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Analysis and comparison of reliability measures of critical points

Keywords

reliability, critical points, reliability index

Abstract

In this work, algorithms have been used to compare the reliability measure of helicopter structure critical points. Reliability index, Cornell's reliability index and Hasofer-Lind's reliability index have been calculated and analysed. Inputs to the algorithms, stress and strength were generated by using the Markov chain model based on actual flight records of helicopters. With this approach, results of methods for determining the reliability of critical points of a helicopter's structure are properly founded.

1. Introduction

Research on safety and therefore conduct of various health risk analyses has made a relatively recent entry into the fields of science. Safety research started with the realization that safety problems in many branches of technics and human life are common in character and therefore can be described in the same ways.

2. Risk index assessments methods

Risk analysis has been done in accordance with the referenced method [1], [2]. First, critical points of helicopter structure have been found, and one of them has been taken under observation. With respect to the laboratory data probability normal distribution has been proposed for steel strength. Value of the maximum principal stress in force mounting elements knots was obtained by using the model and calculations of the MSC Marc program. Based on experimental studies using Markov chain models the probability distribution function of internal stresses in the critical element was determined. Then the cumulative distribution for the probability distribution of stresses and strength were calculated. Reliability index, Cornell's reliability index and Hasofer-Lind's reliability index have been counted.

2.1. Risk index

The basic assumption of the model is the conclusion that the helicopter component is damaged when the stress value is greater than the strength of this element. Stress X_f and strength X_g are random variables with probability distributions defined by the density functions $f(x)$ and $g(y)$. Reliability of helicopter component can be defined as follows:

$$R = P(X_g > X_f) = 1 - P_f \qquad (1)$$

where P_f is the probability of failure. Having a relationship that defines the density function, based on calculations in [1], the expression can be written:

$$R = \int_{-\infty}^{+\infty} f(x) \left\{ \int_{x}^{+\infty} g(y)dy \right\} dx \qquad (2)$$

This model allows to determine the likelihood of damage to the helicopter component when the single force occurs.

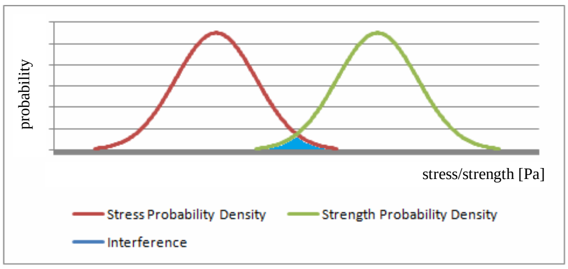

Figure 1. Superimposing the Stress Graph on the Strength Graph shows the Stress/Strength Interference.

A function describing this model is a random variable:

$$g(X_f, X_g) = X_g - X_f \tag{3}$$

According to [1] damage occurs if g ≤ 0. If X_f and X_g is the random variables having normal distributions $N(X_f^0, \sigma_{x_f})$, $N(X_g^0, \sigma_{x_g})$, random variable $g(X_f, X_g)$ has also a normal distribution [6] with mean $g^0 = X_g^0 - X_f^0$ and variance $\sigma_g = (\sigma_{x_g}^2 + \sigma_{x_f}^2)^{\frac{1}{2}}$. Probability of failure is:

$$P_f = \Phi\left(-\frac{g^0}{\sigma_g}\right) = \Phi(-\beta) \tag{4}$$

where Φ is the normal cumulative distribution function (CDF) and $\beta = \frac{g^0}{\sigma_g}$ is risk index.

2.2. Cornell's risk index

Often, when determining the reliability of critical points of a helicopter's structure [6] a problem arises that the distribution of the random vector ***X***[n] is unknown. Vector of mean values of random vector ***X***[n] is known:

$$\mathbf{X}^0 = \begin{bmatrix} X_1^0 \\ X_2^0 \\ \vdots \\ X_n^0 \end{bmatrix} \tag{5}$$

and the covariance matrix is also known:

$$C_x = \begin{bmatrix} \sigma_1^2 & \rho_{12}\sigma_1\sigma_2 & \cdots & \rho_{1n}\sigma_1\sigma_n \\ \rho_{21}\sigma_2\sigma_1 & \sigma_2^2 & \cdots & \rho_{2n}\sigma_2\sigma_n \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{n1}\sigma_n\sigma_1 & \rho_{n2}\sigma_n\sigma_2 & \cdots & \sigma_n^2 \end{bmatrix} \tag{6}$$

where X_i^0 is the mean value, σ_i is the variance of random variable X_i and ρ_{ik} is correlation coefficient of random variables X_i and X_k. Function g(**X**) can be evaluated to Taylor series:

$$g(\mathbf{X}) \approx \bar{g}(\mathbf{X}) = g(\mathbf{X}^0) + \sum_{i=1}^{n} \left.\frac{\partial g(\mathbf{X})}{\partial x_i}\right|_{x=X^0} \left(X_i - X_i^0\right)$$

