...,$ are the conditional reliability functions of the system determined by (4).

Its particular case for the Weibull conditional reliability functions is as follows.

Corollary 1

If the conditional reliability functions of the system subjected to two-state operation process are:

$$R_k^{(b)}(t) = \exp[-\alpha_k^{(b)} t^{\beta_k^{(b)}}]$$
$$t \in \langle 0, \infty), \; k = 0,1,..., \; b = 1,2,... \tag{5}$$

Then, the unconditional reliability function of the system subjected to two-state operation process is given by:

$$\boldsymbol{R}(t) = \sum_{k=0}^{\infty} P(N(t) = k) \exp[-\alpha_k^{(b)} t^{\beta_k^{(b)}}], \; t \in \langle 0,\infty),$$

where the distribution $P(N(t) = k)$, $t \in \langle 0, \infty)$, $k = 0,1,...,$ is determined by (2)–(3). Unfortunately, the fixed analytical results are complex and difficult to apply practically. The problem can also be analyzed by Monte Carlo simulation method.

Monte Carlo approach to system reliability evaluation

We can apply the Monte Carlo simulation method based on the result of Corollary 1, according to a general Monte Carlo simulation scheme presented in figure 2.

At the beginning, we fix the following parameters:

- the number $N \in \boldsymbol{N} \setminus \{0\}$ of iterations (runs of the simulation) equal to the number of the system lifetime realizations;
- the vector of the initial probabilities $[p_b(0)]$, $b = 1,2$, of the system operation process $Z(t)$ states at the moment $t = 0$ defined in Section 2;
- the matrix of the probabilities $[p_{bl}]$, $b,l = 1,2$, $b \ne l$, of the system operation process $Z(t)$ transitions between the various system operation states defined in Section 2.

Next, we generate the realizations of the conditional sojourn times $\theta_{bl}^{(i)}$, $b,l = 1,2$, $b \ne l$, $i = 1,2,...,n$, of the system operation process at the operation states defined in Section 2.

Further, we generate the realizations of the system conditional lifetimes $T_k^{(b)}$, $b = 1,2$, $k = 0,1,...,$ according to the formula (4).

In the next step we introduce:

- $k \in \boldsymbol{N}$ as the number of system operation process states changes;
- $j \in \boldsymbol{N} \setminus \{0\}$ as the subsequent iteration in the main loop and set $j = 1$;
- $t_j \in \langle 0,\infty)$, $j = 1,2,...,N$ as the system unconditional lifetime realization and set $t_j = 0$.

As the algorithm progresses, we draw a random number q from the uniform distribution on the interval $\langle 0,1\rangle$. Based on this random value, the realization $z_b(q)$, $b = 1,2$, of the system operation process initial operation state at the moment $t = 0$ is generated according to the formula:

