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Free Vibrations of Column Built Out Pipe and Rod with Two-Parametric Elastic Connector

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Abstract

In this paper the geometrically nonlinear system subjected to compressive external Euler's load has been investigated. The column is composed of pipe and rod. The rod is concentrically installed in the pipe. Between pipe and rod at given distance from the end of the column the two-parametric elastic connector has been placed. The numerical calculations were performed for different parameters of the system on the basis of free vibration boundary problem. The parameters are as follows: spring stiffness (translational and rotational) which models elastic connector, coefficient of asymmetry flexural rigidity, location of the connector.

Keywords: column, free vibrations, elastic connector

1. Introduction

In the investigations on slender supporting systems the discrete elements (rotational and translational springs, dumpers) are being considered. These elements have an influence on critical or bifurcation load magnitude and natural vibration frequency of the systems. When the non-conservative load is taken into account the discrete elements have an effect on type of instability (see [3-10]). By means of these elements an influence of real life elements on instability and free vibrations can be modeled. In the literature the papers devoted to instability and free vibrations with consideration of elastic and viscoelastic supports can be found (see [1,2]).

In [12] the investigations of one-parametric elastic connector on vibration and instability of a system built out pipe and rod have been presented. Elastic connector has been placed between pipe and rod. It has been shown that the translational stiffness of elastic connector at specific magnitude of coefficient of asymmetry flexural rigidity causes the increase of bifurcation load. This element of elastic connector has an influence on vibration frequency and change of buckling mode. In the case when the system is characterized by the local instability the presence of the elastic connector becomes more significant. The viscoelastic connector has been taken into account in [11]. In this paper an influence of the connector on first vibration frequency in the range of external load from zero up to bifurcation force has been investigated. In the mathematical model the Kelvin - Voigt model of viscoelastic connector was considered. An increase of connector dumping factor causes an increase of the first vibration frequency magnitude.

The main scope to this paper is to study an influence of two-parametric elastic connector on natural vibrations of the system built out pipe and rod. Particularly the parameter of rotational elasticity has been investigated.

2. Boundary problem formulation on the basis of Hamilton's principle

The column composed of pipe and rod is presented in the Figure 1. The system is subjected to Euler's compressive load. Between pipe and rod the two-parametric elastic connector has been modeled. This connector consists of two springs: translational (C_T stiffness) and rotational (C_R stiffness). The length l_{11} describes connector location. The column is hinged on both ends. The model of the system is created by means of four elements. Elements marked as 11 and 22 corresponds to pipe while 21 and 22 stands for rod.

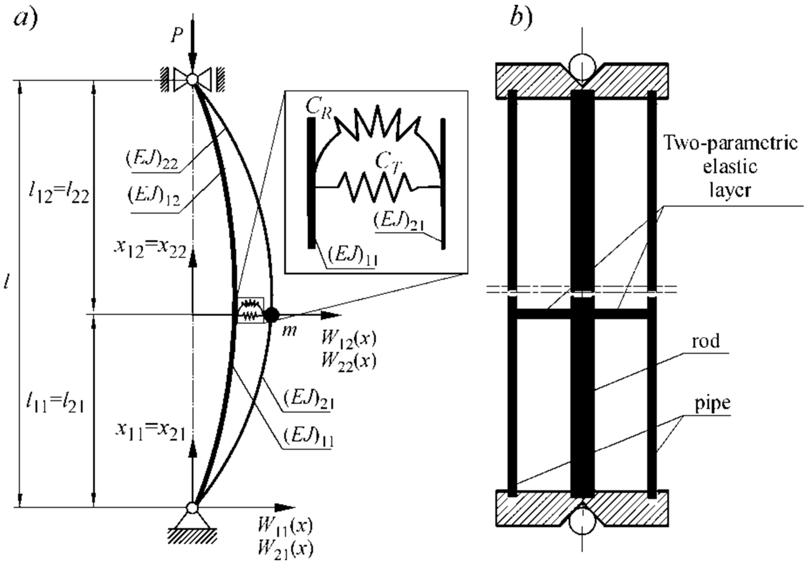

Figure 1. Considered column loaded by Euler's load: a) mathematical model, b) column consists of pipe and rod