Cornell's risk index is defined as:

$$\beta^{MVFOSM} = \frac{\bar{g}^0(\mathbf{X})}{\sigma_{\bar{g}}(\mathbf{X})} \tag{7}$$

where $\bar{g}^0(\mathbf{X}) = g(\mathbf{X}^0)$ and

$$\sigma_{\bar{g}}(\mathbf{X}) = \left(\nabla g^T(\mathbf{X})\big|_{\mathbf{X}=\mathbf{X}^0} C_x \nabla g(\mathbf{X})\big|_{\mathbf{x}=X^0}\right)^{\frac{1}{2}} \tag{8}$$

2.3. Hasofer - Lind's risk index

The ideology [6] is the expansion of the function g in a Taylor series around a point lying on the boundary surface. As a point of linearization the chosen point is lying closest to the origin in standard Gaussian space $\mathcal{U}$. The transformation of space $\mathcal{X}$ to the $\mathcal{U}$ has the form:

$$\boldsymbol{U} = \boldsymbol{L}^{-1} \boldsymbol{D}^{-1} (\boldsymbol{X} - \boldsymbol{X}^0) \tag{9}$$

where $\boldsymbol{D} = [\sigma_{x_i}]$ is a diagonal matrix, and $\boldsymbol{L}$ is a lower triangular matrix obtained from the Cholesky decomposition of a matrix of correlation coefficients $\boldsymbol{\rho} = \rho_{ij}$ and $\boldsymbol{\rho} = \boldsymbol{LL}^T$. Design point $\boldsymbol{u}^*$ corresponds to the largest value of the probability density function

$$\|\boldsymbol{u}^*\| = \min_{G(\boldsymbol{u})=0} \|\boldsymbol{u}\| = \delta^* \tag{10}$$

where $G(U)=g(X^0+DLU)$ is the boundary function transformed to the $\mathcal{U}$ Gauss space, ||·|| is an Euclidean norm. Hasofer-Lind's risk index is defined as:

$$\beta^{FOSM} = sign[G(\mathbf{0})]\delta^* \tag{11}$$

3. Markov Chain model

According to [3] a First-order Markov Chain (FCM) with finite space *E* is a sequence of *E*-valued random variable $(X_n)_{n\in\mathrm{N}}$ such that the conditional distribution of X_{n+1} knowing the discrete-time process $(X_m)_{m\le n}$ is the same as the conditional distribution of X_{n+1} given only X_n:

$$\begin{aligned} &P(X_{n+1} = e_{n+1} \mid (X_n = e_n, X_{n-1} = e_{n-1}, \dots, X_1 = e_1)) = \\ &= P(X_{n+1} = e_{n+1} \mid X_n = e_n) \end{aligned}$$

If K is the number of load classes, the transition probabilities define a K x K - real matrix P such that:

$$P = \begin{pmatrix} p_{1,1} & p_{1,2} & \cdots & p_{1,K} \\ p_{2,1} & p_{2,2} & \cdots & p_{2,K} \\ \cdots & \cdots & \cdots & \cdots \\ p_{K,1} & p_{K,2} & \cdots & p_{K,K} \end{pmatrix} \qquad (12)$$

$$\sum_{j=1}^{K} p_{i,j} = 1, \quad 0 \le p_{i,j} \le 1 \qquad (13)$$

where $i = 1,...K$, $p_{i,j} = P(X_{n+1} = e_j \mid X_n = e_i)$.
In order to generate the next value of the Markov Chain X_{n+1}, it is sufficient to know only the previous value $X_n = e_n$. The probability that the $X_{n+1} = e_{n+1}$ is $p_{n,n+1}$.
Hidden Markov chains Models (HMM) [4,5] are an extension of the concept of Markov chains for which the observation of *X* is not directly the state pertaining to *E* but a probabilistic function of this state. Each state has a probability distribution over the possible outputs. It is a bivariate discrete-time process $\{S_n, X_n\}_{n>0}$. Initially, in order to generate X_{n+1} based on knowledge of the previous value X_n function S_k must be chosen. Based on random function S_k the value e_{n+1} is generated.

