

## MULTI-CRITERIA OPTIMIZATION IN FUEL DISTRIBUTION

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**Abstract** This paper deals with some computational study of multi-criteria optimization in fuel distribution problem. We consider vehicle routing problem, e.g. routing of a fleet of tank trucks, for one of the famous polish petroleum companies. For the purpose of solving this problem, we developed the random search algorithm, which explores the broad range of feasible sets of routes and searches for non-dominated multi-criteria solutions. The problem is examined on real data, which contains distances between 50 petrol stations and one central warehouse (refinery). Obtained results indicate, that it is possible to obtain single solution and satisfy both optimization criteria. Based on the analysis of the collected data, we formulate a number of proposals useful in future for construction of algorithms for multi-criteria fuel distribution optimization.

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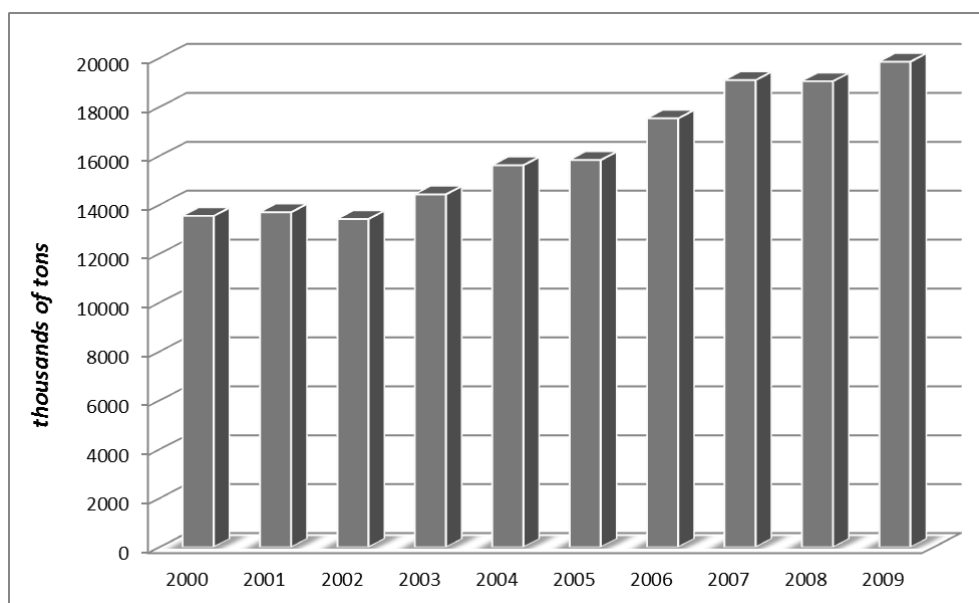
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## 1. INTRODUCTION

Petroleum has played an important role in the social, economic, and political history of any developing country. Global economic slowdown in 2009, influenced the development of the Polish economy, but it did not stop. In the years 2000-2009 the demand for the main four types of liquid fuels increased by 46 per cent., reaching in 2009 a level of almost 20 million tons. However, the consumption of fuels in Poland passed in recent years a significant decline. The decrease was observed in both domestic production and imports of fuels.



**Fig. 1** History of the consumption of liquid fuels in Poland – source POPiHN

High oil prices mean that the Polish petrochemical industry is forced to look for new ways to reduce the prices of liquid fuels. Financial expenditures are pinned to research on, among others, effective methods of producing fuels and reduction of production costs. Apart from advancement in production technologies, there are other ways to decrease costs and increase profit. One of such methods was introduced in 1959 by Dantzig and Ramser, who proposed a model and an algorithm for solving the problem of routing of a fleet of gasoline delivery trucks between a bulk terminal and a large number of service stations supplied by the terminal. Since that time, we have made significant progress in discrete optimization. Both the models and the methods of solving those problems have evolved over time. Currently, trends in vehicle routing include both additional constraints and optimizing multiple criteria simultaneously.

Optimization in transport and logistics translates into an improvement in a number of other factors of production, services and the economy, such as reducing the time of delivery of products, increase of the production efficiency or reduction of costs. Therefore, allows to increase competitiveness of the company.

## 2. LITERATURE OVERVIEW

Researchers approached the multi-objective transportation problem. A few of those approaches will be mentioned here.

One of the promising scalar techniques was proposed by in (Bowerman, Hall & Calamai, 1995). It uses five different sets of weights chosen by a decision-maker. Their heuristic first groups the nodes into clusters that can be served by a vehicle, and then determines a tour for each cluster and the specific stops. It is an allocation-routing-location strategy.

