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CALCULATION OF A TORQUE RIPPLE IN TWO-PHASE ASYNCHRONOUS MOTOR SUPPLIED BY A PWM CONTROLLED INVERTER

METODA OBLICZEŃ PULSACJI MOMENTU W DWUFAZOWYM SILNIKU ASYNCHRONICZNYM ZASILANYM Z PRZEKSZTAŁTNIKA PWM

Abstract: The paper deals with steady state analysis of electromagnetic torque ripples of a two-phase induction machine, which is supplied by an IGBT transistors half-bridge connected inverter. The inverter's output voltage is controlled by a PWM of the input DC voltage. The complex Fourier series analysis of the inverter's output voltage was made, to obtain a spectrum of the harmonic supply voltages. The different voltage harmonic was applied to the two-phase asynchronous machine model to obtain the electromagnetic torque waveforms for various operation conditions.

1. Introduction

Electrical low-power drives (around 100W) which are supplied by a single-phase voltage, used in different industrial and domestic devices are presently increasingly deployed by a two-phase motors.

Two-phase motors by their characteristics no differ from three-phase motors. Their advantage is easier winding, which is of great importance for automated production. The two-phase motors are manufactured as either squirrel cage asynchronous or permanent magnets synchronous motors. They are deployed as drives of pumps in washing machines and dishwashers, but also as the circulating pumps for central domestic heating. A permanent magnet is in this case, water and lye resistant, allows making a pump with an absolute waterproof. Two-phase voltage is produced from the single-phase network and supplied by converters.

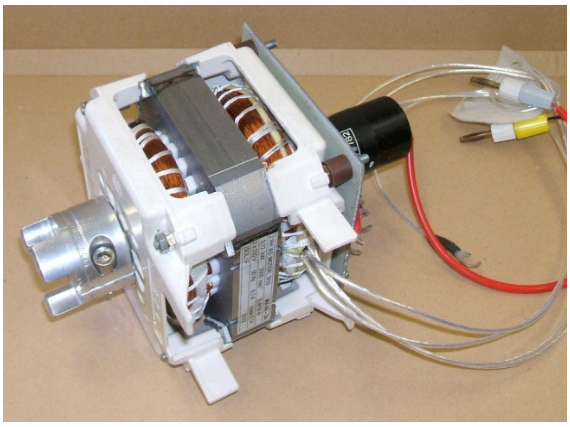

Fig. 1. Prototyp of a two-phase asynchronous motor

Use of two-phase motor has several advantages. The stator winding has most simple form. Three shifted coil windings form one phase. The stator windings can be configured in either a serial or parallel two-phase system.

Normally, the windings are identical. The windings which form one phase are connected to induce opposite magnetic polarity.

Figure 1 shows the construction of a two-phase asynchronous motor.

2. Mathematical model of the supply converter

For inverter's operation study at steady state we consider following idealized conditions:

- power switch, that means the switch can handle unlimited current and blocks unlimited voltage,
- the voltage drop across the switch and leakage current through switch are zero,
- the switch is turned on and off with no rise and fall times,
- sufficiently big capacitance of input voltage capacitor divider, so that input DC voltage may be assumed to be constant.

This assumption helps us to analyze a power circuit and helps us to build a mathematical model for the inverter at steady state. Figure 2 shows two-phase converter circuit layout.

The output voltage level of the inverter can be controlled by a reduction of the DC source voltage. Another form of the voltage control is by a notching, where the transistors in the inverter circuit are turned on and off so as to produce zero periods of equal length.

An improvement to the notched waveform is to vary the on and off periods so that the on-periods are longest at the peak of the wave. This form of control is known as pulse-width modulation (PWM).

It can be observed that area of each pulse corresponds approximately to the area under the sine-wave between the adjacent mid-points of the off-periods. The pulse width modulated wave has much lower order harmonic content than the other waveforms.

If the desired reference voltage is sine-wave, two parameters define the control:

Coefficient of the modulation m - equal to the ratio of the modulation and reference frequency.