The differential equations of motion in the transversal direction of the investigated system are as follows: (comp. [11]):

$$(EJ)_{ij}\frac{\partial^4 W_{ij}(x,t)}{\partial x^4}+S_{ij}(t)\frac{\partial^2 W_{ij}(x,t)}{\partial x^2}+(\rho A)_{ij}\frac{\partial^2 W_{ij}(x,t)}{\partial t^2}=0 \tag{1}$$

Moreover the differential equations of motion in the transversal direction (1) there exists the differential equation of longitudinal displacements. Performing mathematical operations on it allows one to obtain (see [12]):

$$U_{ij}(x_{ij},t)-U_{ij}(0,t)=-\frac{S_{ij}(t)}{(EA)_{ij}}x_{ij}-\frac{1}{2}\int_0^{x_{ij}}\left(\frac{\partial W_{ij}(x_{ij},t)}{\partial x_{ij}}\right)^2 dx_{ij} \tag{2}$$

In equations (1) and (2) the following designations were made: $W_{ij}(x_{ij},t)$, $U_{ij}(x_{ij},t)$ – transversal and longitudinal displacements respectively, $(EJ)_{ij}$, $(EA)_{ij}$, $(\rho A)_{ij}$ – bending, compression stiffness, mass attributable to length unit of each member, S_{ij} – longitudinal force in element of the system. The investigated column is characterized by:

- geometrical boundary conditions:

$$W_{11}(0,t)=W_{11}(0,t)=W_{12}(l_{i2},t)=W_{12}(l_{i2},t)=U_{11}(0,t)=U_{21}(0,t)=0 \qquad \text{(3a-f)}$$

$$\left.\frac{\partial W_{11}(x_{11},t)}{\partial x_{11}}\right|_{x_{11}=0}=\left.\frac{\partial W_{21}(x_{21},t)}{\partial x_{21}}\right|_{x_{21}=0} \qquad \text{(3g)}$$

$$\left.\frac{\partial W_{12}(x_{12},t)}{\partial x_{12}}\right|^{x_{12}=l_{12}}=\left.\frac{\partial W_{22}(x_{22},t)}{\partial x_{22}}\right|^{x_{22}=l_{22}} \qquad \text{(3h)}$$

$$\left.\frac{\partial W_{i1}(x_{i1},t)}{\partial x_{i1}}\right|^{x_{i1}=l_{i1}}=\left.\frac{\partial W_{i2}(x_{i2},t)}{\partial x_{i2}}\right|_{x_{i2}=0},\ W_{i1}(l_{i1},t)=W_{i2}(0,t) \qquad \text{(3i,j)}$$

$$U_{12}(l_{12},t)=U_{22}(l_{22},t),\ U_{i1}(l_{i1},t)=U_{i2}(0,t) \qquad \text{(3k,l)}$$

- natural boundary conditions:

$$\sum_i (EJ)_{i1}\left.\frac{\partial^2 W_{i1}(x_{i1},t)}{\partial x_{i1}^2}\right|_{x_{i1}=0}=0,\quad \sum_i (EJ)_{i2}\left.\frac{\partial^2 W_{i2}(x_{i2},t)}{\partial x_{i2}^2}\right|^{x_{i2}=l_{i2}}=0 \qquad \text{(4a,b)}$$

$$(EJ)_{11}\left.\frac{\partial^3 W_{11}(x_{11},t)}{\partial x_{11}^3}\right|^{x_{11}=l_{11}}-(EJ)_{12}W_{12}^{III}(x_{12},t)\left.\frac{\partial^3 W_{12}(x_{12},t)}{\partial x_{12}^3}\right|_{x_{12}=0}+$$
$$-C_T\left(W_{11}(l_{11},t)-W_{21}(l_{21},t)\right)=0 \qquad \text{(4c)}$$

