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Kołowrocki Krzysztof
Maritime University, Gdynia, Poland

Reliability of large and complex systems - state of art

Keywords

reliability, two-state system, multistate system, ageing, operation process, complex system, optimization, applications

Abstract

The author's latest monograph "Reliability of large and complex systems" published by Elsevier in 2014 is presented. The paper is composed of an introduction, the monograph contents, conclusions and the references the monograph contents is based on.

1. Introduction

The book is concerned with the application of limit reliability functions to the reliability evaluation of large systems. Two-state and multi-state large systems composed of independent components are considered. The main emphasis is on multi-state systems with degrading (ageing) components because of the importance of such an approach in safety analysis, assessment and prediction, and analysing the effectiveness of operation processes of real technical systems.

Many technical systems belong to the class of complex systems as a result of the large number of components they are built of and their complicated operating processes. This complexity very often causes evaluation of system reliability and safety to become difficult. As a rule these are series systems composed of large number of components. Sometimes the series systems have either components or subsystems reserved and then they become parallel-series or series-parallel reliability structures. We meet large series systems, for instance, in piping transportation of water, gas, oil and various chemical substances. Large systems of these kinds are also used in electrical energy distribution. A city bus transportation system composed of a number of communication lines each serviced by one bus may be a model series system, if we treat it as not failed, when all its lines are able to transport passengers. If the communication lines have at their disposal several buses we may consider it as either a parallel-series system or an "*m* out of *n*" system. The simplest example of a parallel system or an "*m* out of *n*" system may be an electrical cable composed of a number of wires, which are its basic components, whereas the transmitting electrical network may be either a parallel-series system or an "*m* out of *n*"-series system. Large systems of these types are also used in telecommunication, in rope transportation and in transport using belt conveyers and elevators. Rope transportation systems like port elevators and ship-rope elevators used in shipyards during ship docking and undocking are model examples of series-parallel and parallel-series systems.

Taking into account the importance of the safety and operating process effectiveness of such systems it seems reasonable to expand the two-state approach to multi-state approach in their reliability analysis. The assumption that the systems are composed of multi-state components with reliability states degrading in time without repair gives the possibility for more precise analysis of their reliability, safety and operational processes' effectiveness. This assumption allows us to distinguish a system reliability critical state to exceed which is either dangerous for the environment or does not assure the necessary level of its operational process effectiveness. Then, an important system reliability characteristic is the time to the moment of exceeding the system reliability critical state and its distribution, which is called the system risk function. This distribution is strictly related to the system multi-state reliability function that is a basic characteristic of the multi-state system.

In the case of large systems, the determination of the exact reliability functions of the systems and the system risk functions leads us to very complicated formulae that are often useless for reliability practitioners. One of the important techniques in this situation is the asymptotic approach to system reliability evaluation. In this approach, instead of the preliminary complex formula for the system reliability function, after assuming that the number of system components tends to infinity and finding the limit reliability of the system, we obtain its simplified form.

The mathematical methods used in the asymptotic approach to the system reliability analysis of large systems are based on limit theorems on order statistics distributions considered in very wide literature ([3], [9]–[11], [15], [22]–[23], [28]–[31], [37]–[41], [44]–[45], [125], [143], [149], [151], [169], [172]). These theorems have generated the investigation concerned with limit reliability functions of the systems composed of two-state components [5], [7], [25]–[29], [48]–[50], [58]–[74], [93, [94], [119]–[125], [135]–[136], [148], [152], [167], [175]. The main and fundamental results on this subject that determine the three-element classes of limit reliability functions for homogeneous series systems and for homogeneous parallel systems have been established by Gniedenko in [41]. These results are also presented, sometimes with different proofs, for instance in subsequent works [7], [15], [23], [30], [65] and [93]. The generalisations of these results for homogeneous "*m* out of *n*" systems have been formulated and proved by Smirnow in [151], where the seven-element class of possible limit reliability functions for these systems has been fixed. Some partial results obtained by Smirnow may be found in [93] and additionally with the solution of the speed of convergence problem in [31]. As it has been done for homogeneous series and parallel systems classes of limit reliability functions have been fixed by Chernoff and Teicher in [23] for homogeneous series-parallel and parallel-series systems. Their results were concerned with so-called "quadratic" systems only. They have fixed limit reliability functions for the homogeneous series-parallel systems with the number of series subsystems equal to the number of components in these subsystems, and for the homogeneous parallel-series systems with the number of parallel subsystems equal to the number of components in these subsystems. These results may also be found for instance in later works [7] and [65].