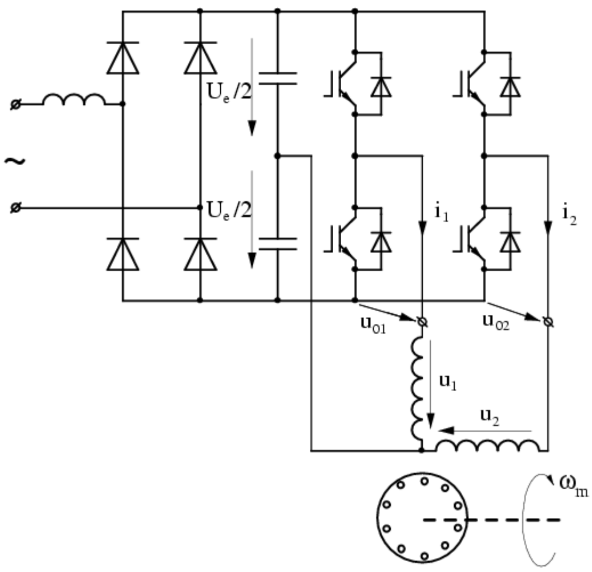

Fig. 2. Supply converter circuit layout

- *Voltage control coefficient* r - equal to the ratio of the desired voltage amplitude and the DC supply voltage.

Mostly the synchronous modulation is used. In synchronic modulation the modulation frequency is an integer multiple of the reference sine-wave. Generally to control the inverter numeric control device is used. The turn on (α) and turn off (β) angles are calculated by the discredit of the reference sine-wave. That means the reference sine-wave is replaced with discrete values. If the coefficient of modulation m is sufficiently great, the difference between real values and discrete values is negligible. The inverter's output voltage of the first branch can be mathematically expressed as a complex Fourier series of the form:

$$u_{01} = U_e \sum_{k=-\infty}^{\infty} \sum_{n=1}^{m} c_{01n} e^{jk\theta} \tag{1}$$

$$\begin{cases} c_{01n} = \dfrac{1}{j2k\pi}\left(e^{-jk\alpha_{01n}} - e^{-jk\beta_{01n}}\right) & for \quad k \neq 0 \\ c_{01n} = \dfrac{\beta_{01n} - \alpha_{01n}}{2\pi} & for \quad k = 0 \end{cases}$$

Similarly for the second branch:

$$u_{02} = U_e \sum_{k=-\infty}^{\infty} \sum_{n=1}^{m} c_{02n} e^{jk\theta} \tag{2}$$

$$\begin{cases} c_{02n} = \dfrac{1}{j2k\pi}\left(e^{-jk\alpha_{02n}} - e^{-jk\beta_{02n}}\right) & for \quad k \neq 0 \\ c_{02n} = \dfrac{\beta_{02n} - \alpha_{02n}}{2\pi} & for \quad k = 0 \end{cases}$$

Calculated waveforms of the branch's voltages are unipolar. Based on the voltage equation, the phase voltages are given as a difference between branch voltages and voltage of the capacitor divider:

$$\begin{aligned} u_1 &= u_{01} - \frac{U_e}{2} = rU_e \sum_{k=-\infty}^{\infty} \sum_{n=1}^{m} c_{01n} e^{jk\theta} - \frac{U_e}{2}; \\ u_2 &= u_{02} - \frac{U_e}{2} = rU_e \sum_{k=-\infty}^{\infty} \sum_{n=1}^{m} c_{02n} e^{jk\theta} - \frac{U_e}{2} \end{aligned} \tag{3}$$

In the Fig. 3 are shown the phase voltages waveforms. The voltages are bi-polar with amplitude equal to half of DC input voltage.