$$(EJ)_{21}\left.\frac{\partial^3 W_{21}(x_{21},t)}{\partial x_{21}^3}\right|^{x_{21}=l_{21}}-(EJ)_{22}\left.\frac{\partial^3 W_{22}(x_{22},t)}{\partial x_{22}^3}\right|_{x_{22}=0}+$$
$$+C_T\left(W_{11}(l_{11},t)-W_{21}(l_{21},t)\right)-m\left.\frac{\partial^2 W_{21}(x_{21},t)}{\partial t^2}\right|^{x_{21}=l_{21}}=0 \qquad \text{(4d)}$$

$$-(EJ)_{11}\left.\frac{\partial^2 W_{11}(x_{11},t)}{\partial x_{11}^2}\right|^{x_{11}=l_{11}}+(EJ)_{12}\left.\frac{\partial^2 W_{12}(x_{12},t)}{\partial x_{12}^2}\right|_{x_{12}=0}+$$
$$-C_R\left(\left.\frac{\partial W_{11}(x_{11},t)}{\partial x_{11}}\right|^{x_{11}=l_{11}}-\left.\frac{\partial W_{21}(x_{21},t)}{\partial x_{21}}\right|^{x_{21}=l_{21}}\right)=0 \qquad \text{(4e)}$$

$$-\left(EJ\right)_{21}\frac{\partial^2 W_{21}\left(x_{21},t\right)}{\partial x_{21}^2}\Bigg|^{x_{21}=l_{21}}+\left(EJ\right)_{22}\frac{\partial^2 W_{22}\left(x_{22},t\right)}{\partial x_{22}^2}\Bigg|_{x_{22}=0}+$$
$$+C_R\left(\frac{\partial W_{11}\left(x_{11},t\right)}{\partial x_{11}}\Bigg|^{x_{11}=l_{11}}-\frac{\partial W_{21}\left(x_{21},t\right)}{\partial x_{21}}\Bigg|^{x_{21}=l_{21}}\right)=0 \tag{4f}$$

$$S_{11}\left(t\right)=S_{12}\left(t\right),\ S_{21}\left(t\right)=S_{22}\left(t\right),\ S_{11}\left(t\right)-S_{22}\left(t\right)-P=0 \tag{4g-i}$$

The equation of longitudinal displacements is a non-linear one. Due to geometrical non-linearities the small parameter method has been used to solve the boundary problem (see [12]). The non-linear equations are being written in a power series of small parameter. Rectilinear and curvilinear forms of static equilibrium are present in the investigated system. The power series for each form are different. In this paper the rectilinear form of static equilibrium has been considered. The power series are as follows:

$$W_{ij}\left(x,t\right)=\sum_{k=1}^{N}\varepsilon^{2k-1}W_{ij(2k-1)}\left(x,t\right)+O\left(\varepsilon^{2(N+1)}\right) \tag{5}$$

$$U_{ij}\left(x,t\right)=U_{ij(0)}\left(x\right)+\sum_{k=1}^{N}\varepsilon^{2k}U_{ij(2k)}\left(x,t\right)+O\left(\varepsilon^{2(N+1)}\right) \tag{6}$$

$$S_{ij}\left(t\right)=S_{ij(0)}+\sum_{k=1}^{N}\varepsilon^{2k}S_{ij(2k)}\left(t\right)+O\left(\varepsilon^{2(N+1)}\right) \tag{7}$$

$$\omega^2=\omega_{(0)}^2+\sum_{k=1}^{N}\varepsilon^{2k}\omega_{(2k)}^2+O\left(\varepsilon^{2(N+1)}\right) \tag{8}$$

where: ω – natural vibration frequency.