All the results so far described have been obtained under the linear normalisation of the system lifetimes. Of course, there is a possibility to look for limit reliability functions of large systems under other than linear standardisation of their lifetimes. In this context, the results obtained by Pantcheva [143] and Cichocki [27] are exemplary. Pantcheva in [143] has fixed the seven-element classes of limit reliability functions of homogeneous series and parallel systems under power standardisation for their lifetimes. Cichocki in [27] has generalised Pantcheva's results to hierarchical series-parallel and parallel-series systems of any order.

The book contains the results described above and their newest generalisations for large two-state systems and their developments for multi-state systems' asymptotic reliability analysis under the linear standardisation of the system lifetimes and the system lifetimes in the state subsets, respectively. Generalisations presented here of the results on limit reliability functions of two-state homogeneous series, and parallel systems for these systems in case they are non-homogeneous, are mostly taken from [93] and [81]. A more general problem is concerned with fixing the classes of possible limit reliability functions for so-called "rectangular" series-parallel and parallel-series systems. This problem for homogeneous series-parallel and parallel-series systems of any shapes, with different number of subsystems and numbers of components in these subsystems, has been progressively solved in [62]–[65], [68] and [70]. The main and new result of these works was the determination of seven new limit reliability functions for homogeneous series-parallel systems as well as for parallel-series systems. This way, new ten-element classes of all possible limit reliability functions for these systems have been fixed. Moreover, in these works it has been pointed out that the type of the system limit reliability function strongly depends on the system shape. These results allow us to evaluate reliability characteristics of homogeneous series-parallel and parallel-series systems with regular reliability structures, i.e. systems composed of subsystems having the same numbers of components. The extensions of these results for non-homogeneous series-parallel and parallel-series systems have been formulated and proved successively in [65], [69]–[72] and [81]. These generalisations additionally allow us to evaluate reliability characteristics of the series-parallel and parallel-series systems with non-regular structures, i.e. systems with subsystems having different numbers of components. In some of the cited works, as well as the theoretical considerations and solutions, numerous practical applications of the asymptotic approach to real technical system reliability evaluation may also be found [29], [48]–[50], [60], [66], [71], [73], [93], [95], [152], [167], [173].

More general and practically important complex systems composed of multi-state and degrading in time components are considered among others in

[1]–[2], [4]–[6], [8], [12], [16]–[21], [32], [35], [43], [53]–[57], [74], [93], [73]–[84], [116]-[117], [122], [126]–[132], [134], [137]– [139], [142], [144], [147], [150] and [174]–[177]. An especially important role they play in the evaluation of technical systems reliability and safety and their operating process effectiveness is defined in the book for large multi-state systems with degrading components. The most important results regarding generalisations of the results on limit reliability functions of two-state systems dependent on transferring them to series, parallel, "*m* out of *n*", series-parallel and parallel-series multi-state systems with degrading components are given in [74]–[84]. Some of these publications also contain practical applications of the asymptotic approach to the reliability evaluation of various technical systems [74]–[79], [81]-[84], [93].

The results concerned with the asymptotic approach to system reliability analysis have become the basis for the investigation concerned with domains of attraction for the limit reliability functions of the considered systems [25], [93], [95], [119]–[120]. In a natural way they have led to investigation of the speed of convergence of the system reliability function sequences to their limit reliability functions ([93]). These results have also initiated the investigation of limit reliability functions of "*m* out of *n*"-series, series-"*m* out of *n*" systems [25], [135]–[136], and systems with hierarchical reliability structures [25]–[27], as well as investigations on the problems of the system reliability improvement and optimisation [121]–[122].

The aim of the book is to deliver a complete elaboration of the state of art on the method of asymptotic approach to reliability evaluation for as wide as possible a range of large systems included mainly in [85]. Pointing out the possibility of this method's extensive practical application in the operating processes of these systems is also an important reason for this book. The book contains complete current theoretical results of the asymptotic approach to reliability evaluation of large two-state and multi-state series, parallel, "*m* out of *n*", series-parallel, and parallel-series systems together with their practical applications to the reliability evaluation of a wide range of technical systems. Some partial results on the asymptotic approach to reliability evaluation of "*m* out of *n*"-series, series-"*m* out of *n*" and hierarchical systems, and their application to large systems reliability improvement are also presented in the book.