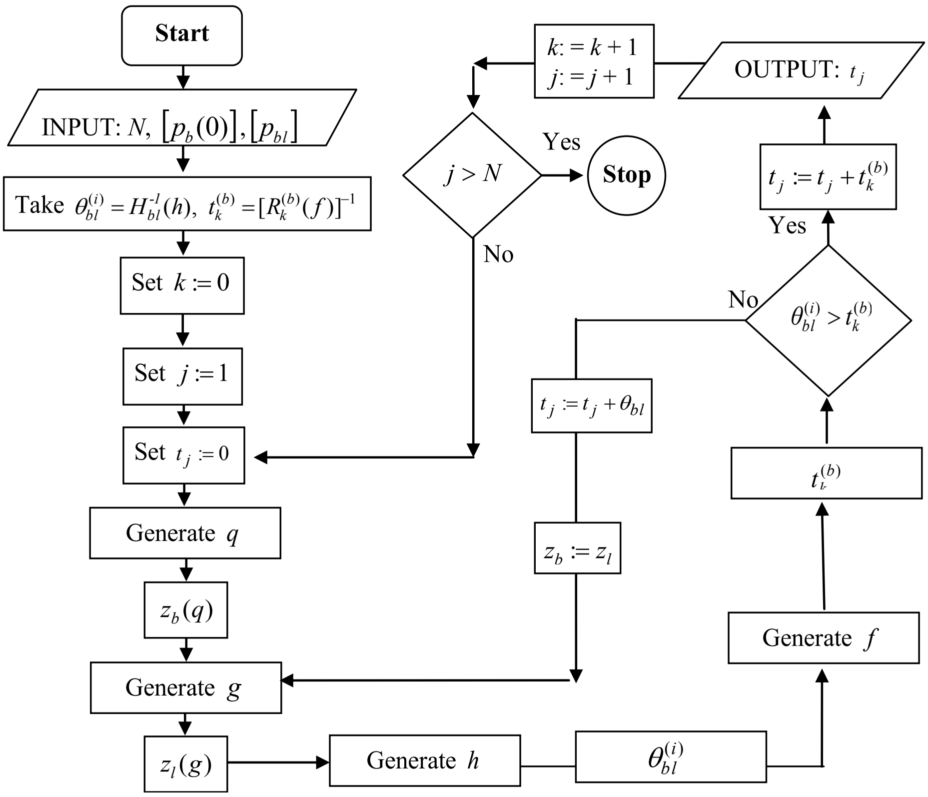

Fig. 2. Monte Carlo algorithm for a system reliability evaluation

$$z_b(q) = \begin{cases} z_1, & 0 \le q < p_1(0), \\ z_2, & p_1(0) \le q < 1. \end{cases}$$

Next, we draw a random number g uniformly distributed on the unit interval. Concerning this random value, the realization $z_l(g)$, $l = 1,2$, $l \neq b$, of the system operation process consecutive operation state is generated according to the formulas:

$$z_{1l}(g) = z_2,$$
$$z_{2l}(g) = z_1.$$

Further, we generate a random number h from the uniform distribution on the interval $\langle 0,1 \rangle$, which we put into the formula $H_{bl}^{-1}(t)$, $b,l = 1,2$, $b \neq l$ obtaining the realization $\theta_{bl}^{(i)}$, $b,l = 1,2$, $b \neq l$, $i = 1,2,...,n$. Subsequently, we generate a random number f uniformly distributed on the unit interval, which we put into the formula (5) obtaining the realization $t_k^{(b)}$, $b = 1,2$. If the realization of the empirical conditional sojourn time is not greater than the realization of the system conditional lifetime, we add to the system unconditional lifetime realization t_j the value $\theta_{bl}^{(i)}$. The realization t_j is recorded and z_l is set as the initial operation state.

We generate another random numbers g, h, f from the uniform distribution on the interval $\langle 0,1 \rangle$ obtaining the realizations $z_l(g)$, $\theta_{bl}^{(i)}$ and $t_k^{(b)}$, $b,l = 1,2$, $b \neq l$. Each time we compare the realization of the conditional sojourn time $\theta_{bl}^{(i)}$ with the realization of the system conditional lifetime $t_k^{(b)}$. If $\theta_{bl}^{(i)}$ is greater than $t_k^{(b)}$, we add to the sum of the realizations of the conditional sojourn times $\theta_{bl}^{(i)}$ the realization $t_k^{(b)}$ and we obtain and record an system unconditional lifetime realization t_j. Thus, we can proceed replacing j with $j + 1$ and shift into the next iteration in the loop if $j < N$. In the other case, we stop the procedure.

Example 1

The input data for the system operation process are:

– the vector of the initial probabilities of the system operation process $Z(t)$ staying at the particular operation states at the moment $t = 0$:

$$[p_b(0)]_{1\times 2} = [0.4, 0.6];$$

– the matrix of the probabilities of the system operation process $Z(t)$ transitions between the operation states:

$$[p_{bl}]_{2\times 2} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix};$$

– the matrix of the conditional distribution functions of the system operation process $Z(t)$ sojourn times $\theta_{bl}^{(i)}$, $b,l = 1,2$, $b \neq l$, $i = 1,2,...,n$ at the operation states:

$$H_{12}(t) = F_{N(m_{12},\sigma_{12})}(t),\ H_{21}(t) = F_{N(m_{21},\sigma_{21})}(t),$$

where:

$$m_{12} = 290,\ \sigma_{12} = 10,\ m_{21} = 70,\ \sigma_{21} = 5.$$

The input data for the system reliability are:

$$R_k^{(1)}(t) = \exp[-0.00000335\frac{2k+1}{k+1}t^2],$$

$$R_k^{(2)}(t) = \exp[-0.00000163\frac{2k+1}{k+1}t^2],$$

$$t \in \langle 0,\infty),\quad k = 0,1,2,.... \qquad (6)$$

The results of simulation calculated for $N = 1{,}000{,}000$ realizations are:

– the mean value of the system unconditional lifetime $\bar{T} \approx 820$ days;
– the standard deviation $\sigma \approx 676$ days.

Conclusions

The discussed problem seems to be very interesting in practice because of the natural omitting the assumption on exponentiality of the system reliability functions at operation states. Both, the analytical method and the simulation method, should be modified and developed to get results better fitting to real technical systems.

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