The power series (5-8) are being introduced into equations (1-2) and boundary conditions. The coefficients at the same power of small parameter are being collected what leads to sequences of equations with corresponding boundary conditions. In this paper the basic vibration frequency $\omega_{(0)}$ has been presented (obtained on the basis of equations at zero and first power of the small parameter). The first components of expansions ($W_{ij(1)}(x_{ij},t)$, $U_{ij(0)}(x_{ij})$, $S_{ij(0)}$) are only considered in computations of basic vibration frequency. Separating space and time variables in the form:

$$W_{ij(1)}\left(x_{ij},t\right)=Y_{ij(1)}\left(x_{ij}\right)\cos\left(\omega t\right) \tag{9}$$

allows one to write the differential equation of transversal displacements:

$$\left(EJ\right)_{ij}\frac{d^4Y_{ij(1)}\left(x_{ij}\right)}{dx_{ij}^4}+S_{ij(0)}\frac{d^2Y_{ij(1)}\left(x_{ij}\right)}{dx_{ij}^2}-\left(\rho A\right)_{ij}\omega_{(0)}^2Y_{ij(1)}\left(x_{ij}\right)=0 \tag{10}$$

The distribution of internal forces $S_{11(0)}$ i $S_{21(0)}$ can be calculated from equation (2). The relation between forces is as follows:

$$S_{12(0)} = S_{22(0)} \frac{(EA)_{12}}{(EA)_{22}} \tag{11}$$

The solution of equations (10) can be presented as a function:

$$\begin{aligned} Y_{ij(1)} &= B1_{ij(1)} \cosh\left(\alpha_{ij(1)} x_{ij}\right) + B2_{ij(1)} \sinh\left(\alpha_{ij(1)} x_{ij}\right) + \\ &+ B3_{ij(1)} \cos\left(\beta_{ij(1)} x_{ij}\right) + B4_{ij(1)} \sin\left(\beta_{ij(1)} x_{ij}\right) \end{aligned} \tag{12}$$

where: $B1_{ij(1)}$, $B2_{ij(1)\,)}$, $B3_{ij(1)\,)}$, $B4_{ij(1)}$ are constants of integration and $\alpha_{ij(1)}$, $\alpha_{ij(1)}$ are quantities obtained from the characteristic differential equations (10). Introducing solution (12) into boundary condition allows on to write system of equations. The determinant of the matrix of coefficients equated to zero leads to transcendental equation on the basis of which the vibration frequency $\omega_{(0)}$ can be calculated.

3. Results of numerical calculations

In Figures 2 and 3 the characteristic curves on the plane external load - natural vibration frequency have been plotted. Numerical calculations were performed at different magnitude of the rotational spring. Graphs presented in Figures 2 and 3 have been created for different magnitudes of coefficient of asymmetry flexural rigidity $\mu = 0.004$ and $\mu = 0.5$. Parameters at which the numerical calculations were performed are as follows:

$$\lambda = \frac{Pl^2}{(EJ)},\ \Omega = \frac{\omega_{(0)}^2 \left(\sum_i (\rho A)_{i1} \right) l^2}{(EJ)},\ \kappa = \frac{E_{21}}{E_{11}},\ \mu = \frac{(EJ)_{21}}{(EJ)_{11}}, \tag{13a-d}$$

$$c_R = \frac{C_R l}{(EJ)},\ c_T = \frac{C_T l^3}{(EJ)},\ \zeta = \frac{l_{11}}{l} \tag{13e-g}$$

The total flexural stiffness of the system (EJ) is constant. It has been shown that with lower magnitude of coefficient of asymmetry flexural rigidity the mode of free vibration is changing. The mode of free vibration depends on rotational spring stiffness c_R. At smaller stiffness of c_R the investigated system is characterized by buckling mode M3. At greater rotational spring stiffness the mode M1 is present. The characteristic curve corresponding to mode M3 may cross the ones related to modes M2 and M1. At greater magnitude of coefficient of asymmetry flexural rigidity the mode of free vibrations does not change. The modes of free vibrations were plotted in the Figure 4. An influence of the rotational spring stiffness on investigated parameters is greater at lower magnitude of coefficient μ. The characteristic curves related to modes M1 and M2 do not depend on rotational spring stiffness.