Additionally, the book delivers a very wide discussion on the reliability modelling, prediction and optimization of complex multistate systems operating at variable conditions with changing at time their reliability structures and their components reliability parameters. The discussion is based on the results included in [13]-[14], [47], [86]-[93], [96]-[115], [123]-[124], [157]-[159], [161], [163]-[166]. Moreover, the asymptotic approach to the large complex systems at varying operation conditions is presented and illustrated with some applications [154]-[156], [160], [162].

The following construction of the book has been assumed. In chapters concerned with two-state systems the results and theorems are presented without the proofs but with exact reference to the literature where their proofs may be found. Moreover, the procedures of the results' practical applications are described and applied to the model two-state systems reliability evaluation. In chapters concerned with multi-state systems the recent theorems about their multi-state limit reliability functions are formulated and briefly justified. Next, the procedures of the result applications are presented and applied to real technical systems reliability and risk evaluation. Moreover, the possibility of the computer aided reliability evaluation of these systems is suggested and its use is presented.

The book contains complete actual solutions of the formulated problems for the considered large systems reliability evaluation in the case of any reliability functions of the system components.

2. Contents

The book consists of Preface partly presented in the above Introduction, ten Chapters described in this section, Summary given in the Conclusions and Bibliography included in References.

In Chapter 1, some basic notions necessary for further considerations are introduced. The asymptotic approach to the system reliability investigation and the system limit reliability function are defined.

In Chapter 2 two-state homogeneous and non-homogeneous series, parallel, "*m* out of *n*", series-parallel and parallel-series systems are defined. Their exact reliability functions are also determined.

Basic notions of the system multi-state reliability analysis are introduced in Chapter 3. Further the multi-state homogeneous and non-homogeneous series, parallel, "*m* out of *n*", series-parallel and parallel-series systems with degrading components are defined and their exact reliability functions are determined. Moreover, the notions of the multi-state limit reliability function of the system, its risk function and other multi-state system reliability characteristics are introduced.

Chapter 4 is concerned with limit reliability functions of two-state systems. Three-element classes of limit reliability functions for homogeneous and non-homogeneous series systems are fixed. Some auxiliary theorems that allow us to justify facts on the methods of those systems' reliability evaluation are formulated and proved. The chapter also contains the application of one of the proven facts to the reliability evaluation of a non-homogeneous gas pipeline that is composed of components with Weibull reliability functions. The accuracy of this evaluation is also illustrated. Three-element classes of possible limit reliability functions for homogeneous and non-homogeneous parallel systems are fixed as well. Some auxiliary theorems that allow us to justify facts on the methods of these systems' reliability evaluation are formulated and proved. The chapter also contains the application of one proved fact to the reliability evaluation of a homogeneous energetic cable used in the overhead electrical energy distribution that is composed of components with Weibull reliability functions. The accuracy of this evaluation is illustrated in a table and a figure. The class of limit reliability functions for a homogeneous "*m* out of *n*" system is fixed and the "16 out of 35" lighting reliability is evaluated in this chapter. Chapter 4 contains also the results of investigations on limit reliability functions of two-state homogeneous and non-homogeneous series-parallel systems. Apart from formulated and proved auxiliary theorems that allow us to justify facts on the methods of these systems' reliability evaluation, their ten-element classes of possible limit reliability functions are fixed. In this chapter, in the part concerned with applications there are two formulated and proved facts that determine limit reliability functions of series-parallel systems in the cases where they are composed of components having the same and different Weibull reliability functions. On the basis of those facts the reliability characteristics of a homogeneous gas pipeline composed of two lines of pipe segments and a non-homogeneous water supply system composed of three lines of pipe segments are evaluated. The results of investigations on limit reliability functions of two-state homogeneous and non-homogeneous parallel-series systems are given in this chapter as well. Theorems, which determine ten-element classes of possible limit reliability functions for these systems in the cases where they are composed of identical and different components, are formulated and justified. Moreover, some auxiliary theorems that are necessary in practical reliability evaluation of real technical systems are formulated and proved. In the part concerned with applications one fact is formulated and proved and then applied to evaluation of the reliability of a model homogeneous parallel-series system. Moreover, the procedure of the reliability evaluation of more complicated large systems that structures are compositions of large two-state systems considered in this chapter previous sections is presented. This procedure application is illustrated in the exact and approximate reliability evaluation of the two-state non-homogeneous system composed of a large series-parallel system and a large series-"*m* out of *k*" system linked in series and the two-state homogeneous system composed of three large "*m* out *of l*" systems linked in series.