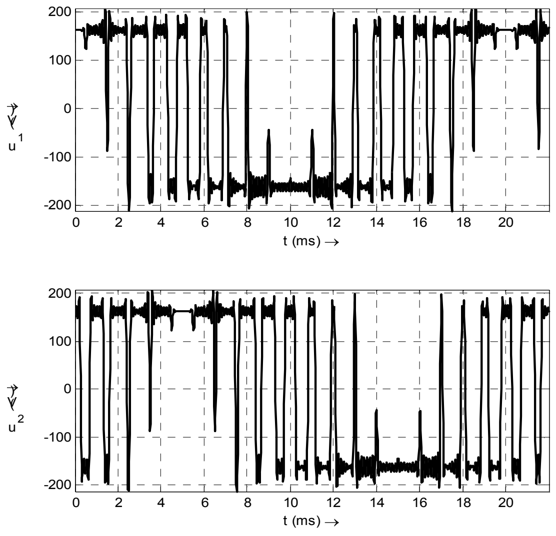

Fig. 3. Waveforms of the phase voltages

3. Harmonic analysis of the supply voltages

On the base of Fourier series formulas of the supply voltages, a harmonic analysis of the supply waveforms can be made.

The amplitude of each harmonic is calculated on the base of equations (3). Amplitude of k[th] harmonic is given:

$$A_k = \sum_{n=1}^{m} \left(c_{01n}^{k} + c_{01n}^{-k} \right) \tag{4}$$

The Fig.4 depicts a harmonic analysis of the PWM output voltage for frequency of $50\,Hz$ and modulation frequency of $1\,kHz$.

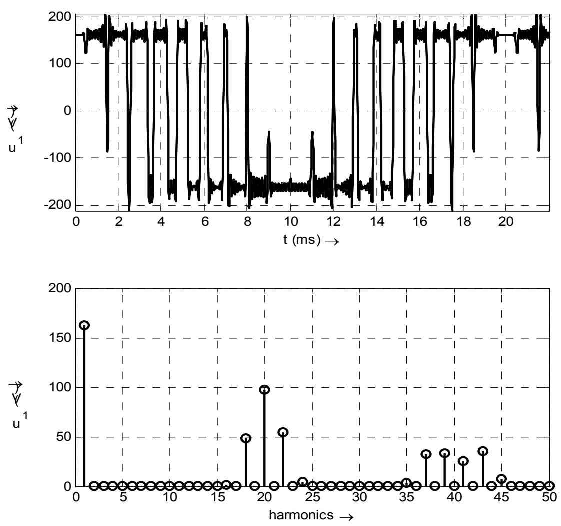

Fig. 4. Harmonic analysis

Tab.1 The main harmonics

harmonics	Amplitude	1. phase	2.phase
1	162,3 V	cos	sin
18	48,4 V	-cos	cos
20	97,8 V	cos	cos
22	54,2 V	-cos	cos
37	32,6 V	cos	sin
39	33,3 V	-cos	sin
41	25,7 V	-cos	-sin
43	25,4 V	-cos	-sin

In the Tab.1 are given the parameters of the main harmonics.

4. Torque ripple calculation

To calculate the electromagnetic torque ripple the dynamic model of the two-phase asynchronous motor was build.