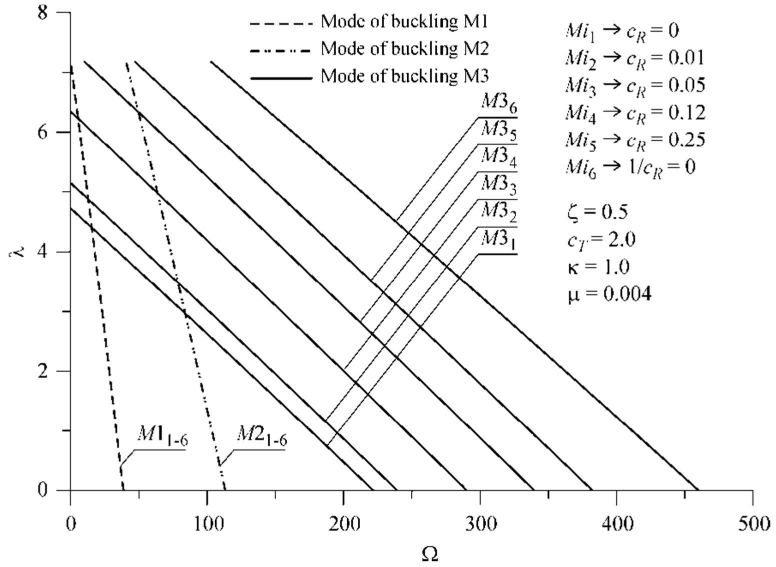

Figure 2. Parameter of loading force λ in relation to parameter of free vibration frequency Ω at $\mu = 0.004$

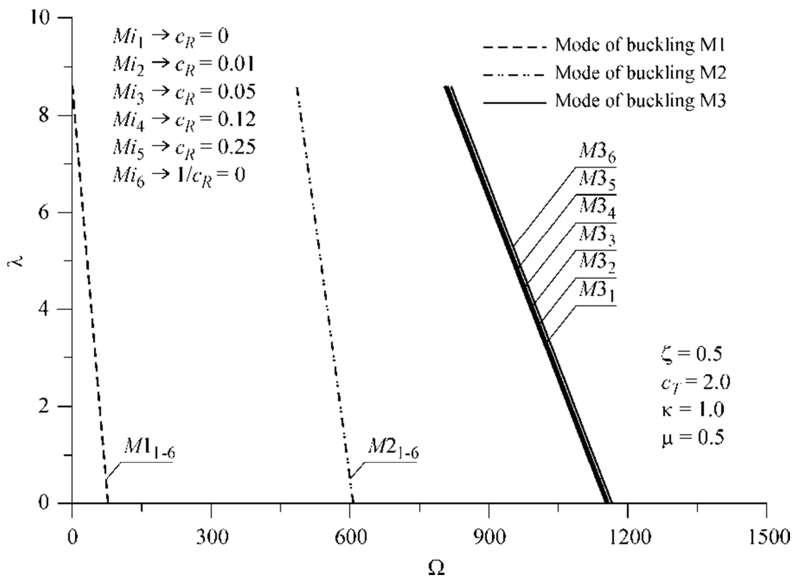

Figure 3. Parameter of loading force λ in relation to parameter of free vibration frequency Ω at $\mu = 0.5$

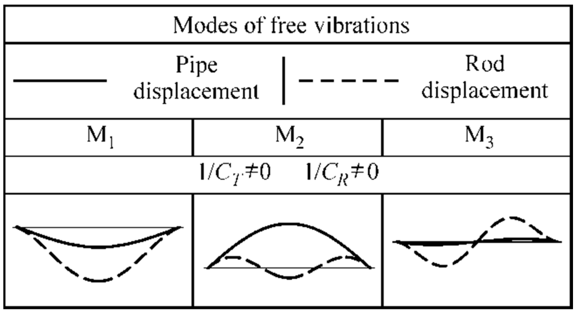

Figure 4. Modes of free vibrations

4. Conclutions

In this paper the result of theoretical study and numerical calculations of slender system built out pipe and rod have been presented. Between pipe and rod the two-parametric elastic connector is placed. The main purpose of numerical studies was to describe an influence of rotational stiffness of elastic connector on vibration frequency. It can be concluded that at smaller magnitude of coefficient of asymmetry flexural rigidity an influence of rotational stiffness of elastic connector on vibration frequency becomes intensified than at greater magnitudes of μ coefficient. There exist modes of vibrations irrespective of considered stiffness.

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