Generalisations of the results of Chapter 4 on limit reliability functions of two-state systems consisting in their transferring to multi-state series, parallel, "*m* out of *n*", series-parallel and parallel-series systems are done in Chapter 5. The classes of all possible limit reliability functions for these systems in the cases when they are composed of identical and different (in the reliability sense) components are fixed. The newest theorems that allow us to evaluate the reliability of large technical systems of those kinds are formulated and proved in Chapter 5 as well. Apart from the main theorems fixing the classes of multi-state limit reliability functions of the considered system, some auxiliary theorems and corollaries allowing their direct applications to reliability evaluation of real technical objects are also formulated and proved. Moreover, in this chapter there are wide applications depending on the results applying to the evaluation of reliability characteristics and risk functions of different multi-state transportation systems. The results concerned with multi-state series systems are applied to the reliability evaluation and risk function determination of homogeneous and non-homogeneous pipeline transportation systems, a homogeneous model telecommunication network and a homogeneous bus transportation system. The results concerned with multi-state parallel systems are applied to reliability evaluation and risk function determination of an energetic cable used in an overhead electrical energy distribution network and to reliability and durability evaluation of a three-level steel rope used in rope transport. Results on limit reliability functions of a homogeneous multi-state "*m* out of *n*" system are applied to durability evaluation of a steel rope. A model homogeneous series-parallel system and homogeneous and non-homogeneous series-parallel pipeline systems composed of several lines of pipe segments are estimated as well. Moreover, the reliability evaluation of the model homogeneous parallel-series electrical energy distribution system is performed. This chapter is completed by presentation of the procedure of the reliability evaluation of more complicated large systems that structures are compositions of large multi-state systems considered in the previous sections. This procedure application is illustrated in the exact and approximate reliability evaluation of the non-homogeneous multistate system composed of a large series-parallel system and a large series-“*m* out of *k*” system linked in series and the homogeneous multistate system composed of three large “*m* out of *l*” systems linked in series.

Chapter 6 is devoted to the multi-state asymptotic reliability analysis of port and shipyard transportation systems. Theoretical results of this chapter and Chapter 5 are applied to the reliability evaluation and the risk functions determination of some selected port transportation systems. The results of the asymptotic approach to reliability evaluation of non-homogeneous multi-state series-parallel systems are applied to the transportation system used in the Baltic Grain Terminal of the Port of Gdynia for transporting grain from its elevator to the rail carriages. The results of the asymptotic approach to the reliability evaluation of the non-homogeneous multi-state series-parallel systems are applied to the piping transportation system used in the Oil Terminal in Dębogorze. This transportation system is set up to take the oil from the tankers that deliver it to the unloading pier located at the breakwater of the Port of Gdynia. The results of the asymptotic approach to reliability evaluation of non-homogeneous multi-state series-parallel and series systems are applied to the transportation system used in the Baltic Bulk Terminal of the Port of Gdynia for loading bulk cargo on the ships. The results of this chapter and Chapter 5 are also applied to reliability evaluation and risk function determination of the shipyard transportation system. Namely, the results of the asymptotic approach to reliability evaluation of homogeneous multi-state parallel-series systems are applied to the ship-rope transportation system used in the Naval Shipyard of Gdynia for docking ships coming for repair. The reliability analysis performed on the considered systems in this chapter is based on the data concerned with the operation processes and reliability of their components coming from experts, from component technical norms and from their producer’s certificates.

In Chapter 7 the classes of possible limit reliability functions are fixed for the considered systems in the case where their components have exponential reliability functions. Theoretical results are represented in the form of a very useful guide containing algorithms placed in tables and giving sequential steps for proceeding in the reliability evaluation in each of the possible cases of the considered system shapes. The application of these algorithms for reliability evaluation of the multi-state non-homogeneous series transportation system, the multi-state model homogeneous series-parallel, the multi-state non-homogeneous series-parallel pipeline transportation system and the multi-state non-homogeneous parallel-series bus transportation system is illustrated. The evaluation of reliability functions, risk functions, mean values of sojourn times in subsets of states and mean values of sojourn times in particular states for these systems is carried out. The calculations are performed using a computer program based on the algorithms, so allowing automatic evaluation of the reliability of large real technical systems.