4. Calculation and results

4.1. Calculation of the probability curve of strength

Samples of steel with the symbol 30HGSNA in the form of a rod with a diameter of 5-8 mm were tested. Yield stress was obtained as follows [7]:

Table 1. The results of yield stress:

No. samples	2/05 /2	2/05 /3	2/05 /4	2/05 /5	2/05 /36
$R_{0.05}$[MPa]	1160	1145	1150	1150	1200

No. samples	2/05 /61	2/05 /62	2/05 /63	2/05 /64	2/05 /132
$R_{0.05}$[MPa]	1230	1230	1240	1175	1175

No. samples	2/05 /133	2/05 /134	2/05 /135	2/05 /136	2/05 /137
$R_{0.05}$[MPa]	1235	1280	1290	1330	1220

No. samples	2/05 /138	2/05 /139	2/05 /140	2/05 /141	2/05 /142
$R_{0.05}$[MPa]	1225	1235	1240	1225	1250

Based on the survey it can be stated that the yield stress of steel 30HGSNA, obtained on samples heat treated under ITWL is: $R_{0.05}$ = 1220 MPa ± 1,82%. Based on these results one can propose a normal probability distribution of strength with the mean equal μ = 1219 and variance $\sigma^2 = 110^2$.

$$g(y) = \frac{1}{\sqrt{2\pi 20^2}} \exp\left(\frac{-(x-1219)^2}{2 \cdot 110^2}\right) \qquad (14)$$

4.2. Calculation of the probability curve of stress

Critical element of the helicopter Mi-24 No. 8 was scanned using a 3D scanner ATOS III (Advanced Topometric System) [8]. Based on photogrammetric measurements the shapes of force elements were reproduced in a CAD/CAM environment. Models of geometric elements of strength were performed in Unigraphics.
A finite-element method (FEM) model applied a unit axial force and the calculations were performed by using the MSC Marc programme. The maximum value of the maximum principal stress under the influence of force per unit amounted to 2880 Pa.

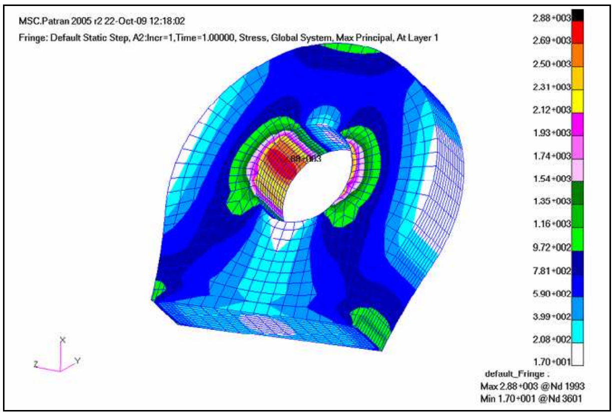

Figure 2. Maximum principal stress distribution in a single node of the holes for mounting the elements.

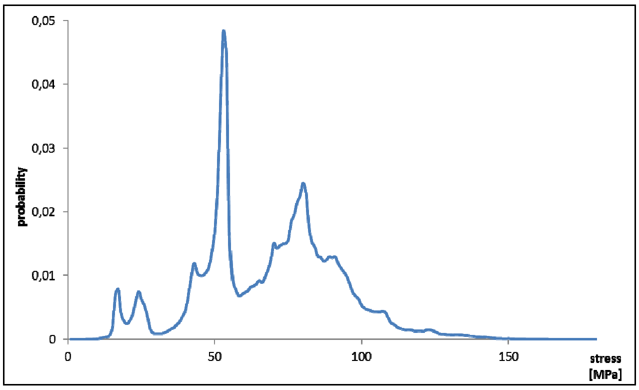

Figure 3. Histogram of stress value

Based on measurements of the strength of the flight test a stress histogram was generated.
Based on these results one can propose a normal probability distribution of stress with the mean equal $\mu = 70$ and variance $\sigma^2 = 25^2$.

$$f_1(x) = \frac{1}{\sqrt{2\pi 25^2}} \exp\left(\frac{-(x-70)^2}{2 \cdot 25^2}\right) \tag{15}$$

Based on the shape of the histogram of stress levels, the data set was split into 8 c_k-classes. For the FCM model, all stress of load sequences pertaining to a given class c_i were replaced by a unique stress value s_i.