An insertion algorithm was used in (Lee & Ueng, 1998). In each iteration it adds one node to the vehicle with the shortest work time using a saving criterion. Another insertion heuristic was proposed in (Zografos & Androustopoulos, 2004), although its origin was in a method proposed by Solomon. It differs in the selection of the customers to be inserted, allowing both routed and unrouted demand points to be inserted.

Another scalar approach uses the  $\varepsilon$ -constraint method. In this strategy, only one objective is optimized and the others are considered as constraints, expressed as  $f_i \leq \varepsilon_i$ . In (Pacheco & Marti, 2006), authors optimize the makespan objective for every possible value of the second objective and then use a Tabu search algorithm to solve each problem. Similar strategy was used in paper (Corberan, Fernandez & Laguna, 2002), but instead of Tabu search, they used scatter search approach.

In multi-objective vehicle routing problems, the Pareto concept is frequently used within an evolutionary framework. One of the works used a memetic algorithm proposed in (Żelazny, 2012). This genetic algorithm in each iteration uses local search method on non-dominated solutions in order to further improve the Pareto frontier approximation. In later work (Żelazny & Jagiełło, 2013), a GPU implementation of Tabu Search algorithm was proposed. Significant speedup and improvement of the approximation of the Pareto front were observed.

Pareto dominance has also been used in a simulated annealing technique called Multi-Objective Simulated Annealing – MOSA (Ulungu, Teghem, Fortemps & Tuytens, 1999). In (Paquete & Chiarandini, 2004) authors have called upon Pareto Local Search techniques. These techniques are based on the principle that the next current solution is chosen from the non-dominated solutions of the neighborhood.

In paper (Jozefowicz, Semet & Talbi, 2008), an interesting survey of multi-criteria vehicle routing problems was presented. Different modifications and variations were described, as well as some methods and criteria of optimization.

### 3. PROBLEM DESCRIPTION

The petroleum company has a homogenous fleet consisting of  $f$  tank trucks (vehicles) forming the set  $F = \{1, \dots, f\}$ . Each tank truck is able to carry multiple products or products for multiple clients at once due to compartmentation of the tank into  $x = 5$  tank containers. The basic unit of the order is one container. Of course, the customer can order more than one fuel tank, and less than one full tank. In the second case, for safety reasons, the tank is not supplemented with fuel destined for another client. The client orders generate the set of transportation tasks  $J = \{1, \dots, n\}$ . The transportation task consists of supplying specific fuel from the central warehouse to the place specified by the customer. Travel times (equivalent to distance)  $d_{i,j} > 0$ , for  $i, j = 0, 1, \dots, n$ , between all loading and unloading nodes are given and by 0 we denote the central warehouse. We assume that: (i) we transport one type of fuel in single vehicle, (ii) the fleet is homogenous, hence the vehicles are identical, (iii) the unloading time is constant and equals  $t_u = 30$  min, and (iv) the average vehicle speed is  $s = 60$  km/h.

First, each of the transport tasks from the set  $J = \{1, \dots, n\}$  must be assigned to the vehicle. Next, for each vehicle we must determine its route. The allocation of tasks to vehicles and vehicle routes can be clearly described by a set of permutations of  $r = (r_1, \dots, r_f)$ , where  $r_i = (r_i(1), \dots, r_i(n_i))$  is a permutation describing the route of  $i$ -th vehicle, while  $n_i$  is the number of clients served by this vehicle. Thus, the feasible solution (a set of routes) must fulfill the following constraints:

$$\bigcup_{k=1}^f T_k = J, \quad (1)$$

$$T_l \cap T_k = \emptyset, \quad l \neq k, \quad l, k \in F, \quad (2)$$

where  $T_i = \{r_i(1), \dots, r_i(n_i)\}$  denotes the set of tasks assigned to the truck  $i \in F$ . The constraint (1) denotes that all jobs must be assigned to the trucks, whereas the constraint (2) denotes that each task can be assigned only to one truck.

The driver's work-time is equal to the sum of unloading times and travel times. For any given route  $r_i = (r_i(1), \dots, r_i(n_i))$  of truck  $i \in F$ , the work-time of a driver  $i$  is calculated as follows:

$$w(t_i) = n_i \cdot t_u + t_{0,r_i(1)} + \sum_{s=1}^{n_i-1} t_{r_i(s),r_i(s+1)} + t_{r_i(n_i),0}, \quad (3)$$

where  $t_{x,y} = (d_{x,y} / s)$  is travel time from place  $x$  to place  $y$ .