In Chapter 8 the open problems related to the topics considered in the book are presented. The domains of attraction for previously fixed limit reliability functions of the series, parallel, “*m* out of *n*”, series-parallel and parallel-series systems are introduced. More exactly, there are formulated theorems giving conditions which reliability functions of the components of the system have to satisfy in order that the system limit reliability function is one of the functions from the system class of all limit reliability functions. Some examples of the result application for series systems are also illustrated. The practically very important problem of the speed of convergence of system reliability function sequences to their limit reliability functions is investigated as well. An exemplary theorem is presented, which allows the differences between the system limit reliability functions and the members of their reliability function sequences to be estimated. Next, an example of the speed of convergence evaluations of reliability function sequences for a homogeneous series-parallel system is given. Partial results of the investigation on the asymptotic approach to reliability evaluation of “*m* out of *n*”-series, series-“*m* out of *n*” and hierarchical systems and on system reliability improvement are presented. These result applications are illustrated graphically as well.

Chapter 9 is devoted to modelling, prediction and optimization of the operation and reliability of technical systems related to their operation processes. The emphasis is on multistate systems composed of ageing components and changing their reliability structures and their components reliability parameters during their operation processes that are called the complex systems. The integrated general model of complex systems’ reliability, linking their reliability models and their operation processes models and considering variable at different operation states their reliability structures and their components reliability parameters is constructed. This system reliability general model together with the linear programming are proposed to operation and reliability optimization of complex technical systems. Those theoretical tools are applied to modelling, prediction and optimization of the operation processes and reliability characteristics of the multistate non-homogeneous system composed of a series-parallel and a series-“*m* out of *k*” subsystems linked in series, the multistate homogenous “*m* out of *l*”-series system and the port grain transportation system composed of three multistate non-homogeneous series-parallel subsystems linked in series changing their reliability structures and their components reliability parameters at variable operation conditions.

In Chapter 10 the combination of the results on the reliability of complex multi-state systems related to their operation processes and the results concerning the limit reliability functions of the multistate systems is proposed, to obtain the results on the asymptotic approach to the evaluation of the large complex multi-state systems reliability at the variable operation conditions. The asymptotic approach to the large complex system reliability evaluation and the large complex system limit reliability function are defined. Limit reliability functions of selected large complex systems composed of components having exponential reliability functions are fixed. The way of using this results is illustrated by their application to the approximate evaluation of reliability characteristics of the large multistate non-homogeneous system composed of a series-parallel and a series-“*m* out of *k*” subsystems linked in series, the large multistate homogenous “*m* out of *l*”-series system and the port grain transportation system composed of three large multistate non-homogeneous series-parallel subsystems linked in series changing their reliability structures and their components reliability parameters at variable operation conditions. The accuracy of the approximation of the exact reliability characteristics fixed for those systems in Chapter 9 by their approximate evaluation obtained in this chapter is discussed as well.

The book is completed by the Summary that contains the evaluation of the presented results, the formulation of open problems concerned with large and complex systems’ reliability and the perspective of further investigations on the considered problems.

3. Conclusions

In this book the asymptotic approach to the reliability evaluation of homogeneous and non-homogeneous series and parallel systems, homogeneous “*m* out of *n*” systems and homogeneous and non-homogeneous regular series-parallel and parallel-series systems has been completely analysed. For these systems, in the case where their components are two-state as well in the case where they are multi-state, the classes of limit reliability functions have been fixed. Moreover, the auxiliary theorems useful for finding limit reliability functions of real technical systems composed of components having any reliability functions have been formulated and motivated. The series-parallel and parallel-series systems have been considered in the case where their reliability structures are regular. However, this fact does not restrict the completeness of the performed analysis, since by conventional joining of a suitable number of failed components in parallel subsystems of the non-regular parallel-series systems we get the regular non-homogeneous parallel-series systems considered in the book. Similarly, conventional joining of a suitable number of components which do not fail, in series sub-systems of the non-regular series-parallel systems, leads us to the regular non-homogeneous series-parallel systems considered in the book. Thus the problem has been analysed exhaustively.