Table 2. Probability distribution function after the application of the FCM model

x_i	15	30	45	60
p_i	0,0034	0,0629	0,0698	0,2699

x_i	75	95	140	180
p_i	0,1733	0,3094	0,1084	0,0025

For the HMC model the finite space *E* was divided into 5 parts. After the calculation of stress cycles the resulting probability distribution can be estimated by following function f_3:

$$f_3(x) = \begin{cases} 0 & x < 10 \\ \frac{0,03}{\sqrt{2\pi 1,5^2}} \exp\left(\frac{-(x-17)^2}{2 \cdot 1,5^2}\right) & 10 \le x < 20 \\ \frac{0,03}{\sqrt{2\pi 1,5^2}} \exp\left(\frac{-(x-24)^2}{2 \cdot 1,5^2}\right) & 20 \le x < 30 \\ \frac{0,38}{\sqrt{2\pi 3,6^2}} \exp\left(\frac{-(x-53)^2}{2 \cdot 3,6^2}\right) & 30 \le x < 60 \\ \frac{0,6}{\sqrt{2\pi 12^2}} \exp\left(\frac{-(x-80)^2}{2 \cdot 12^2}\right) & x \ge 60 \end{cases}$$

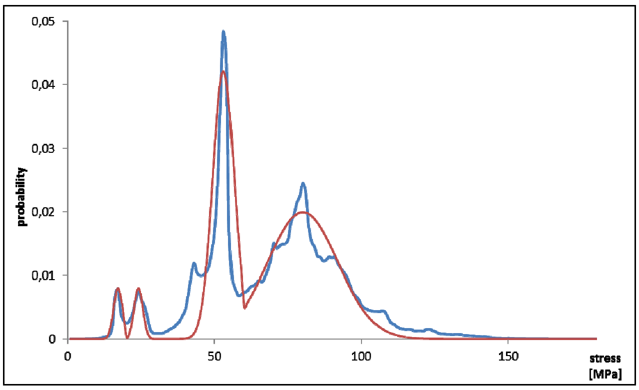

Figure 4. Histogram of stress value and its estimate

4.3. Results

Risk index β can only be calculated from a normal probability distribution of X_i. Based on a normal probability distribution of strength with the mean equal $\mu = 1219$ and variance $\sigma^2 = 110^2$, a normal probability distribution of stress with the mean equal $\mu = 70$ and variance $\sigma^2 = 25^2$, one can propose risk index $\beta = 10{,}2$. This value corresponds to probability of failure $P_f < 1 \cdot 10^{-7}$.
Cornell's risk index β^{MVFOSM} depends on how the data stress was generated. If the data from real flights is used (f_1), Cornell's risk index is $\beta^{MVFOSM} = 2{,}07$. This value corresponds to probability of failure $P_f = 1{,}92 \cdot 10^{-2}$. When the stress data is obtained using Markov chain models, I received the following index and probability of failure: $\beta^{MVFOSM} = 1{,}33$, $P_f = 9{,}18 \cdot 10^{-2}$ and $\beta^{MVFOSM} = 2{,}94$, $P_f = 1{,}64 \cdot 10^{-3}$ for First-order (f_2) and Hidden Markov Chains (f_3).
Hasofer-Lind's risk index β^{FOSM} also depends on how the data stress was generated. If the data real from flights is used (f_1), Hasofer-Lind's risk index is $\beta^{FOSM} = 0{,}0037$. When the stress data is obtained using Markov chain models, I received the following index and probability of failure: $\beta^{FOSM} = 0{,}0016$ and $\beta^{FOSM} = 0{,}0076$ for First-order (f_2) and Hidden Markov Chains (f_3). All those values correspond to probability of failure $P_f = 0{,}48 \div 0{,}5$.

5. Conclusions

Risk index and Cornell's risk index models allow to determine the likelihood of damage to the helicopter component when a single force occurs. Based on these assumptions, it will not apply in aviation where a stress on a specific element varies with time.
In these models the aging process of the element is not included. Taking into account of this phenomenon would decrease strength, and thus decrease the number of cycles till failure.
Models are similar regardless of how the stress was generated from the basis of experimental data. They differ slightly from those obtained with standard models.
Results obtained from Hasofer-Lind's risk index are not satisfactory. The major drawback of this model is not taking into account the boundary curve shape.

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