We consider two optimization criteria:

1.  $f_1(t) = \sum_{i \in F} w(t_i)$  – sum of work-times,
2.  $f_2(t) = \max_{i \in F} w(t_i)$  – maximal work-time.

The first criterion minimizes total work-time of all drivers and total distance driven by cars, whereas the second criterion minimizes the maximal work-time and balances the driver's work-time.

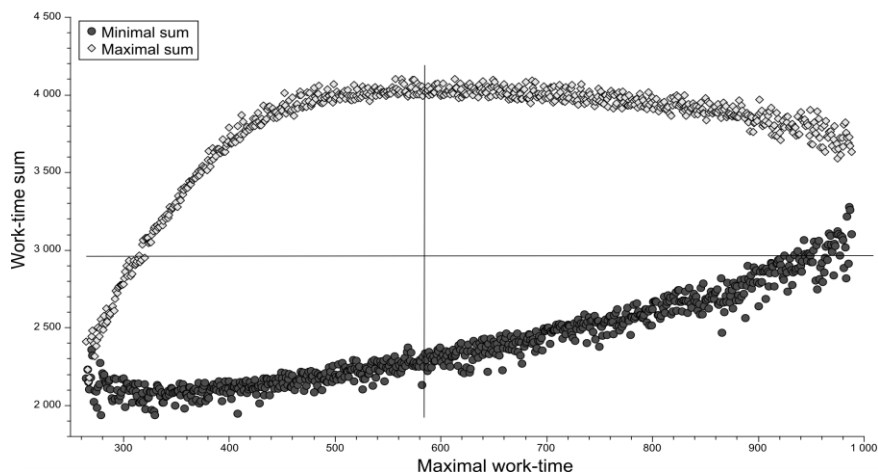
#### 4. CASE STUDY

The case study concerns one of the famous polish petroleum companies. The company supplies fuel to hundreds of petrol stations scattered throughout Poland. In our study, we focus on the one distribution point and 50 nearest petrol stations and  $n = 50$  orders i.e. each petrol station requests exactly one order. The distances between location were determined using the Google API services. The minimal number of trucks is equal to  $\lceil n/x \rceil = 10$ , i.e. each truck serves around 5 orders.

In order to explore a broad range of different solutions, we developed a random walk algorithm RW. In each iteration of the algorithm, based on current set of routes  $t$ , new set of routes  $t'$  is generated. The solution  $t'$  arise from  $t$  by exchanging two randomly selected orders.

The algorithm RW was run for 100 million of iterations. Each generated route was stored in set  $S$ . Based on routes from the set  $S$  we determine two functions with maximal work-time argument:

3.  $Max(x) = \max\{f_2(t) : f_1(t) = x, x \in S\}$  – maximal sum of work-times for all solutions with maximal work-time equal to  $x$ , i.e.  $f_1(t)=x$ ,
4.  $Min(x) = \min\{f_2(t) : f_1(t) = x, x \in S\}$  – minimal sum of work-times for all solutions with maximal work-time equal to  $x$ , i.e.  $f_1(t)=x$ .



**Fig. 2** Minimal and maximal work-time sums in regard to maximal work-time

Figure 2 shows course of the algorithm and values of both maximal and minimal sums of work-times. Detailed observation shows that the focus is solely on *maximal work-time* may lead to the spreading the values of the *work-time sum* function from around 2400 to even 4000, while concentrating on *work-time sum* leads to scattering values of *maximal work-time* from around 320 to even 980.

It should be noted that both criteria converge to the optimum almost simultaneously, despite the observed dispersion of values of a single criterion for certain constant value of the second criterion. Moreover, in this case study of the fuel supply problem, we managed to obtain a single Pareto-optimal solution. Although it can be assumed that for other similar problem cases this number may be higher than one, our experiences with multi-criteria optimization problems indicate that the number of Pareto-optimal solutions will be significantly lower than the number of all feasible solutions.

## 5. CONCLUSIONS

In practice, multi-criteria optimization has not found such wide application as single criterion methods. In this paper, we have showed the use of an algorithm based on Pareto-optimality for the practical problem of supplying fuel from refineries to gas stations. Convergence of criteria, observed in this work, can be used as the basis in designing of even more efficient methods. We conclude, that the focus should be put on optimizing the maximal work-time, while the sum of work-times should be considered as a secondary criterion. In future work, we will extend existing methods, which optimize maximal work-time, and use properties of the problem to propose efficient algorithms for its multi-criteria case.

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## BIOGRAPHICAL NOTES

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