In addition to the general solutions, a practically important case when the components of the considered systems have exponential reliability functions has been considered separately. In this case the class of limit reliability functions for the considered multi-state systems has been fixed and other practically useful theorems have been proposed. The results obtained in this case may play the role of an easy-to-use guide necessary in quick reliability evaluations of real large technical systems, as well as during their operations and when they are designed. There are proposed algorithms presented in the form of tables giving simple sequential steps in systems reliability evaluation. To make these algorithms an easy and useful tool for reliability practitioners their usage is illustrated by various examples of their application to the evaluation of the real system reliability characteristics. Thinking about the practitioners using computers, on the basis of the algorithms the computer program has been elaborated and the results of its practical use have been illustrated in the book.

Theoretical results have been illustrated by many practical examples of their application in reliability evaluation of an extensive range of large systems. The evaluations of the real technical systems presented here have been obtained on the basis of non-precise component reliability data and therefore first of all they should be treated as an illustration of a wide possibility of applications of the proposed asymptotic approach in system reliability analysis. Reliability data come from experts and from the literature and are concerned with component’s mean lifetimes and their expected reliability function types. These evaluations, despite not being precise may be a very useful, simple and quick tool in approximate reliability evaluation, especially during the design of large systems, and when planning and improving their safety and effectiveness operation processes.

Optimisation of the reliability structures [61], [110], [123]-[124] of large systems with respect to their safety and costs is complicated and often not possible to perform by practitioners because of the mathematical complexity of the exact methods.

The proposed method offers enough simplified formulae to allow significant simplifying of reliability optimising calculations. This is testified by the recent partial and preliminary results presented in the last chapter. Especially important are results concerned with exponential systems. Their joining with non-precise data coming from experts concerned with the component mean lifetimes and expected reliability function only allow the constructors to evaluate and to optimise the system structures and their operation processes during designing and before including them in everyday practice [51]-[52], [141].

The results presented in the book have become the basis of investigations on domains of attraction of system limit reliability functions and initiated the problem of the speed at which system reliability function sequences reach their limit reliability functions. The problem of the domains of attraction for fixed limit reliability functions, presented partly in the book, has been completely solved for two-state systems in [120] and generalised to multi-state systems in [93]. The solutions deliver the necessary and sufficient conditions that the reliability functions of the system particular components have to satisfy in order that the system limit reliability function is one of the reliability functions from the fixed class of possible limit reliability functions for this system. This way, on the basis of data about the types of system component reliability functions and the system shape it is possible to predict which of the reliability functions is its limit reliability function.

Another significant problem in applying the proposed method to the reliability evaluation of real technical system is its accuracy. This problem has been completely solved for the considered two-state systems in [93]. Practical examples presented in the book testify that the mistakes in the approximation of the exact system reliability functions by their limiting forms are not significant in practice, often for not very large systems. Moreover, in the asymptotic approach to system reliability evaluation it is possible to get lower and upper bounds of the exact system reliability characteristics, which is illustrated in the investigation of the ship-rope elevator. The complete solution of the proposed method, i.e. the evaluation of the speed of convergence of the system exact reliability sequences to their limiting forms presented partly in the book for two-state systems, is easy to transfer to multi-state systems. The way of proceeding in this transfer is commented on in the book and solved in [93]. The asymptotic method accuracy evaluations are complicated, so it is probably not possible to use them in everyday practice by reliability practitioners. However, it seems to be practically possible to make the accuracy evaluation supported by computer calculations.

Additionally, the main results of the book have initiated and become the basis for further investigations on limit reliability functions. Especially the investigations on limit reliability functions of practically important large series-“*m* out of *n*” and “*m* out of *n*”-series systems and hierarchical systems have been significantly developed [27], [136], [162]. Another problem concerned with the methods of the improving of large systems reliability, which are briefly presented in the book, has been completely solved for series systems in [122].

All the results of the investigations on limit reliability functions have initiated practically very important aspect of joining them with the reliability models of the complex system subjected to their operation processes considered widely in [110]. This problem have been formulated in the book and the latest results on operation and reliability modelling, prediction and optimization of large complex multistate exponential systems changing their reliability structures and their component reliability parameters at variable operation conditions have been presented. Unfortunately, there is not possible to find analytically general solutions after omitting the assumption on multistate system components’ exponential reliability functions and Monte Carlo simulation method [178] can be suggested as an sometimes useful tool in solving this complex problem [94], [118].

The results presented in the book can be very helpful backgrounds for very important investigations on safety of complex systems [106]-[110], [158], safety of critical infrastructures [92], and safety of nanosystems [33], which currently are the main research areas in the world science [112].

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