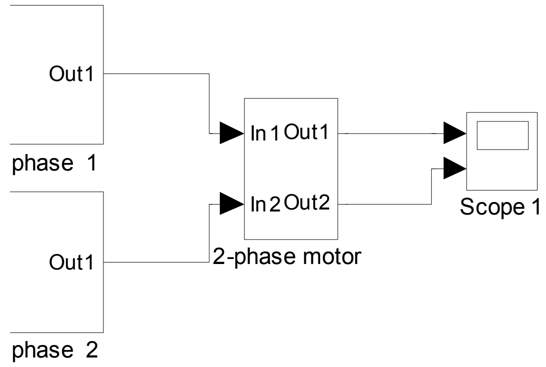

Fig. 5. Matlab-Simulink block diagram

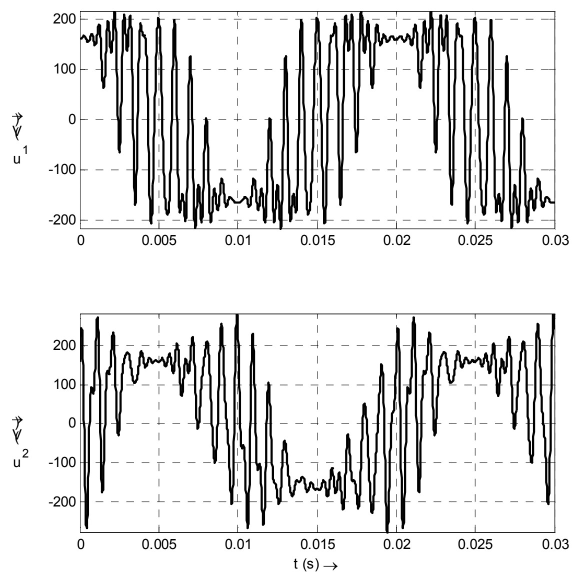

Fig. 6. Supply harmonic voltages

The dynamic model is supplied by a series of harmonic voltages whose parameters are given in Tab.1. The Fig.6 shows the supply voltage waveforms which were created by main harmonic voltages.

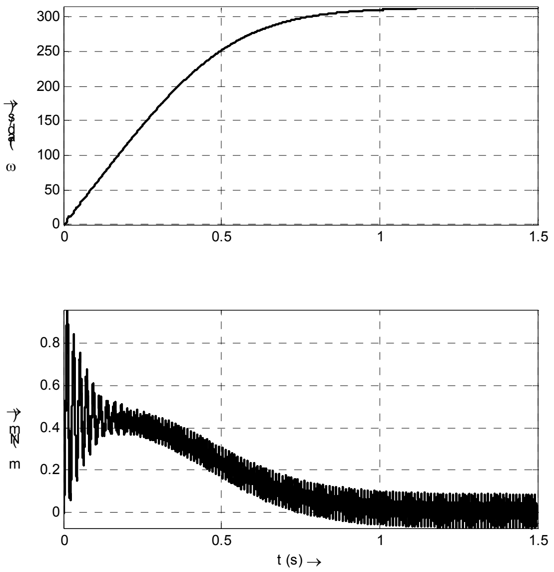

Fig. 7. Start of two-phase motor

Fig.7 depicts a start up no loaded motor by direct connection to the frequency of $50\,Hz$.

In Fig.6 are shown the stator current and electromagnetic torque waveforms.

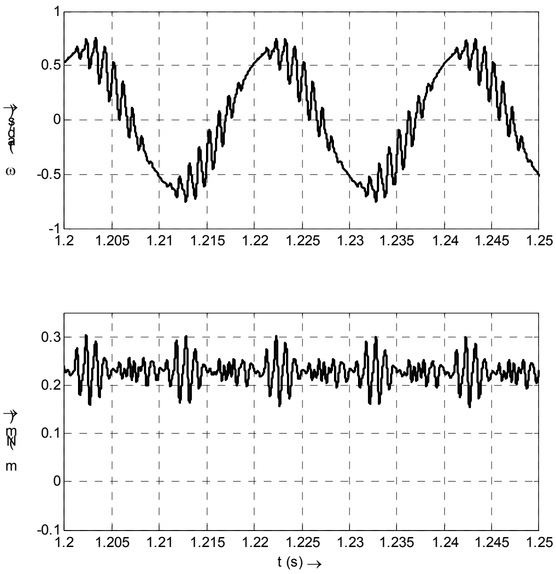

Fig. 6. Current and torque waveforms

Conclusion

Method of using complex Fourier series to investigate torque ripple waveform of a two-phase asynchronous motor is presented in the paper. The electromagnetic torque waveform shows, that the greatest influence on the output torque ripple the paper has a harmonic which is created by modulation frequency. In practical application of the iinverter it is necessary to use highest possible frequency for modulation purposes.

Acknowledgment

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